Tchórzewska-Cieślak Barbara

Pietrucha-Urbanik Katarzyna

Szpak Dawid
Rzeszow University of Technology, Rzeszow, Poland

Development of cause-effect dependence model of undesirable events using Bayes network

Keywords

bayes network, risk, security

Abstract

In the paper the method of cause and effect analysis of undesirable events using the Bayesian networks is presented. For the analysis, due to the complexity of the calculations, it is proposed to use Java Bayes program as a free and simple tool to support Bayesian analysis. Bayesian estimation allows to identify the probability of the event occurrence. For this reason its use was proposed to determine the safety probability. Using Bayes' theorem is also possible to modify initial judgement about the situation with the use of a priori probability so that a new situation described by a posteriori probability arises. In this sense, by Bayes' theorem the data can be sequentially processed, including considerations for newer information, and thereby create a more reliable basis for decision making for the system operator. In the paper, the methodology was presented, which can be extended in order to improve the detection and monitoring of undesirable events in infrastructure.

1. Introduction

Technical progress and the development of civilization cause that the requirements for technical objects are increasing. Man as a user of these systems aims to ensure that the objects (systems) which he uses were more durable, reliable, safe, ergonomic and simple to operate [2], [11], [18].

Daily use of technical systems is inseparably linked with the possibility of the occurrence of various types of undesirable events [1], [5], [7]. Proper assessment of the reliability of the technical system should guarantee making the right decisions concerning the choice of the best solutions in terms of technical, economic and reliability aspects, at the stage of design, construction and operation [4], [19], [21] also in hazard and interconnection analysis in port [8], [9], [24].

Indicators and measures that can be used in the process of risk analysis of technical systems, in general, are divided into:

- statistical - determined in accordance with accepted principles of mathematical statistics based on historical data from the operating system,
- probabilistic - determined on the basis of the theory of probability,
- linguistic - describing the risk parameters by means of the so-called linguistic variables, expressed in natural language using words like small, medium, large.

Random nature of the formation of failure causes that related to it research is complex and is based primarily on the analysis of operational data and experts opinions. The idea of data exploration involves the use of information technology to find information in databases. There are many data exploration techniques derived directly from mathematical statistics and machine learning.

Analysis of the risks associated with the operation of technical systems requires a lot of detailed information on individual risk factors and their identification and the determination of losses that may occur as a result of the occurrence of undesirable events. Such an analysis is performed under conditions of uncertainty, caused undoubtedly by the complexity of the technical systems and individual components, the degree of dependence between them, as well as difficulties in obtaining the necessary information. The most effective form of knowledge about the uncertain environment is conditional independence which is described by the Bayes' formula.

The main aim of this work is the cause and effect analysis and assessment of undesirable events in technical systems, with particular emphasis on critical infrastructure, using Bayesian networks. Dependencies between individual events are expressed by means of conditional probabilities. The use of Bayesian networks allows determining the probability of the top event and sub-events in the network, which is a basic information for the evaluation of the system safety. The technical system should be monitored in terms of operating parameters and patrolled by teams of operating services. During repair and modernization unauthorized persons should not have access. The developed method can be a support tool for operators as to increase the security level and used to expand the system of monitoring and detection of undesirable events in the considered infrastructure. In the work, the example of application of the developed method was presented.

2. Theoretical basis of Bayes network

The Bayesian networks - BRA (Bayes Risk Analysis) are used in risk analysis due to the ability to model the dependent events. The Bayesian network is upgraded by means of experience and acquired knowledge. The network is modelled by a directed acyclic graph in which vertices represent events and edges represent causal connections between these events. In addition, the Bayesian network is not limited to two states: up state or down state (as in the event tree method and the fault tree method) and may be used for analysing the intermediate states [17], [20], [23].

The occurrence of the event X_j (cause) has some impact on the occurrence of the event X_i (effect). If the impact is not "certain" and can only be determined by the probability, then such an arrangement of events and relations between them can be modelled by a directed graph D [3], [6].

Each event is represented as a vertex of the graph. Relations between events are represented by edges. If the occurrence of the event X_j has some impact on the occurrence of the event X_i (X_i depends on X_j), then there is an edge (X_j, X_i) in the graph model, exiting the X_j and entering the X_i (direction is indicated by the arrow). The vertex X_j is called 'parent' of the vertex X_i. The set of all 'parents' of the vertex X is marked as $\pi(X)$. Figure 1 shows a general schematic diagram of Bayesian networks [22].

