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An algorithmic tool for supporting risk and root-cause analysis of critical incidents in Baltic Sea Region ports

Keywords

Critical incident, cause-effect relation, cascade of events, initiating event, secondary event, stochastic modeling, Poisson process, risk analysis, root-cause analysis

Abstract

This paper is both a summarization and extension of [6] and [7], where a stochastic model of interacting operations carried out in a generic Baltic Sea Region port was proposed and analyzed. Each operation involves a number of possible unwanted events (critical incidents) whose instances occur randomly and can cause instances of other events affecting this or other operations. This can lead to a cause-effect chain of events affecting one or multiple operations. The model presented in [6] is somewhat complex, therefore it was downgraded to a simpler, application-oriented version demonstrated in [7], where an algorithm computing the risks of critical incidents is constructed and then applied to a real-life example. The current paper, apart from presenting a method of computing the risks of critical incidents, occurring by themselves or resulting from the cascade effect, also features a method of root-cause analysis of such incidents. First, the formulas for the root-cause probabilities are derived, where such a probability quantifies the likelihood that a critical incident occurring in step h of a cascade was caused by another incident that initiated this cascade. Second, an algorithm computing the root-cause probabilities, based on the derived formulas, is constructed. This algorithm is illustrated by its application to the example given in [7]. The presented results can be used as a tool for fault propagation analysis and fault diagnosis applied not only to a port environment, but to any complex industrial system.

1. Introduction

The current paper summarizes in a condensed form the results obtained by the author (Jacek Malinowski) during the realization of the tasks undertaken as a participant of the HAZARD project. These results were presented in greater detail in the three following reports: "Modeling hazard-related interactions between the processes realized in and around the Baltic Sea Region ports", "A simple tool for evaluating risks related to hazardous interactions between the processes realized in the Baltic Sea Region's port areas" and "An algorithmic tool for supporting root-cause analysis of critical incidents in the Baltic Sea Region ports". The first two reports were published in JPSRA – see [6] and [7]. In the first report a probabilistic model of hazard-related interactions between different operations carried out in a (generic) Baltic Sea Region port is presented. Each such operation, considering its hazardous aspect, can be defined as a series of undesired events (critical incidents) occurring at random instants, i.e. the operation is modeled by a random point process. An event can occur with different strengths measured by a discrete scale, however, in the second report, where the model is simplified for the sake of greater applicability, only binary events are considered, i.e. they either occur or not, strengths being irrelevant. An event can be primary (occurring by itself) or secondary (caused by another event in the same or another process). The processes interact in the sense that an event in one process can cause a cascade of events (a chain of events linked by cause-effect relation) propagating through multiple processes. The direct cause-effect relations between the events are expressed by the first-grade cause-effect probabilities defined in Notation section. The formulas for cause-effect probabilities of higher grades, expressing non-direct relations, are given in Section 3.

The cause-effect probabilities are used for two main purposes. First, they serve to compute the risks of undesired events. Such a risk is defined as the probability that a given number of instances of one event occur in a given time interval. The respective formulas are given in Section 4. Second, they are used to obtain formulas for the root-cause probabilities which can be referred to as "reverse cause-effect probabilities". Such a probability expresses the likelihood that the cascade in which a given event occurred, was initiated by another given event. The respective formulas are to be found in Section 5. Root-cause probabilities are applied in the root-cause analysis of critical incidents.

The presented results can be classified as methods of fault propagation analysis or fault diagnosis (areas of reliability engineering). Many authors use the Bayes network approach to address these issues – surveys of recent relevant developments can be found in [2] and [4]. In the current paper the process-oriented approach to the considered issues is pursued – see [1], [3], [5], [8] and [9] for a broader scope of the topic.

This summary is focused on the practical results of the work on the project. They consist of two algorithms the first of which computes the risks of different critical incidents that can happen in the port area, and the second one computes the root-cause probabilities of critical incidents occurring in a certain step of a cascade. The algorithms written in a pseudo-code are to be found in Sections 4 and 5. They are illustrated with their application to a real-life example in Section 6.

2. Notation, definitions, and fundamental assumptions

General notation:

$O^{(1)},\ldots, O^{(n)}$ – the stochastic processes representing different operations carried out in the considered port area; $O^{(i)}$ represents the series of critical incidents that occur during the respective operation, i=1,…,n

$E_1^{(i)},\ldots,E_{m(i)}^{(i)}$ – different events that can occur in process $O^{(i)}$; m(i) – the number of these events

$\lambda_1^{(i)},\ldots,\lambda_{m(i)}^{(i)}$ – the intensities of occurrences of $E_1^{(i)},\ldots,E_{m(i)}^{(i)}$ as primary events; $\lambda_a^{(i)}\cdot(t-s)$ is the average number of occurrences of $E_a^{(i)}$ as a primary event in the time interval (s, t], which means that the sequence of primary events $E_a^{(i)}$ is a Poisson process with the intensity $\lambda_a^{(i)}$

$\coprod_{i=1,\ldots,r} x_i$ – the "inverted pi" operation on numbers from the [0, 1] interval, defined as follows:

$$\coprod_{i=1,\ldots,r} x_i = 1 - \prod_{i=1,\ldots,r}(1 - x_i)$$

The "inverted pi" operation computes the probability of a sum of independent events, i.e. $\Pr(\bigcup_{i=1,\ldots,r} A_i) = \coprod_{i=1,\ldots,r} P(A_i)$ if the events $A_1,\ldots,A_r$ are independent.

