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# Methods for integrated failure risk analysis of water network in terms of water supply system functioning management

### Keywords

water supply system, failure indicators, risk analysis.

### Abstract

In the paper the issues connected with failure risk analysis related to the functioning of this system were presented. In the analysis of water supply system it is important to assess the safety of its operation, both from the producer and the consumer point of view. The paper presents the method of analysis of consumers acceptance of water costs that companies bear for the risk reduction. The proposed method can be the basis of comprehensive program of risk failure management and in decision-making on water supply operation.

## 1. Introduction

Being consumers of drinking water we deal with the operation of water supply system every day. Daily use of the water supply system is bound up with the possibility of the occurrence of various types of failures so the recipients can experience the nuisance of interference in its functioning [3, 4]. Random nature of the formation of failure makes the research in this area very complex, also it is based primarily on the analysis of operating data [30, 37, 38].

Recently the problem of objects and technical systems reliability occupies a significant position in the process of design and operation. The term reliability appears in many references, the differences depend on the technical system which is examined [11, 12, 20, 23, 26].

The risks associated with the operation of the water supply should be controlled at the level of the water supply company in order to provide consumers water with the appropriate level of safety [15]. The standard should be comfort of the consumers because of the fact that water used by them is not a danger to their health. It also involves trust to water companies and such inspection services as sanitary-epidemiological stations [14].

Unfortunately, the functioning of the water supply system is always subjected to risk associated with the possibility of the occurrence of various types of undesirable events, which may result in deterioration of water quality or its lack [16, 18, 29]. Managing the risk associated with the operation of the water supply system can be defined as the process of coordinating work of the water supply system elements and system operators, with the use of available means, in order to achieve the level of tolerated risk [5, 8, 10]. Proper control system and regulation should allow for the identification of risk factors important in terms of system operation and safety [6, 7, 28, 40].

Proper assessment of the functioning of the water supply network should guarantee making the right decisions regarding the choice of the best solutions in terms of technical, economic and reliability, at the stage of design, construction and operation [21, 22, 32, 35].

Failure occurrence in water supply network is often associated with inappropriate operational procedures and a lack of the water supply network monitoring, resulting in a lack of response to small water leaks, also not taking into account scenarios of emergency water supply [17, 24, 25, 27]. The causes of the failures occurring in the water supply network can be defective factory material, faulty sealing or anticorrosion coating, environmental causes, such as weather conditions, including temperature changes, landslides and causes resulting from the operation to

protect the water supply network through its monitoring [19, 33, 34, 39].
In the paper the method of risk analysis of water supply system failure through the method of prioritizing risk, the application of the Dempster-Shafer theory and the methods for accepting costs connected with risk reduction, were presented.
The presented methodology for the analysis of failure risk in water network can be used to assess new strategies in the management of the water supply system safety.

## 2. Method of risk prioritizing in water Supply network

Method of Risk Prioritizing - MRP - involves selecting the factors affecting the risk level of failure in the water supply network. The proposed method is based on the classification of risk factors for failure of the water supply network and assigning them points values - functional criteria - $FC_i$ and point weights - assessment criteria - $AC_i$, and then calculating the index of risk prioritizing - *IRP*.
In this way, a value of the index of risk prioritizing - IRP - is calculated according to the formula:

$$I_{RP} = \sum_{i=1}^{n} FC_i \cdot AC_i \qquad (1)$$

where *IRP* is the index of failure vulnerability, $FC_i$ means functional criteria, $AC_i$ means assessment criteria and n is the number of criteria taken into account in the considered method.
Each functional criterium $FC_i$, depending on the degree of influence of the factor on the risk prioritizing index, has assigned a point value in the following way as shown in Table 1 (from 0 to 1 - neglected, from 2 to 3 - unimportant, from 4 to 6 - the average important, 7 and 8 – important, from 9 to 10 - very important).
The values of assessment criteria $AC_i$ are adopted depending on the importance of the damaged pipe, according to the following scale: 1 - low, 2 - medium, 3 - high or 4 - very high.
If the given factor is not present in the analysis, the values of $FC_i$ and $AC_i$ are assumed as 1.
In *Table 1* the classes for factors were proposed to analyse and identify risk areas of water supply network failure and $FC_i$ and $AC_i$ values.