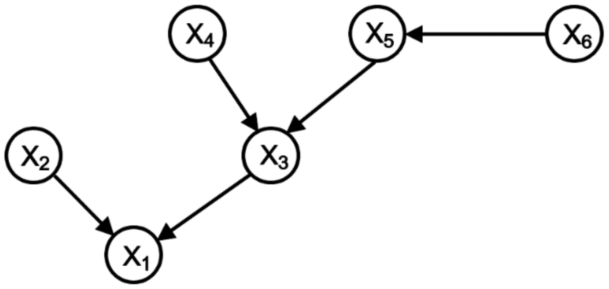

Figure 1. An example of Bayesian networks

For the graph D (*Figure 1*) the dependencies between the events are as follows: $\pi(X_1) = \{X_2, X_3\}$, $\pi(X_2) = \{\varnothing\}$, $\pi(X_3) = \{X_4, X_5\}$, $\pi(X_4) = \{\varnothing\}$, $\pi(X_5) = \{X_6\}$, $\pi(X_6) = \{\varnothing\}$.

The basic assumption in the Bayesian networks is independence of each vertex from the vertices which are not its parents, for example, X_1 is independent of X_4, X_5, X_6. Most often every event is identified with the corresponding random variable having the same name, on the assumption that all the random variables corresponding to the events are bivalent (1 - an event that occurs, 0 – an event opposed to the event that occurs).The relations between the vertices (events) are expressed by means of the conditional probability. For the vertex X, whose parents are in the set $\pi(X)$, these relations are represented by the conditional probability tables (CPT). In CPT, for the variable X, all the probabilities $P(X|\pi(X))$ (for all the possible combinations of variables from the set $\pi(X)$ must be specified. The table for the vertex that does not have parents includes the probabilities that the random variable X will take its particular values [10], [22]

The Bayes' theorem has the form:

$$P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B)}, \tag{1}$$

where $P(A)$ is a priori probability of the occurrence of event A, $P(B)$ is a priori probability of the occurrence of event B, $P(A/B)$ is a conditional probability of the occurrence of event A under the condition of the occurrence of event B. It is also called a posterior probability because it derives or depends on the value of B. $P(B/A)$ is a conditional probability of the occurrence of event B on condition of the occurrence of event A.

If the network has n vertices, X_1, ..., X_n, the total probability distribution of all the random variables is shown as the relation [6], [22]:

$$P(X_1,..., X_n) = \prod_{n=1}^{n} P(X_i|\pi(X_i)), \tag{2}$$

For the network in Figure 1, a combined probability distribution is as follows:

$$P(X_1, X_2, X_3, X_4, X_5, X_6) = P(X_1 \mid \mathrm{X}_2, \mathrm{X}_3) \cdot \mathrm{P}(\mathrm{X}_2) \cdot P(X_3 \mid \mathrm{X}_4, \mathrm{X}_5) \cdot \mathrm{P}(\mathrm{X}_4) \cdot P(X_5 \mid \mathrm{X}_6) \cdot \mathrm{P}(\mathrm{X}_6) \quad , \qquad (3)$$

To determine the total probability distribution without using the Bayesian network it is necessary to know all the values of $P(X_1, ..., X_n)$ for all the possible combinations of variables $X_1, ..., X_n$, which gives 2^n values of the probabilities. Using the Bayesian network it is sufficient to know the conditional probabilities for each vertex. With given values of its direct ancestors (parents) the total number of required values is given by the formula:

$$\mathrm{LP} = \sum_{i=1}^{n} 2^{|\pi(X_i)|}, \qquad (4)$$

where n is the number of vertices of the Bayesian network and $|\pi(X_i)|$ is the number of elements of the set $\pi\ (X_i)$.

3. Analysis of the risk of interference in the functioning of the seaport using bayesian networks

The Bayesian network can be used in decision-making model for risk analysis of interference of the complex technical systems. In this work the risk analysis model that can be used in making decisions by the seaport companies (concerning the modernization or renovation), is presented. The model was introduced to the program JavaBayes.

Figure 2 shows the developed Bayesian network diagram that can be used to analyse the risk of interference of the sea port.