The cause-effect probabilities:

$p^{(i,j)}(a, b, 1)$ – probability that an occurrence of $E_a^{(i)}$ directly causes an occurrence of $E_b^{(j)}$ (first grade cause-effect probability)

$p^{(i,j)}(a, b, h)$ – probability that an occurrence of $E_b^{(j)}$ takes place in step h (and not less-than-h) of a cascade initiated by $E_a^{(i)}$ (cause-effect probability of grade h), h≥2

$P^{(i,j)}(a, b)$ – probability that an occurrence of $E_b^{(j)}$ takes place in any step of a cascade initiated by $E_a^{(i)}$

In stochastic terms, $p^{(i,j)}(a, b, h)$, h≥1, can be interpreted as the conditional probability that an instance of $E_b^{(j)}$ occurs in a cascade in its step h (and not less-than-h), provided that the cascade is initiated by $E_a^{(i)}$. The above definitions yield that

$$P^{(i,j)}(a,b) = \sum_{h\geq 1} p^{(i,j)}(a,b,h) \quad (1)$$

In practice, it is sufficient to compute the above sum for $h\leq h_{max}$, where $p^{(i,j)}(a, b, h)$ are negligibly small if $h>h_{max}$.

Intensities of critical incidents:

$\lambda_b^{(j)}(h)$ – intensity of occurrences of $E_b^{(j)}$ in step h (and not less-than-h) of cascades initiated by any primary events

$\Lambda_b^{(j)}$ – intensity with which occurrences of $E_b^{(j)}$ take place as primary or secondary events

Risks of critical incidents:

$R_b^{(j)}(k, s, t, 0)$ – probability that exactly k instances of $E_b^{(j)}$ as a primary event occur in the time interval (s, t]

$R_b^{(j)}(k, s, t, h)$ – probability that exactly k instances of $E_b^{(j)}$ occur in the time interval (s, t], where each occurrence takes place in step h (and not less-than-h) of a cascade initiated by some primary event, h≥1

$R_b^{(j)}(k, s, t)$ – probability that exactly k instances of $E_b^{(j)}$ as a primary or secondary event occur in the time interval (s, t]

Root-cause probabilities:

$r\text{-}c^{(j,i)}(b, a \mid h)$ – probability that an instance of $E_b^{(j)}$, provided that it occurs in step h (and not less-than-h) of a cascade, was caused by an instance of $E_a^{(i)}$ initiating this cascade, h≥1

$r\text{-}c^{(j,i)}(b, a, h)$ – probability that an instance of $E_b^{(j)}$ occurs in step h (and not less-than-h) of a cascade and was caused by an instance of $E_a^{(i)}$ initiating this cascade, h≥1

R-$C^{(j,i)}$(b, a) – probability that an instance of $E_b^{(j)}$, occurring as a secondary event in a certain step of a cascade, was caused by an instance of $E_a^{(i)}$ initiating this cascade, (j,b)≠(i,a)

R-$C^{(j,j)}$(b, b) – probability that an instance of $E_b^{(j)}$ occurs as a primary event

In stochastic terms, $c^{(j,i)}(b, a \mid h)$, h≥1, can be interpreted as the conditional probability that $E_a^{(i)}$ initiates a cascade, given that $E_b^{(j)}$ occurs in its step h (and not less-than h). In turn, $c^{(j,i)}(b, a, h)$ is the (unconditional) probability that $E_b^{(j)}$ occurs in a cascade in its step h (but not less-than-h), and the cascade was initiated by $E_a^{(i)}$. The first three probabilities in the above group quantify in three different ways the likelihood that $E_a^{(i)}$ is the root cause of $E_b^{(j)}$, whence their name. The last probability is not actually a root-cause one, but it can be regarded as such if we assume that an event can be a root-cause of itself.

This section ends with the list of assumptions upon which the considered model of interacting operations carried out in a port area is constructed. Some of these assumptions are repeated from the Introduction for the sake of self-containment.

1) Critical incidents occurring in the considered environment are modeled by n random processes denoted as $O^{(i)}$, i=1,…,n.

2) m(i) different events can occur in process $O^{(i)}$; they are denoted as $E_a^{(i)}$, a=1,…,m(i).

3) An event can be either primary (occurring by itself) or secondary (caused by another event).

4) All primary events are mutually independent.

5) The instances of primary event $E_a^{(i)}$ in process $O^{(i)}$ occur according to a Poisson process with known intensity $\lambda_a^{(i)}$.

6) The probability that event $E_a^{(i)}$ in process $O^{(i)}$ directly causes event $E_b^{(j)}$ in process O_j , is known for each combination of the indices i, j, a, b used for numbering the processes and events. It is referred to as first grade cause-effect probability and denoted as $p^{(i,j)}(a, b, 1)$. Clearly, if no cause-effect relation exists between $E_a^{(i)}$ and $E_b^{(j)}$, then $p^{(i,j)}(a, b, 1)=0$.