*Table 1*. Impact of factors $FC_i$ and $AC_i$

| Impact factor | | $AC_i$ | $FC_i$ |
|---|---|---|---|
| Type of water network ($WN_T$) | water supply connection | 1 | 10 |
| | distribution network | 2 | |
| | mains | 3 | |
| Water network age ($WN_A$) | to 10 years | 1 | 9 |
| | from 10 to 30years | 2 | |
| | from 30 to 60years | 3 | |
| | above 60 years | 4 | |
| Water network material | plastics | 1 | 6 |
| | steel | 2 | |
| | grey cast iron | 3 | |
| Hydrogeological conditions | good | 1 | 8 |
| | average | 2 | |
| | poor | 3 | |
| Network monitoring | above-standard | 1 | 5 |
| | standard | 2 | |
| | none | 3 | |
| Corrosion protection | full | 1 | 4 |
| | standard | 2 | |
| | none | 3 | |
| The density of underground infrastructure in the area where the network is situated | small | 1 | 3 |
| | average | 2 | |
| | big | 3 | |
| Dynamic loads, including the difficulty of repairs in the area where the network is situated | pipeline in the not urbanized areas | 1 | 3 |
| | pipeline in the pedestrian traffic (pavements) | 2 | |
| | pipeline in the street | 3 | |
| Failure rate, λ | 0,5 $km^{-1} \cdot a^{-1}$ | 1 | 10 |
| | from 0,5 $km^{-1} \cdot a^{-1}$ to 1,0 $km^{-1} \cdot a^{-1}$ | 2 | |
| | > 1,0 $km^{-1} \cdot a^{-1}$ | 3 | |
| Size of possible losses resulting from failure occurrence | financial loss up to $10^4$ EUR | 1 | 6 |
| | financial loss from $10^4$ EUR to $10^5$ EUR | 2 | |
| | financial loss above $10^5$ EUR | 3 | |
| The difficulty to repair damages | repair brigades are organized and equipped appropriately and they are in full readiness for 24 hours | 1 | 5 |
| | basic equipment to repair a failure, one shift work | 2 | |

| | | | |
|---|---|---|---|
| | lack of mechanized equipment to repair a failure | 3 | |

Explanations:
Above-standard network monitoring: full monitoring of water pipe network by measuring the water pressure and water flow rate, possession of a specialized apparatus to detect water leaks by acoustic methods, unrestricted communication with the public through the phone line active 24 hours, monitoring of water quality in water network by means of protection and warning system.
Standard, simplified monitoring of water- pipe network with the use of pressure measurement, inability to respond to small water leaks, water quality tests in water- pipe network are conducted.
None, lack of monitoring of water-pipe network and water quality.
For the MRP method a five - step scale was adopted as shown in *Table 2.*

*Table 2.* Categories of the $I_{RP}$

| Values of the $I_{RP}$ | Interpretation of risk levels |
|---|---|
| above 70 | neglected risk ($r_Z$) |
| from 70 to 100 | tolerable risk ($r_T$) |
| from 100 to 170 | controlled risk ($r_C$) |
| above 170 | unacceptable risk ($r_{UNA}$) |

The value of the $I_{RP}$ obtained through performed analysis helps to make decisions concerning the operation or modernization of the system. Neglected risk - no further action is required and system operates in proper and reliable way. In case of obtaining tolerable risk preventive action in the system is not needed. Controlled risk means that the system is allowed to operate but under the condition that modernization or repair will be undertaken. If unacceptable level occurs an immediate action should be taken to reduce the $I_{RP}$.
Example of application
Section of the distribution network with a total length of five kilometres which is part of the system of water supply network in the considered city supplying 50 thousands of inhabitants.
Considered water pipe is characterized by the following parameters (respectively $AC_i$ and $FC_i$):