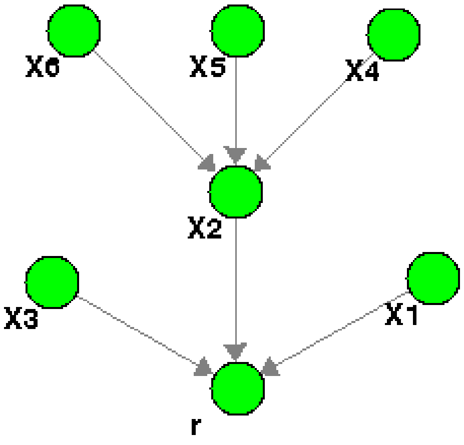

Figure 2. Bayesian networks for analysis of the risk of interference of the sea port

Symbols used in *Figure 1* and *2* mean :

- r – the risk of interference in the operation of the seaport in the five-point scale:
  - neglected risk $r = r_Z$,
  - tolerable risk $r = r_T$,
  - controlled risk $r = r_K$,
  - intolerable risk $r = r_{NT}$,
  - unacceptable risk $r = r_{NA}$,
- X_1 – ship collision with the port construction
- X_2 – seaport fault
  - X_4 – technical failure,
  - X_5 – failure of the control system,
  - X_6 – operator error,
- X_3 – seaport protection against existing threat
  - very little – x_{31},
  - little – x_{32},
  - medium – x_{33},
  - large – x_{34},
  - very large – x_{35}.

In the study it was assumed that the event in the given node takes exactly one of the possible values:

- 1 – event occurs,
- 0 – event does not occur.

For each of the vertices belonging to the developed Bayesian network the CPT is defined: $P(r \,/X_1, X_2, X_3)$, $P(X_1)$, $P(X_2 \,/X_4, X_5, X_6)$, $P(X_3)$, $P(X_4)$, $P(X_5)$, $P(X_6)$. Using the formula (2) the values of probability for each risk category, ie. $P(r = r_Z, r_T, r_K, r_{NT}, r_{NA})$ were calculated. Aggregation is performed according to the general formula:

$$P(r = r_Z, r_T, r_K, r_{NT}, r_{NA}) = P\left(r = r_i \,\middle|\, X_1 = X_j, X_2 = X_k, X_3 = X_l\right) \sum \cdot P\left(X_1 = X_j\right) \cdot P(X_2 = X_k) \cdot P(X_3 = X_l), \qquad (5)$$

where r_i is a risk value, $i = r_Z, r_T, r_K, r_{NT}, r_{NA}$, X_j is the occurrence or lack of occurrence of the event X_1; $j = 1,0$, X_k is occurrence or lack of occurrence of the event X_2; $j = 1,0$, and X_l is a given value of the event X_3; $j = \mathrm{x}_{31}, \mathrm{x}_{32}, \mathrm{x}_{33}, \mathrm{x}_{34}, \mathrm{x}_{35}$.

The individual probability values are:

- The probability of the occurrence of event X_2

$P(X_2=1)=$
$=P(X_2=1 \,/X_4=1,X_5=1,X_6=1) \cdot P(X_4=1) \cdot P(X_5=1) \cdot P(X_6=1)$
$+P(X_2=1 \,/X_4=1,X_5=1,X_6=0) \cdot P(X_4=1) \cdot P(X_5=1) \cdot P(X_6=0)$
$+P(X_2=1 \,/X_4=1,X_5=0,X_6=0) \cdot P(X_4=1) \cdot P(X_5=0) \cdot P(X_6=0)$
$+ P(X_2=1 \,/X_4=0, X_5=0,X_6=0) \cdot P(X_4=0) \cdot P(X_5=0) \cdot (X_6=0)$
$+P(X_2=1 \,/X_4=0,X_5=1,X_6=0) \cdot P(X_4=0) \cdot P(X_5=1) \cdot P(X_6=0)$
$+P(X_2=1 \,/X_4=1,X_5=0,X_6=1) \cdot P(X_4=1) \cdot P(X_5=0) \cdot P(X_6=1)$
$+P(X_2=1 \,/X_4=0,X_5=0,X_6=1) \cdot P(X_4=0) \cdot P(X_5=0) \cdot P(X_6=1)$
$+P(X_2=1 \,/X_4=0,X_5=1,X_6=1) \cdot P(X_4=0) \cdot P(X_5=1) \cdot P(X_6=1)$