7) The intensities $\lambda_a^{(i)}$ and first-grade cause-effect probabilities $p^{(i,j)}(a, b, 1)$ can be obtained from the available statistical data and/or by way of expert elicitation. The probabilities $p^{(i,j)}(a, b, h)$, h≥2, can be computed using the method presented in Section 3.

8) Events can occur in a cause-effect chain called a cascade; the events in a cascade succeed each other instantaneously, thus cascades, like their initiating primary events, are mutually independent.

3. Computing the cause-effect probabilities of grades greater than one

3.1. The basic lemma

The cause-effect probabilities of higher grades can be computed with the use of the following lemma which provides a recursive formula relating the cause-effect probabilities of grade h to those of grade h–1, h≥2 :

Lemma 1
For h≥1 we have:

$$p^{(i,i)}(a,a,h) = 0 \qquad (2)$$

while for h≥2 and (j,b) ≠ (i,a) the following recursive formula holds:

$$p^{(i,j)}(a,b,h) = \left[1 - p^{(i,j)}(a,b,1)\right] \times$$
$$\times \coprod_{\substack{k=1,\dots,n \\ c=1,\dots,m(k)}} \begin{bmatrix} p^{(i,k)}(a,c,1) \times \\ p^{(k,j)}(c,b,h-1) \end{bmatrix} \qquad (3)$$

Proof: Let $E_{c,k,b,j}^{(h)}$, h≥1, be an event defined as follows:

$E_{c,k,b,j}^{(h)}$ = { an instance of $E_b^{(j)}$ occurs in step h, but not in step <h, of the cascade triggered by an instance of $E_c^{(k)}$ }

The equality (2) holds for h≥1, i=1,…,m, a=1,…,m(i), because the underlying events $E_{a,i,a,i}^{(h)}$ are impossible. Indeed, in order for $E_{a,i,a,i}^{(h)}$ to occur, the triggering event $E_a^{(i)}$ should take place at step 0 of the cascade, but the definition of $E_{a,i,a,i}^{(h)}$ yields that $E_a^{(i)}$ cannot occur at step <h, hence $E_{a,i,a,i}^{(h)}$ is an impossible event for h≥1.

Let now h≥2 and (j,b)≠(i,a). The event $E_{a,i,b,j}^{(h)}$ takes place if
1) the event $E_{a,i,b,j}^{(1)}$ does not occur and
2) a pair of consecutive events $E_{a,i,c,k}^{(1)}$ and $E_{c,k,b,j}^{(h-1)}$ occurs, where k=1,…,n, c=1,…,m(k), (k,c)≠(j,b).
The first condition and the inequality (k,c)≠(j,b) in the second condition are equivalent. They ensure that $E_b^{(j)}$ does not occur in step 1, which is required in the definition of $E_{a,i,b,j}^{(h)}$. We thus have:

$$p^{(i,j)}(a,b,h) =$$
$$= \Pr\left[\begin{array}{c} \left(\neg E_{a,i,b,j}{}^{(1)}\right) \cap \\ \left(\bigcup_{\substack{k=1,\dots,n \\ c=1,\dots,m(k) \\ (k,c)\neq(j,b)}} \left[E_{a,i,c,k}{}^{(1)} \cap E_{c,k,b,j}^{(h-1)} \right] \right) \end{array} \right] \qquad (4)$$

As follows from assumptions 4 and 8 in the last part of Section 2, the triggering events along with the triggered cascades are mutually independent, hence (3) is a direct consequence of (4). Let us note that (k,c)=(j,b) can be included in the range of the operator "inverted pi" in (3), because, in view of (2), if (k,c)=(j,b) then $p^{(i,k)}(a, c, 1)\cdot p^{(k,j)}(c, b, h-1) = p^{(i,j)}(a, b, 1)\cdot p^{(j,j)}(b, b, h-1) = 0$.

Remark 1: When applying (3) it should be taken into account that, according to (2), $p^{(i,k)}(a, c) = 0$ for (k,c)=(i,a).

Remark 2: The proof of Lemma 1 was provided in [7], but it is repeated here in order that the report be self-contained.

3.2. The matrix of cause-effect probabilities

The behavior of the considered multi-process environment can be described by the collection of matrixes $\pi^{(i,j)}(h)$, i,j=1,…,n, h≥1, where $\pi^{(i,j)}(h)[a, b] = p^{(i,j)}(a, b, h)$ is the element in row a and column b of matrix $\pi^{(j,i)}(h)$, a=1,…,m(i), b=1,…,m(j). The matrix $\pi^{(i,j)}(h)$ expresses the impact of events occurring in process p_i on the events in process p_j, where the latter events occur in step h (and not less-than-h) of cascades initiated by the former events. For a fixed h, the matrixes $\pi^{(i,j)}(h)$, i,j=1,…,n, can be arranged in the matrix π(h) as shown in Fig. 1.

$$\pi(h) = \begin{bmatrix} \pi^{(1,1)}(h) & \pi^{(1,2)}(h) & \cdots & \pi^{(1,n)}(h) \\ \pi^{(2,1)}(h) & \pi^{(2,2)}(h) & \cdots & \pi^{(2,n)}(h) \\ \vdots & \vdots & \ddots & \vdots \\ \pi^{(n,1)}(h) & \pi^{(n,2)}(h) & \cdots & \pi^{(n,n)}(h) \end{bmatrix}$$

Fig. 1. Matrix π(h) composed of matrixes $\pi^{(i,j)}(h)$, i,j=1,…,n.