- distribution network (2;10),
- network age - 29 years (2;9),
- pipe material - PE (1;6),
- hydrogeological conditions - poor, not very stable ground, (3;8),
- standard network monitoring (2;5),
- corrosion protection – not applicable (1;1),
- the density of underground infrastructure in the area where the network is situated - small (1;3),
- pipeline in the not urbanized areas (1;3),
- failure rate $\lambda = 0{,}34\ km^{-1}\cdot a^{-1}$ (1;10),
- size of possible losses resulting from failure occurrence $10^2$ EUR (1;6),
- basic equipment to repair a failure, one shift work (2;5).

According to *Table 2*, the value of IRP was 111 which corresponds to the category of controlled level of risk prioritizing.

## 3. Application of the dempster shafer evidence theory in the analysis of failure risk of water supply network

The DS theory is treated as a generalization of Bayesian probability theory. For different hypotheses or evidences the probabilities are assigned using the so-called belief function referred to as BPA ( Basic Probability Assignment) or *m* [1, 3, 13, 31]. The DST also provides the possibility of combining different hypotheses and, on that basis, determining the baseline probability. The main difference between the probabilities lies in the fact that the m function does not need to be specified on all elements of the event and only on some of the subsets [41].
In case of having more than one value of the belief function for a given set of hypotheses (e.g. one expert determined that the probability of failure due to mechanical failure of a section of the water supply system is 0.5 and the second expert stated that the probability of failure of the same network section, but as a result of processes corrosion, is 0.4).
Several problems with the interpretation of combining rules of the DS were considered in the work of Zadeh [42], who pointed to the fact that only hypotheses that are not in conflict are considered and contradictory hypotheses are ignored by standardization. Modification of combining rules of the DS can be found, among others, in [9].
Analysis of the failure risk with the use of the DST is based on the analysis of opinions of experts, who assess (giving hypothesis and the values of the belief function) the possibility of risk level.
The lower and upper limit of the belief degree function ($m$) about the truth of the hypothesis H concerning the set (or its components) $X$ determine the measures [31]: belief: bel($X$) and plausibility (credibility): pl( $X$).
The belief function $m(X_i)$ is characterized by $m(X_i)$ $\rightarrow$ [0,1] and $m(\emptyset)$=0 for each $\sum_{X_i \subseteq X} m(X_i) = 1$ for *i*-1,2,3…n, n is the number of hypotheses on the set $X$, and j is the number of elements of the set $X$.

The dependencies between the individual belief functions are as follows:

$$bel(X) \leq P(X) \leq pl(X), \quad (1)$$

$$bel(X_i) = \sum_{X_k \subseteq X_i} m(X_k) \quad (2)$$

$$bel(\varnothing) = 0, \ bel(X) = 1, \quad (3)$$

$$pl(X_i) = \sum_{X_k \cap X_i \neq} m(X_k), \quad (4)$$

$$pl(\varnothing) = 0, pl(X) = 1, \quad (5)$$

$$pl(X_i) = 1 - bel(\neg X_i) \quad (6)$$

The Dempster-Shafer Rule of Combination (DSRC) is as follows:

$$m_{1\text{-}2}(\varnothing) = 0 \quad (7)$$

$$m_{1\text{-}2}(X_i) = \frac{\sum_{X_a \cap X_b = X_i} m_1(X_a) \cdot m_2(X_b)}{1 - \sum_{X_a \cap X_b = \Theta} m_1(X_a) \cdot m_2(X_b)} \quad (8)$$

wheras the degree of conflict $C_f$ is determined through:

$$C_f = \sum_{X_a \cap X_b = \Theta} m_1(X_a) \cdot m_2(X_b) \quad (9)$$

Then, the process of combining the rules using dependences 8 and 9 must be made. For the first (1st) and the second expert (2nd), a matrix scheme to combine the rules is shown in *Table 3*.