- The probability that the risk is at the neglected level

$P(r = r_Z) =$
$=P(r=r_Z /X_1=1,X_2=1,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_Z /X_1=1,X_2=1,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+ P(r=r_Z /X_1=1,X_2=1,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_Z /X_1=1,X_2=1,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3= x_{34})$
$+P(r=r_Z /X_1=1,X_2=1,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_Z /X_1=1,X_2=0,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_Z /X_1=1,X_2=0,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_Z /X_1=1,X_2=0,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
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$+P(r=r_Z /X_1=1,X_2=0,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{35})$
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$+P(r=r_Z /X_1=0,X_2=1,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_Z /X_1=0,X_2=1,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_Z /X_1=0,X_2=0,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_Z /X_1=0,X_2=0,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_Z /X_1=0,X_2=0,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_Z /X_1=0,X_2=0,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_Z /X_1=0,X_2=0,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{35})$

- The probability that the risk is at the tolerable level

$P(r = r_T) =$
$=P(r=r_T /X_1=1,X_2=1,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_T /X_1=1,X_2=1,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_T /X_1=1,X_2=1,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_T /X_1=1,X_2=1,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_T /X_1=1,X_2=1,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_T /X_1=1,X_2=0,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_T /X_1=1,X_2=0,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_T /X_1=1,X_2=0,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_T /X_1=1,X_2=0,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_T /X_1=1,X_2=0,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{35})$
$+P(r=r_T /X_1=0,X_2=1,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_T /X_1=0,X_2=1,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_T /X_1=0,X_2=1,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_T /X_1=0,X_2=1,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_T /X_1=0,X_2=1,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_T /X_1=0,X_2=0,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_T /X_1=0,X_2=0,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_T /X_1=0,X_2=0,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_T /X_1=0,X_2=0,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_T /X_1=0,X_2=0,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{35})$

- The probability that the risk is at the controlled level

$P(r = r_K) =$
$=P(r=r_K /X_1=1,X_2=1,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_K /X_1=1,X_2=1,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_K /X_1=1,X_2=1,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_K /X_1=1,X_2=1,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_K /X_1=1,X_2=1,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_K /X_1=1,X_2=0,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3= x_{31})$
$+P(r=r_K /X_1=1,X_2=0,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_K /X_1=1,X_2=0,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
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$+P(r=r_K /X_1=1,X_2=0,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{35})$
$+P(r=r_K /X_1=0,X_2=1,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
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$+P(r=r_K /X_1=0,X_2=1,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_K /X_1=0,X_2=0,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_K /X_1=0,X_2=0,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_K /X_1=0,X_2=0,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_K /X_1=0,X_2=0,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_K /X_1=0,X_2=0,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{35})$

- The probability that the risk is at the intolerable level

$P(r = r_{NT}) =$
$=P(r=r_{NT} /X_1=1,X_2=1,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_{NT} /X_1=1,X_2=1,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_{NT} /X_1=1,X_2=1,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_{NT} /X_1=1,X_2=1,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_{NT} /X_1=1,X_2=1,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_{NT} /X_1=1,X_2=0,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_{NT} /X_1=1,X_2=0,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_{NT} /X_1=1,X_2=0,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_{NT} /X_1=1,X_2=0,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_{NT} /X_1=1,X_2=0,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{35})$
$+P(r=r_{NT} /X_1=0,X_2=1,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_{NT} /X_1=0,X_2=1,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_{NT} /X_1=0,X_2=1,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_{NT} /X_1=0,X_2=1,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_{NT} /X_1=0,X_2=1,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_{NT} /X_1=0,X_2=0,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_{NT} /X_1=0,X_2=0,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_{NT} /X_1=0,X_2=0,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_{NT} /X_1=0,X_2=0,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_{NT} /X_1=0,X_2=0,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{35})$