Let us note that π(h) is a square matrix, because $\pi^{(i,j)}(h)$ has m(i) rows and m(j) columns, hence π(h) has $\sum_{i=1,\ldots,n}$ m(i) rows and $\sum_{j=1,\ldots,n}$ m(j) columns, and these sums are equal. It should also be noted that, according to (2), $p^{(i,j)}(a, b, h)$ is computed using the elements from row a of $\pi^{(i,k)}(1)$ and column b of $\pi^{(k,j)}(h-1)$, where k changes from 1 to n. This means that in order to obtain π(h)[m(1)+…+m(i–1)+a, m(1)+…+m(j–1)+b], i.e. the element of π(h) located in row m(1)+…+m(i–1)+a and column m(1)+…+m(j–1)+b , we use the elements from row m(1)+…+m(i–1)+a of π(1) and column m(1)+…+m(j–1)+b of π(h–1), which is analogous to obtaining the elements of a product of two matrices. As known from matrix algebra, such a product is computed using the following formula:

$$(A \times B)[q, r] = \prod_{s=1,\ldots,K} A[q, s]\cdot B[s, r] \quad (5)$$

where × is the matrix multiplication operator and K is the number of A's columns or B's rows. Clearly, the number of A's columns must be equal to the number of B's rows.

By analogy to the operation × let us define the matrix operation ⊗ as follows:

$$(A \otimes B)[q, q] = 0 \quad (6)$$

and

$$(A \otimes B)[q, r] = = (1 - A[q, r])\cdot \coprod_{s=1,\ldots,\kappa(A)} A[q, s]\cdot B[s, r] \quad (7)$$

for q≠r, where ∐ is the "inverted pi" operation defined in Notation section. Let us note that (2) and (3) can be replaced with (6) and (7) if we put A=π(1) and B=π(h–1). We can thus write (2) and (3) in a much simpler form:

$$\pi(h) = \pi(1) \otimes \pi(h-1),\ h\geq 2 \quad (8)$$

As follows from the previous paragraph, the element in row q and column r of π(h) is obtained using the elements in row q of π(1) and column r of π(h–1), similarly as in matrix multiplication. However, comparing (5) with (6) and (7) we see that ⊗ is not the matrix multiplication operation. Nonetheless, the numerical complexity of the operations × and ⊗ is almost the same and formula (8) is far less complicated and more convenient for computer implementation than formulas (2) and (3). The obtained matrix of cause-effect probabilities is used to easily compute the intensities of secondary events that are also needed for computing the root-cause probabilities. The formulas for those intensities are presented in the next section.

4. Computing the intensities of secondary events and their risks

This section starts with one assertion and one theorem. This theorem was already given in [4], but here it is presented in a simplified version, without strength of events being taken into account.

Assertion 1
Primary events $E_b^{(j)}$ constitute a Poisson process with the intensity $\lambda_b^{(j)}$.

The above assertion repeats of one of the assumptions formulated in Section 2. In turn, the theorem given below pertains to the instances of one event such that each instance occurs in step h (and not less-than-h) of a certain cascade, h≥1. This theorem states that the events in question also constitute a Poisson process.

Theorem 1

The instances of $E_b^{(j)}$, each occurring in step h (and not less-than-h) of a cascade triggered by an instance of $E_a^{(i)}$ as a primary event, constitute a Poisson process with the intensity $\lambda_{a,b}^{(i,j)}(h) = \lambda_a^{(i)} \cdot p^{(i,j)}(a, b, h)$, where the probabilities $p^{(i,j)}(a, b, h)$ are given by (2) and (3) or by (8).

Further, the instances of $E_b^{(j)}$, each occurring in step h (and not less-than-h) of a cascade triggered by any primary event in any process, constitute a Poisson process with the intensity given by the following formula:

$$\lambda_b^{(j)}(h) = \sum_{\substack{i=1,\dots,n \\ a=1,\dots,m(i)}} \lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \tag{9}$$

where, according to (2), $p^{(i,j)}(a, b, h)=0$ for (a,i)=(b,j).

Proof: $N_a^{(i)}(s, t)$ be the number of primary instances of $E_a^{(i)}$ in interval (s, t]. Let $N_{a,b}^{(i,j)}(s, t, h)$ be the number of instances of $E_b^{(j)}$ occurring in interval (s,t] in step h (and not less-than-h) of cascades triggered by primary instances of $E_a^{(i)}$. We have:

$$\Pr\left[N_{a,b}^{(i,j)}(s,t,h) = r\right] =$$

$$\sum_{q=r}^{\infty} Pr\left(\begin{matrix} the\ events\ E_a^{(i)} \\ cause\ r\ events\ E_b^{(j)} \\ in\ step\ h \\ of\ cascades \\ initiated\ by\ E_a^{(i)} \end{matrix}\middle| N_a^{(i)}(s,t) = q\right) \times$$

$$\times \Pr\left(N_a^{(i)}(s,t) = q\right) =$$

$$\sum_{q=r}^{\infty} \binom{q}{r} \left[p^{(i,j)}(a,b,h)\right]^r \left[1 - p^{(i,j)}(a,b,h)\right]^{q-r} \times$$