*Table 3.* Matrix of rules of the DST

| 1st expert | $H_1$ | … | $H_i$ |
|---|---|---|---|
| 2nd expert | $m_{E1}(r_{H1})\cdot m_{E2}(r_{Hi})$ | | |
| $H_1$ | $m_{E1}(r_{H1})\cdot m_{E2}(r_{H1})$ | … | $m_{E1}(r_{Hi})\cdot m_{E2}(r_{H1})$ |
| ... | … | ... | … |
| $H_i$ | $m_{E1}(r_{H1})\cdot m_{E2}(r_{Hi})$ | … | $m_{E1}(r_{Hi})\cdot m_{E2}(r_{Hi})$ |

If the information is received from *n* independent experts, thus matrix is obtained of data of belief degree functions recorded in the form [31]:

$$M_{E1,..En} = \begin{vmatrix} m_{E1}(r_i) \\ \vdots \\ m_{En}(r_i) \end{vmatrix} \quad (10)$$

For risk parameter set of its possible levels are expressed as: R = {neglected risk ($r_Z$), tolerable risk ($r_T$), controlled risk ($r_C$) oraz unacceptable risk ($r_{UNA}$)}, k = 4. Number of possible hypotheses $H_n$ is: $n = 2^k = 2^4 = 16$.
Table 1 shows the section of the characteristics of all the hypotheses concerning the possibility of occurrence of this type of risk.

For the risk parameter the set of its possible levels are expressed as: $R$ = {neglected risk ($r_Z$), tolerable risk ($r_T$), controlled risk ($r_C$) and unacceptable risk ($r_{UNA}$)}, $k = 4$. Number of possible hypotheses $H_n$ is: $n = 2^k = 2^4 = 16$. *Table 4* shows the fragment of the characteristics of all the hypotheses concerning the possibility of the occurrence of given type of risk.

*Table 4*. Fragment of the characteristics of the hypotheses for the DS theory

| $H_n/R_i$ | 1st hypothesis | 2nd hypothesis | 3rd hypothesis | … | 15th hypothesis | 16th hypothesis |
|---|---|---|---|---|---|---|
| $r_Z$ | - | + | - | … | - | + |
| $r_T$ | - | - | + | … | + | + |
| $r_C$ | - | - | - | … | + | + |
| $r_{UNA}$ | - | - | - | … | + | + |

Then the information was received from two experts concerning the risk of failure of the water supply network. Part of the results in which the outcome is different from zero was shown.

They are following :

- 1st expert assumed the following hypothesis:
  - 3rd hypothesis: $m_1(r_T) = 0,3$,
  - 4th hypothesis: $m_1(r_C) = 0,3$,
  - 10th hypothesis: $m_1(r_{TvC}) = 0,4$,
- 2nd expert assumed the following hypothesis:
  - 3rd hypothesis: $m_1(r_T) = 0,4$,
  - 4th hypothesis: $m_1(r_C) = 0,5$,
  - 5th hypothesis: $m_1(r_{TvCvUNA}) = 0,1$.

The matrix of data of the belief degree function obtained in such a way was recorded as:

$$M = \begin{vmatrix} m_1(r_T) & m_1(r_C) & m_1(r_{TvC}) \\ m_2(r_T) & m_2(r_C) & m_2(r_{TvCvUNA}) \end{vmatrix} \quad (11)$$

and

$$M = \begin{vmatrix} 0,3 & 0,3 & 0,4 \\ 0,4 & 0,5 & 0,1 \end{vmatrix} \quad (11)$$

In the Table 5 the matrix of combining rules of DS for the results different from zero was shown.

*Table 5*. Matrix rules DS

| The results of combining | | 1st expert | | |
|---|---|---|---|---|
| | | $r_T$ | $r_C$ | $r_{TvC}$ |
| 2nd expert | $r_T$ | 0,12 (T) | 0,12 | 0,16 (T) |
| | $r_C$ | 0,15 | 0,15 (C) | 0,2 (C) |
| | $r_{TvCvUNA}$ | 0,03 (T) | 0,03 (C) | 0,04 (TvC) |

where the underlined values represent the conflict information - $C_f$.

On the basis of formulas 1-9 a degree of conflict information and values of the belief function and bel measure were calculated.

- $C_f = 0,27$
- $1 - C_f = 0,73$

- $m_{1-2}r_T = (0,12 + 0,16 + 0,03)/0,73 = 0,42$
- $m_{1-2}r_C = (0,15 + 0,03 + 0,2)/0,73 = 0,52$
- $m_{1-2}r_{TvC} = 0,04/0,73 = 0,054$

All other belief functions for other combined rules are equal to zero.

- $bel_{1-2}r_T = m_{1-2}r_T = 0,42$
- $bel_{1-2}r_C = m_{1-2}r_C = 0,52$
- $bel_{1-2}r_{TvZ} = m_{1-2}r_T + m_{1-2}r_Z + m_{1-2}r_{TvZ} = 0,42$
- $bel_{1-2}r_{ZvC} = m_{1-2}r_Z + m_{1-2}r_C + m_{1-2}r_{ZvC} = 0,52$
- $bel_{1-2}r_{TvC} = m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{TvC} = 0,94$
- $bel_{1-2}r_{TvUNA} = m_{1-2}r_T + m_{1-2}r_{UNA} + m_{1-2}r_{TvUNA} = 0,42$
- $bel_{1-2}r_{CvUNA} = m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{CvUNA} = 0,52$
- $bel_{1-2}r_{ZvTvC} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{ZvT} + m_{1-2}r_{ZvC} + m_{1-2}r_{TvC} + m_{1-2}r_{ZvTvC} = 0,42 + 0,52 + 0,054 = 0,994$
- $bel_{1-2}r_{ZvTvUNA} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_{UNA} + m_{1-2}r_{ZvT} + m_{1-2}r_{ZvUNA} + m_{1-2}r_{TvUNA} + m_{1-2}r_{ZvTvUNA} = 0,42$
- $bel_{1-2}r_{ZvCvUNA} = m_{1-2}r_Z + m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{ZvC} + m_{1-2}r_{ZvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{ZvCvUNA} = 0,52$
- $bel_{1-2}r_{TvCvUNA} = m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{TvCvUNA} = 0,42 + 0,52 + 0,054 = 0,994$
- $bel_{1-2}r_{ZvTvCvUNA} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvZ} + m_{1-2}r_{TvUNA} + m_{1-2}r_{ZvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,52 + 0,054 = 0,994$