$$\times \frac{\left[\lambda_a^{(i)} \cdot (t-s)\right]^q}{q!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] =$$

$$\frac{\left[\lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \cdot (t-s)\right]^r}{r!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] \times$$

$$\times \sum_{q=r}^{\infty} \begin{matrix} \frac{q!}{(q-r)!}\left[1 - p^{(i,j)}(a,b,h)\right]^{q-r} \times \\ \times \frac{\left[\lambda_a^{(i)} \cdot (t-s)\right]^{q-r}}{q!} \end{matrix} =$$

$$\frac{\left[\lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \cdot (t-s)\right]^r}{r!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] \times$$

$$\times \sum_{q=r}^{\infty} \left[1 - p^{(i,j)}(a,b,h)\right]^{q-r} \frac{\left[\lambda_a^{(i)} \cdot (t-s)\right]^{q-r}}{(q-r)!} =$$

$$\frac{\left[\lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \cdot (t-s)\right]^r}{r!} \exp\left[-\lambda_a^{(i)} \cdot (t-s)\right] \times$$

$$\times \exp\left(\left[1 - p^{(i,j)}(a,b,h)\right]\left[\lambda_a^{(i)} \cdot (t-s)\right]\right) =$$

$$\frac{\left[\lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \cdot (t-s)\right]^r}{r!} \times$$

$$\times \exp\left[-\lambda_a^{(i)} \cdot p^{(i,j)}(a,b,h) \cdot (t-s)\right]$$

Thus, the first of the above expressions is equal to the last one, which yields the first part of the theorem.

Let now $N_b^{(j)}(s, t, h)$ be the number of instances of $E_b^{(j)}$ occurring in interval (s,t] in step h (and not less-than-h) of cascades triggered by primary instances of any event different than $E_b^{(j)}$. Due to the assumption that primary events in all the processes are mutually independent, and cascades of events occur instantaneously, the above instances of $E_b^{(j)}$ constitute a Poisson process which is a superposition of independent Poisson processes with the intensities $\lambda_a^{(i)}$, a=1,…,n, i=1,…,m(i), (a,i)≠(b,j). The second part and the whole theorem are thus proved.

Remark: The proof of (9) is also provided in [6], in the extended version taking into account the strengths of events.

In order to shorten the notation and simplify the computer implementation, (9) can be expressed in the following simpler form:

$$\lambda(h) = \lambda(0) \times \pi(h) \tag{10}$$

where × is the usual matrix multiplication operation, π(h) is defined in Fig. 1, while λ and λ(h) are one-row matrices defined as follows:

$$\lambda(0) = [\ \lambda_1^{(1)}, \dots, \lambda_{m(1)}^{(1)}; \lambda_1^{(2)}, \dots, \lambda_{m(2)}^{(2)}; \dots ;$$
$$\lambda_1^{(n)}, \dots, \lambda_{m(n)}^{(n)}\ ] \tag{11}$$

$$\lambda(h) = [\lambda_1^{(1)}(h),\dots, \lambda_{m(1)}^{(1)}(h); \lambda_1^{(2)}(h),\dots, \lambda_{m(2)}^{(2)}(h);$$
$$\dots ; \Lambda_1^{(n)}(h),\dots, \lambda_{m(n)}^{(n)}(h)\ ] \tag{12}$$

Thus, λ and $\lambda(h)$ are composed of the intensities of all events in all processes, where the events respectively occur as primary or in step h (and not less-than-h). It should be noted that formulas (9), (10) and (12) hold for $h\geq1$.

Let us now define the stochastic process $O^{(b,j)}(t)$ as the (random) number of instances of $E_b^{(j)}$, no matter whether primary or not, in the time interval (0, t]. The following theorem holds:

Theorem 2

$O^{(b,j)}(t)$ is a Poisson process with the following intensity:

$$\Lambda_b^{(j)} = \lambda_b^{(j)} + \sum_{h\geq1} \lambda_b^{(j)}(h) \tag{13}$$

In practice, the sum in (13) is only computed for several values of h, i.e. for $h\leq h_{max}$, where $\Lambda_b^{(j)}(h)$ are negligibly small for $h>h_{max}$.

Proof: Due to the assumption that primary events in all the processes are mutually independent, and cascades of events occur instantaneously, $O^{(b,j)}(t)$ is a superposition of independent processes $O^{(b,j,h)}(t)$ $h\geq0$, where $O^{(b,j,h)}(t)$ is defined as the (random) number of instances of $E_b^{(j)}$ in the interval (0, t], where each instance is a primary event if h=0, or occurs in step h (and not less-than-h) of a cascade triggered by an instance of any event different than $E_b^{(j)}$. From Assertion 1 and Theorems 1 and 2 it follows that $O^{(b,j)}(t)$ is a Poisson process with the intensity given by (13), Q.E.D.