- $pl_{1-2}r_T = m_{1-2}r_T + m_{1-2}r_{TvC} + m_{1-2}r_{TvZ} + m_{1-2}r_{TvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,054 = 0,474$
- $pl_{1-2}r_C = m_{1-2}r_C + m_{1-2}r_{TvC} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvCvUNA} = 0,52 + 0,054 = 0,574$
- $pl_{1-2}r_{TvZ} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_{TvC} + m_{1-2}r_{TvZ} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,054 = 0,574$
- $pl_{1-2}r_{ZvC} = m_{1-2}r_Z + m_{1-2}r_C + m_{1-2}r_{TvC} + m_{1-2}r_{TvZ} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,52 + 0,054 = 0,574$
- $pl_{1-2}r_{TvC} = m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{TvC} + m_{1-2}r_{TvZ} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,52 + 0,054 = 0,994$
- $pl_{1-2}r_{TvUNA} = m_{1-2}r_T + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvZ} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,054 = 0,474$
- $pl_{1-2}r_{CvUNA} = m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{TvZ} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,52 + 0,054 = 0,574$
- $pl_{1-2}r_{ZvTvC} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{TvZ} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,52 + 0,054 = 0,994$
- $pl_{1-2}r_{ZvTvUNA} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,054 = 0,474$
- $pl_{1-2}r_{ZvCvUNA} = m_{1-2}r_Z + m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,52 + 0,054 = 0,574$
- $pl_{1-2}r_{TvCvUNA} = m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,52 + 0,054 = 0,994$
- $pl_{1-2}r_{ZvTvCvUNA} = m_{1-2}r_Z + m_{1-2}r_T + m_{1-2}r_C + m_{1-2}r_{UNA} + m_{1-2}r_{TvC} + m_{1-2}r_{TvUNA} + m_{1-2}r_{CvUNA} + m_{1-2}r_{CvZ} + m_{1-2}r_{ZvCvUNA} + m_{1-2}r_{TvCvUNA} + m_{1-2}r_{ZvTvC} + m_{1-2}r_{ZvTvUNA} + m_{1-2}r_{ZvTvCvUNA} = 0,42 + 0,52 + 0,054 = 0,994$

*Table 6* shows the result of the DSRA analysis for the risk of failure of the considered water supply network. The highest values of the belief function $m_{1-2r}$ (when combined hypotheses from two experts) indicate the failure risk level.

Measures of belief $bel_{1-2}r$ and plausibility $pl_{1-2}r$ characterize the so-called the range of uncertainty as to the interpretation of proposed hypotheses. Uncertainty factor nP is the difference between belief and plausibility for the proposed hypothesis.

*Table 6*. The results of the analysis of the possible risk of failure of the water supply network

| Description | $m_{1-2}r$ | $bel_{1-2}r$ | $pl_{1-2}r$ | nP |
|---|---|---|---|---|
| Ø | 0 | 0 | 0 | 0 |
| Z | 0 | 0 | 0 | 0 |
| T | 0.42 | 0.42 | 0.474 | 0.054 |
| C | 0.52 | 0.52 | 0.574 | 0.054 |
| UNA | 0 | 0 | 0 | 0 |
| ZvT | 0 | 0.42 | 0.574 | 0.154 |
| TvC | 0.054 | 0.94 | 0.994 | 0.054 |
| ZvUNA | 0 | 0 | 0 | 0 |
| TvUNA | 0 | 0.42 | 0.474 | 0.054 |
| CvUNA | 0 | 0.52 | 0.574 | 0.054 |
| CvZ | 0 | 0.52 | 0.574 | 0.054 |
| ZvTvC | 0 | 0.994 | 0.994 | 0 |
| ZvCvUNA | 0 | 0.52 | 0.574 | 0.054 |
| TvCvUNA | 0 | 0.994 | 0.994 | 0 |
| ZvTvUNA | 0 | 0.42 | 0.474 | 0.054 |
| ZvTvCvUNA | 0 | 0.994 | 0.99 | 0 |

The highest value of the belief function was obtained for the hypothesis that describes the level of controlled risk for $m_{1-2rC}$ = 0,52 and for bel function $bel_{1-2rC}$ = 0,52.
The probability of plausibility of the truth of this hypothesis is $pl_{1-2rC}$ = 0.57. Uncertainty to the truth of this hypothesis is 0.05. Other high value of the belief function describes the hypothesis of tolerated risk, in such case it is recommended to conduct the analysis again involving other experts.

## 4. The method of analysis of the acceptance of costs of risk reduction by water consumers

The process of reducing the risk of failure in the water distribution system (WDS) requires some financial investment, which consequently has an impact on the price of drinking water, which should be accepted by water consumers [36].
The level of acceptance for expenses related to risk management in the water utility company depends on various factors such as the rate of quality of life, consumer awareness of water on the risks resulting from the lack of risk management procedures, the degree of confidence in the water supply company.
Acceptance index AI was proposed to assess costs incurred by the water company on the procedures for implementing the risk management methods. This ratio is calculated by multiplying Index of Water Quality Service (IWQS) and Quality of Life Index (QLI).
Quality of Life Index has been developed to reflect the standard of living and life satisfaction in each country.
The method combines the results of subjective life satisfaction surveys with objective factors of life quality. The level of life quality can be analysed, providing both economic indicators, eg. gross national product, income per capita income, social indicators and subjective characteristics.
It proposed
The appointment of *QLI* index based on the survey results on the quality of residents life was proposed. A five rating scale was adopted. Indicator *QLI* is determined according to the formula:

$$QLI = \frac{\sum_{j=1}^{n_{QLI}} w_{javg}}{n_{QLI}} \qquad (12)$$

where $w_{jav}$ is the average value of the numerical evaluation from 1 to 5 for given indicators of life quality, j = 1 - health care, 2 - safety, 3 - education, 4 – income, 5 – qualities of natural environment, etc. $n_{QLI}$ is the number of indicators taken into account in the assessment of the *QLI*.

The Index of Water Quality Service (*IWQS*) can be determined according to the proposed formula:

$$IWQS = P_{WQ} \cdot P_{AWP} \cdot P_C \qquad (13)$$

where $P_{WQ}$ is a parameter associated with assessing water quality by consumers, $P_{AWP}$ is a parameter associated with accepting water price by consumers, $P_C$ is a parameter associated with the degree of consumer confidence to the activities performed by the water company, including trust that tap water is safe for health.
The individual parameters are as follows (point weights):
- $P_{WQ}$, degree of assessment: big (3), average (2), small (1),
- $P_{AWP}$, degree of acceptance: good (3), average (2), poor (1),
- $P_C$, degree of confidence: high (3), average (2), low(1).

The IWQS takes values from 1 to 27, while the AI takes values from 1 to 135.
It has been proposed to describe the level of acceptance of the incurred costs according to the following scale: low from 1 to 25, average above 25 to 50, high above 50 to 135.
Assessment analysis takes into account the following rules.
If in given scale the value of *AI* is classified as high, it means that the costs incurred by the water company are accepted by water consumers.

If in the given scale the value of *AI* is classified as average, it means that the costs incurred by the water company are tolerated by water consumers. The company, however, should conduct information action in order to convince consumers as to the necessity of its undertakings.

If in the given scale the value of *AI* is classified as low, it means that the costs incurred by the water company are not accepted by water consumers, the company should verify its plan of works and make some corrections in the design phase.

## 5. Example of application of the analysis of the acceptance index

For each water company a priority action is to ensure the continuity of supply drinking water with appropriate quality. The ageing water distribution systems and the increasing quality requirements make it necessary to spend large amounts of money on the modernization of water supply system, which affects the water cost.

The process of communicating with water consumers, marketing and information are an integral part of the water company management, including risk management. Managing directors should know whether their measures to reduce the risk associated with water distribution are accepted by water consumers.

In order to assess the rate of acceptance the survey was conducted for assessing by the water consumers the quality of the water services and the life quality. Survey results were used to determine the value of the *AI*.

Conducted research belong to the regional research and are preliminary partial studies used to assess the parameters that are included in the *AI*. The choice of a representative random sample was made in accordance with the procedures described in the literature [36].

Respondents rated the quality of water services for:

- tap water quality in their place of residence,
- company operating water supply in their locality,
- accepting the water price in their locality,
- knowledge of the nearest alternative source of water in case of the lack of water or water pollution in the water supply network,
- confidence in the water supply company that tap water is safe,
- accepting the activities of water supply company such as modernization, extension and renovation of water supply system in their locality.

Some questions were also asked in order to obtain indicative information on consumer awareness of water supply in the city:

- experience of difficulties related to the lack of tap water,
- experience of symptoms associated with the consumption of tap water of inadequate quality,
- water saving,
- assessment of the quality of water resources in the region.