Theorem 3

Formula (13) can be written in the following simpler form:

$$\Lambda = \lambda(0) + \lambda(0) \times \sum_{h\geq1} \pi(h) \tag{14}$$

where + and $\sum$ are the usual addition operations on matrices, and Λ is a one-row matrix composed of the intensities $\Lambda_b^{(j)}$, and defined as follows:

$$\Lambda = [\, \Lambda_1^{(1)},\ldots, \Lambda_{m(1)}^{(1)},\ldots, \Lambda_1^{(n)},\ldots, \Lambda_{m(n)}^{(n)} \,] \tag{15}$$

In practice, the sum in (14) is only computed for several values of h, i.e. for $h\leq h_{max}$, where the elements of $\pi(h)$ are negligibly small for $h>h_{max}$.

Proof: Formula (13) and the definition of Λ given by (15) yield:

$$\Lambda = \lambda(0) + \sum_{h\geq1} \lambda(h) \tag{16}$$

In view of (10) and distributivity of matrix multiplication w.r.t. addition, the above equality converts to:

$$\Lambda = \lambda(0) + \sum_{h\geq1} \lambda(0) \times \pi(h) = \lambda(0) + \lambda(0) \times \sum_{h\geq1} \pi(h) \tag{17}$$

Q.E.D.

Corollary:

With the use of (14) the elements of Λ are computed significantly faster than by using (13) or (16). If (13) or (16) along with (9) or (10) is applied, then $\lambda(h)$ is computed individually for each $h\leq h_{max}$, i.e. h_{max} matrix multiplications are executed. In turn, (14) only requires the execution of one matrix multiplication and $h_{max}-1$ additions, and adding $\pi(h)$ to $\pi(h-1)$ is numerically less complex than multiplying λ by $\pi(h)$. However, we need $\lambda(h)$ if we want to compute the root-cause probabilities related to step h of cascades, as shown in the next section.

Theorems 1 and 2 yield the following formulas for the risks of critical incidents whose definitions can be found in Section 2:

$$R_b^j(k,s,t,0) = \frac{\left[\lambda_b^{(j)}\cdot(t-s)\right]^k}{k!}\cdot \exp\left[-\lambda_b^{(j)}\cdot(t-s)\right] \tag{18}$$

$$R_b^j(k,s,t,h) = \frac{\left[\lambda_b^{(j)}(h)\cdot(t-s)\right]^k}{k!}\cdot \exp\left[-\lambda_b^{(j)}(h)\cdot(t-s)\right] \tag{19}$$

$$R_b^j(k,s,t) = \sum_{h\geq0} R_b^j(k,s,t,h) = \frac{\left[\Lambda_b^{(j)}\cdot(t-s)\right]^k}{k!}\cdot \exp\left[-\Lambda_b^{(j)}\cdot(t-s)\right] \tag{20}$$

Based on the results of Sections 3 and 4, the risks of critical incidents can be computed with the use of the following algorithm:

Algorithm 1

1. Arrange the input data into the matrixes $\lambda(0)$ and $\pi(1)$ defined respectively by (11) and Fig. 1
2. Using (8), determine the matrices $\pi(h)$, $h\geq2$
3. Using (10), determine the matrices $\lambda(h)$, $h\geq1$ defined by (12)
4. Using (13) or (14), determine the matrix Λ defined by (15)
5. Compute from (20) the matrix of risks $R_b^{(j)}(k, s, t)$, j=1,…,n, b=1,…,m(j) for different k, s and t. If a more detailed analysis is needed, also compute from (18) and (19) the matrixes of risks $R_b^{(j)}(k, s, t, h)$, $h\geq0$

5. Computing the root-cause probabilities of critical incidents

The root-cause probabilities defined in Section 2 are necessary to perform the root-cause analysis of critical incidents occurring in processes $O^{(1)},\ldots,O^{(n)}$. These probabilities are calculated using the formulas given in Theorem 4 which is a consequence of the following lemma.

Lemma 2
Let $A_1,\ldots,A_m$, occurring independently with the intensities $\mu_1,\ldots, \mu_m$ respectively, be the cascade initiating events which can cause (directly or not) an event B in step h of the initiated cascade. Then, the root cause probability that B was caused by A_i. is given by the following formula:

$$\Pr(A_i|B) = \\ = \mu_i \Pr(B|A_i)/\textstyle\sum_{k=1}^{m} \mu_k \Pr(B|A_k), \quad (21)$$

where i=1,…,m.

Proof: Let us redefine $A_1,\ldots,A_m$ and B as follows:
A_i = {a cascade is initiated by A_i}
B = {B occurs in step h of a cascade, h≥1}
The law of total probability yields:

$$\Pr(B) = \textstyle\sum_{k=1}^{m} \Pr(B|A_k)\Pr(A_k) \quad (22)$$

Applying the Bayes theorem and using (22), we obtain

$$\Pr(A_i|B) = \Pr(B|A_i)\Pr(A_i)/\Pr(B) = \\ = \Pr(B|A_i)\Pr(A_i)/\textstyle\sum_{k=1}^{m} \Pr(B|A_k)\Pr(A_k) \quad (23)$$

Since the instances of A_i occur according to a Poisson process with the intensity μ_i, and the processes are mutually independent for i=1,…,m, it holds that

$$\Pr(A_i) = \frac{\mu_i}{\mu_1+\cdots+\mu_m} \quad (24)$$

Replacing $\Pr(A_i)$ and $\Pr(A_k)$ in (23) according to (24), yields formula (21). This completes the proof.

Corollary:
Using the "forward" probabilities $\Pr(E|A_i)$, expressing the "forward" stochastic relations between causes and their possible effects, we can compute, using (21), the "backward" probabilities $\Pr(A_i|E)$ expressing the "backward" stochastic relations between effects and their possible causes.

Now the theorem mentioned in the beginning of this section can be formulated.

Theorem 4
The root-cause probabilities defined in Section 2 are given by the following formulas:

$$c^{(j,i)}(b,a|h) = \\ = \lambda_a^{(i)} p^{(i,j)}(a,b,h)/\lambda_b^{(j)}(h), \; h \geq 1 \quad (25)$$

$$c^{(j,i)}(b,a,h) = \\ = \lambda_a^{(i)} p^{(i,j)}(a,b,h)/\Lambda_b^{(j)}, \; h \geq 1 \quad (26)$$

$$C^{(j,i)}(b,a) = \\ = \lambda_a^{(i)} P^{(i,j)}(a,b)/\Lambda_b^{(j)}, \; (i,a) \neq (j,b) \quad (27)$$

$$C^{(j,j)}(b,b) = \lambda_b^{(j)}/\Lambda_b^{(j)} \quad (28)$$

where $P^{(i,j)}(a,b)$ in (27) is given by (1). It should be noted that $p^{(i,j)}(a, b, h)=0$ for (i,a)=(j,b), hence there is no need to assume in (25) and (26) that (i,a)≠(j,b). However, such assumption is necessary in (27), because $P^{(i,,j)}(a,b)=0$ for (i,a)=(j,b), thus (28) is not a special case of (27).

Proof: The probability that $E_b^{(j)}$ occurs in step h of a cascade, h≥1, is equal to $\lambda_b^{(j)}(h)/\Lambda_b(j)$, and the probability that $E_b^{(j)}$ is a primary event is equal to $\lambda_b(j)/\Lambda_b(j)$. This follows from the fact that intensities can be regarded as frequencies with which the respective events occur. Thus, (28) holds. Let us note that $E_c^{(k)}$, 1≤k≤n, 1≤c≤m(k), can be cascade initiating events, each capable of causing event $E_b^{(j)}$ in step h of a cascade initiated by itself, where h≥1 and (k,c)≠(j,b). Hence, in view of Formula 9 (with i and a replaced with k and c), (25) is a straightforward consequence of Lemma 2. In turn, the unconditional probability $c^{(j,i)}(b, a, h)$ is obtained by multiplying the conditional probability $c^{(j,i)}(b, a | h)$ by $\lambda_b^{(j)}(h)/\Lambda_b(j)$, yielding (26). Finally, (1) and (26) result in (27), which completes the proof.

The results of Sections 3 through 5 yield the following algorithm for computing the root-cause probabilities of critical incidents:

Algorithm 2
1. Arrange the input data into the matrixes λ(0) and π(1) defined respectively by (11) and Fig. 1
2. Using (8), determine the matrices π(h), h≥2
3. Using (10), determine the matrices λ(h), h≥1 defined by (12)
4. Using (13) or (14), determine the matrix Λ defined by (15)
5. For each $E_b^{(j)}$ compute from (27) and (28) the matrix of root-cause probabilities $C^{(j,i)}(b,a)$, i=1,…,n, a=1,…,m(i), including the probability $C^{(j,j)}(b,b)$. If a more detailed analysis is needed, also compute from (25) and (26) the matrices of probabilities $c^{(j,i)}(b,a|h)$ and $c^{(j,i)}(b,a,h)$ for h≥1

6. A real-life example and the obtained numerical results

Let us consider three following processes realized in a port area including oil and container terminals:

$O^{(1)}$ – vessel traffic to and from the harbor,
$O^{(2)}$ – crude oil transfer to or from tankers in the oil terminal,
$O^{(3)}$ – truck traffic to and from the container terminal.

The following events can occur in the individual processes:

In $O^{(1)}$:
$E_1^{(1)}$ – vessel collision with another vessel or a wharf,
$E_2^{(1)}$ – spill of burning oil in the port waters,
$E_3^{(1)}$ – vessel on fire,
m(1)=3

In $O^{(2)}$:
$E_1^{(2)}$ – pipeline or hose damage and/or ignition,
$E_2^{(2)}$ – onshore tank on fire,
m(2)=2

In $O^{(3)}$:
$E_1^{(3)}$ – truck accident,
m(3)=1

Let us assume that the following cause-effect relations hold between the above events:

$E_1^{(1)} \to E_2^{(1)}$, $E_2^{(1)} \to E_3^{(1)}$, $E_2^{(1)} \to E_1^{(2)}$
$E_1^{(2)} \to E_2^{(2)}$, $E_2^{(2)} \to E_1^{(2)}$, $E_1^{(2)} \to E_2^{(1)}$
$E_1^{(3)} \to E_1^{(2)}$

We also assume that only $E_1^{(1)}$ (vessel collision), $E_3^{(1)}$ (vessel on fire), $E_2^{(2)}$ (tank on fire) and $E_1^{(3)}$ (truck accident) can occur as primary events, i.e. $\lambda(0) = [\ \lambda_1^{(1)}, 0, \lambda_3^{(1)};\ 0, \lambda_2^{(2)};\ \lambda_1^{(3)}\ ]$. However, as follows from the previous assumption, $E_3^{(1)}$ (vessel on fire) and $E_2^{(2)}$ (tank on fire) can also be secondary events.