The survey results allow for the following conclusions:

- about 58% of the respondents assess the quality of tap water, the work of the water supply company and the quality of water resources well and very well,
- about 76% of the respondents did not have any difficulties due to the lack of water or its poor quality,
- more than 73% of the respondents declared that they save water,
- about 81% of the respondents did not know where there are alternative sources of water.

Questionnaire for the assessment of life quality related to:

- satisfaction with their income,
- assessment of the quality of health service in their locality,
- assessment of the quality of education and the access to it in their location,
- assessment of their own and their relatives safety in their locality,
- assessment of the natural environment in their region.

In order to determine the Index of Water Quality Service (*IWQS*) its various parameters were estimated.

About 76% of consumers assessed the quality of drinking water in the city ($P_{WQ}$) as well and very well. Based on it the degree of acceptance of the parameter was defined as good (point weight: 3).

Consumer acceptance of the water price ($P_{AWP}$) indicates that about 33% of the respondents approve the water price in the city and about 48% of the respondents do not accept it or rather not accept. It is assumed that the parameter A is at low level (point weight: 1).

Consumer confidence and communication with the water supply company is expressed by the $P_C$ index. From the study it can be concluded that about 76% of the respondents accept the activities performed by the water company very much. The survey results showed also that about 81% of the respondents do not have knowledge about water alternative sources in the city. Based on the results of the analyses it was assumed that the parameter Z takes the average value (point weight: 2). The *IWQS* calculated according to the formula 13 is 6.

To estimate the value of the *QLI* five parameters characterizing the quality of life were taken into account: health, safety, education, income, qualities of natural environment. Each parameter was evaluated by the respondents with the use of a five-point scale. The results were used to assign the individual parameters the following values: income (3.5), health (2.5), education (3.5), qualities of natural environment (4) and safety (4). On this basis, the QLI was calculated according to the formula 12, and it was 3.5.

After entering the appropriate values the acceptance index *AI* for costs incurred by the company to reduce risk was obtained, it is 45.5. This index is in the range of average which means that water consumers in the distinguished area tolerate the costs that the company incurs on modernization, protection and repair of water supply system in order to reduce the risk of failure.

The analysis showed that the water company should pay more attention to the necessity of informing consumers about existing risk and way of behaviour when it occurs, the possibilities of protection, crisis prevention, as well as the way of establishing water price and the necessity of losses reduction and water saving.

## 6. Conclusions

Reducing the probability of failure of the water supply system can be done by planning modernization projects, as well as the procedures of prevention, taking into account the active protection requiring operator supervision.

The DS theory gives the possibility to combine the opinions of various experts and consequently the risk assessment of failure in the WDS. The theory of mathematical evidence allows assigning each premise not one but two values.

Apart from modelling the uncertainty it makes possible to obtain a numerical value. The proposed method can be used when operating data are not sufficient for statistical and probabilistic analysis, but can be the basis (together with the experience and knowledge) for expert opinion regarding the level of failure risk.

The combining of the information contained in the two sets of experts opinions through two hypotheses gives the opportunity to update their knowledge, which has a beneficial effect on decision-making and failure risk management. The result is a new subset of possible hypotheses with the new values characterizing the possibility of individual risk categories. This process can be continued as long as the information is coming from the experts, as to obtain the most reliable results.

Application of the theory of mathematical evidence to failure risk analysis of water supply network should be used in the process of risk management, in particular based on the collection, verification and grouping data and hypotheses. The proposed method is an alternative to other methods of assessing and managing the risk of failure of water supply network and its use is justified in the case of having subjective assessments of risk parameters.

The method belongs to the group of experts methods and may also be a part of the decision- making process concerning modernization plans in the WDS.

The presented method of analysis of consumer acceptance of water costs that companies incur on the risk reduction, based on surveys, should be a part of proper company policy in the context of consultation with the local community.

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