The algorithms from Sections 4 and 5 applied to the above example produced the following results:
INPUT DATA PRINTOUT:

Number of processes: 3
Maximum cascade grade: 6 (assumed value of h_{max})
Number of events in process 1: 3
Number of events in process 2: 2
Number of events in process 3: 1

Matrix λ[j,b], j=1,...,n, b=1,...,m(j):

0.5000 0.0000 0.5000
0.0000 0.5000
0.5000

Matrix π(1):

0.0000 0.5000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.9000 0.4000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.9000 0.0000 0.0000 0.9000 0.0000
0.0000 0.0000 0.0000 0.5000 0.0000 0.0000
0.0000 0.0000 0.0000 0.8000 0.0000 0.0000

RESULTS PRINTOUT; MATRICES OF INTENSITIES RELATED TO SUCCESSIVE STEPS OF CASCADES

Matrix Λ_1[j,b]:

0.0000 0.2500 0.0000
0.6500 0.0000
0.0000

Matrix Λ_2[j,b]:

0.0000 0.5850 0.2250
0.1000 0.3600
0.0000

Matrix Λ_3[j,b]:

0.0000 0.0000 0.5265
0.0000 0.0900
0.0000

Matrix Λ_4[j,b]:

0.0000 0.0263 0.0081
0.0000 0.0130
0.0000

Matrix Λ_5[j,b]:

0.0000 0.0000 0.2490
0.0000 0.0032
0.0000

Matrix Λ_6[j,b]:

0.0000 0.0012 0.0038
0.0000 0.0005
0.0000

RESULTS PRINTOUT; RISK MATRICES FOR DIFFERENT TIME AND QUANTITY PARAMETERS

Matrix R[j][b](2.00 years, 0 events):

0.3679 0.1782 0.0486
0.2231 0.1447
0.3679

Each of the above values subtracted from 1 is the probability that at least one respective $E_b^{(j)}$ (whether primary or not) occurs in a 2-year period.

Matrix R[j][b](2.00 years, 1 event):

0.3679 0.3073 0.1469
0.3347 0.2797
0.3679

Matrix R[j][b](2.00 years, 2 events):

0.1839 0.2651 0.2222
0.2510 0.2704
0.1839

Matrix R[j][b](2.00 years, 3 events):

0.0613 0.1524 0.2240
0.1255 0.1742
0.0613

Matrix R[j][b](2.00 years, 4 events):

0.0153 0.0657 0.1694
0.0471 0.0842
0.0153

Matrix R[j][b](2.00 years, 5 events):

0.0031 0.0227 0.1025
0.0141 0.0326
0.0031

Matrix R[j][b](2.00 years, 6 events):

0.0005 0.0065 0.0517
0.0035 0.0105
0.0005

Matrix R[j][b](2.00 years, 7 events):

0.0001 0.0016 0.0223
0.0008 0.0029
0.0001

Matrix R[j][b](2.00 years, 8 events):

0.0000 0.0003 0.0084
0.0001 0.0007
0.0000

Matrix R[j][b](2.00 years, 9 events):

0.0000 0.0001 0.0028
0.0000 0.0002
0.0000

Matrix R[j][b](2.00 years, 10 events):

0.0000 0.0000 0.0009
0.0000 0.0000
0.0000

RESULTS PRINTOUT; MATRICES OF ROOT-CAUSE PROBABILITIES FOR DIFFERENT RESULTING EVENTS

$C^{(1,1)}(1,1)=1$; $E_1^{(1)}$ (vessel collision) can only be a primary event

Matrix $C^{(1,i)}(2,a)$ for the resulting event $E_2^{(1)}$ (burning spill):

0.2899 0.0000 0.0000
0.0000 0.2731
0.4370

Matrix $C^{(1,i)}(3,a)$ for the resulting event $E_3^{(1)}$ (vessel on fire):

0.1567 0.0000 0.3306
0.0000 0.1972
0. 3155

Matrix $C^{(2,i)}(1,a)$ for the resulting event $E_1^{(2)}$ (pipeline or hose damage):

0.1333 0.0000 0.0000
0.0000 0.3333
0.5333

Matrix $C^{(2,i)}(2,a)$ for the resulting event $E_2^{(2)}$ (onshore tank on fire):

0.0965 0.0000 0.0000
0.0000 0.5172
0.3863

$C^{(3,3)}(1,1)=1$; $E_1^{(3)}$ (truck accident) can only be a primary event

Remarks:
1. For clarity of presentation, the intensities $\lambda_b^{(j)}(h)$, $h \geq 0$ are arranged in matrices rather than vectors (cf. formulas 11 and 12).
2. For each j=1, 2, 3 and b=1,…,m(j), the root-cause probabilities $C^{(j,i)}(b,a)$ are arranged in a matrix with 3 rows (i=1, 2, 3) and m(i) elements in row i (a=1,…,m(i)).
3. The probability that the respective event occurs as primary one is underlined in each matrix $C^{(j,i)}(b,a)$.

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