- The probability that the risk is at the unacceptable level

$P(r = r_{NA}) =$
$=P(r=r_{NA} /X_1=1,X_2=1,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_{NA} /X_1=1,X_2=1,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_{NA} /X_1=1,X_2=1,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_{NA} /X_1=1,X_2=1,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_{NA} /X_1=1,X_2=1,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_{NA} /X_1=1,X_2=0,X_3=x_{31})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3= x_{31})$
$+P(r=r_{NA} /X_1=1,X_2=0,X_3=x_{32})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_{NA} /X_1=1,X_2=0,X_3=x_{33})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_{NA} /X_1=1,X_2=0,X_3=x_{34})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_{NA} /X_1=1,X_2=0,X_3=x_{35})\cdot P(X_1=1)\cdot P(X_2=0)\cdot P(X_3=x_{35})$
$+P(r=r_{NA} /X_1=0,X_2=1,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{31})$
$+P(r=r_{NA} /X_1=0,X_2=1,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{32})$
$+P(r=r_{NA} /X_1=0,X_2=1,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{33})$
$+P(r=r_{NA} /X_1=0,X_2=1,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{34})$
$+P(r=r_{NA} /X_1=0,X_2=1,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=1)\cdot P(X_3=x_{35})$
$+P(r=r_{NA} /X_1=0,X_2=0,X_3=x_{31})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{31})$
$+P(r=r_{NA} /X_1=0,X_2=0,X_3=x_{32})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{32})$
$+P(r=r_{NA} /X_1=0,X_2=0,X_3=x_{33})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{33})$
$+P(r=r_{NA} /X_1=0,X_2=0,X_3=x_{34})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{34})$
$+P(r=r_{NA} /X_1=0,X_2=0,X_3=x_{35})\cdot P(X_1=0)\cdot P(X_2=0)\cdot P(X_3=x_{35})$

The model allows to determine the probability of the particular risk level. The result of modelling are the probability values for each risk level. The risk assessment is based on the interpretation of the result (application of risk with the highest and lowest probability of occurrence).

The developed model enables also determining the partial probabilities for the events included in the defined Bayesian network.

The model may be modified or extended depending on the specifics of the analysed technical system.

4. Example of application

For risk analysis of disruption in the seaport functioning the model using Bayesian networks (Bayes Risk Analysis – BRA), developed in step 3, was used.

Calculations were performed using JavaBayes program, to which the developed model was introduced. For each of the vertices of the Bayesian network shown in *Figure 2* the conditional probability tables are defined, which are presented in *Tables 1-4.*

Table 1. Table of conditional probability for *r*

X_1	X_2	X_3					$P(r \mid X_1, X_2, X_3)$				
		x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	r_Z	r_T	r_K	r_{NT}	r_{NA}
1	1	1	0	0	0	0	0,05	0,05	0,2	0,3	0,4
1	1	0	1	0	0	0	0,1	0,1	0,3	0,3	0,2
1	1	0	0	1	0	0	0,2	0,2	0,3	0,2	0,1
1	1	0	0	0	1	0	0,3	0,4	0,2	0,1	0
1	1	0	0	0	0	1	0,4	0,3	0,3	0	0
1	0	1	0	0	0	0	0,1	0,1	0,3	0,3	0,2
1	0	0	1	0	0	0	0,2	0,2	0,3	0,2	0,1
1	0	0	0	1	0	0	0,3	0,4	0,2	0,1	0
1	0	0	0	0	1	0	0,4	0,3	0,3	0	0
1	0	0	0	0	0	1	0,5	0,4	0,1	0	0
0	1	1	0	0	0	0	0,1	0,1	0,3	0,3	0,2
0	1	0	1	0	0	0	0,2	0,2	0,3	0,2	0,1
0	1	0	0	1	0	0	0,3	0,4	0,2	0,1	0
0	1	0	0	0	1	0	0,4	0,3	0,3	0	0
0	1	0	0	0	0	1	0,5	0,4	0,1	0	0
0	0	1	0	0	0	0	0,6	0,4	0	0	0
0	0	0	1	0	0	0	0,65	0,35	0	0	0
0	0	0	0	1	0	0	0,7	0,3	0	0	0
0	0	0	0	0	1	0	0,8	0,2	0	0	0
0	0	0	0	0	0	1	0,9	0,1	0	0	0

Table 2. Table of conditional probability for X_2

$P(X_2 \mid X_4, X_5, X_6)$	X_4	X_5	X_6
0,01	0	0	0
0,25	1	0	0
0,20	0	1	0
0,10	0	0	1
0,60	1	1	0
0,50	1	0	1
0,50	0	1	1
0,80	1	1	1

Table 3. Table of conditional probability for dla X_1, X_4, X_5, X_6

P(X)	X_1	X_4	X_5	X_6
P(X = 1)	0,01	0,01	0,01	0,01
P(X = 0)	0,99	0,99	0,99	0,99

Table 4. Table of conditional probability for X_3

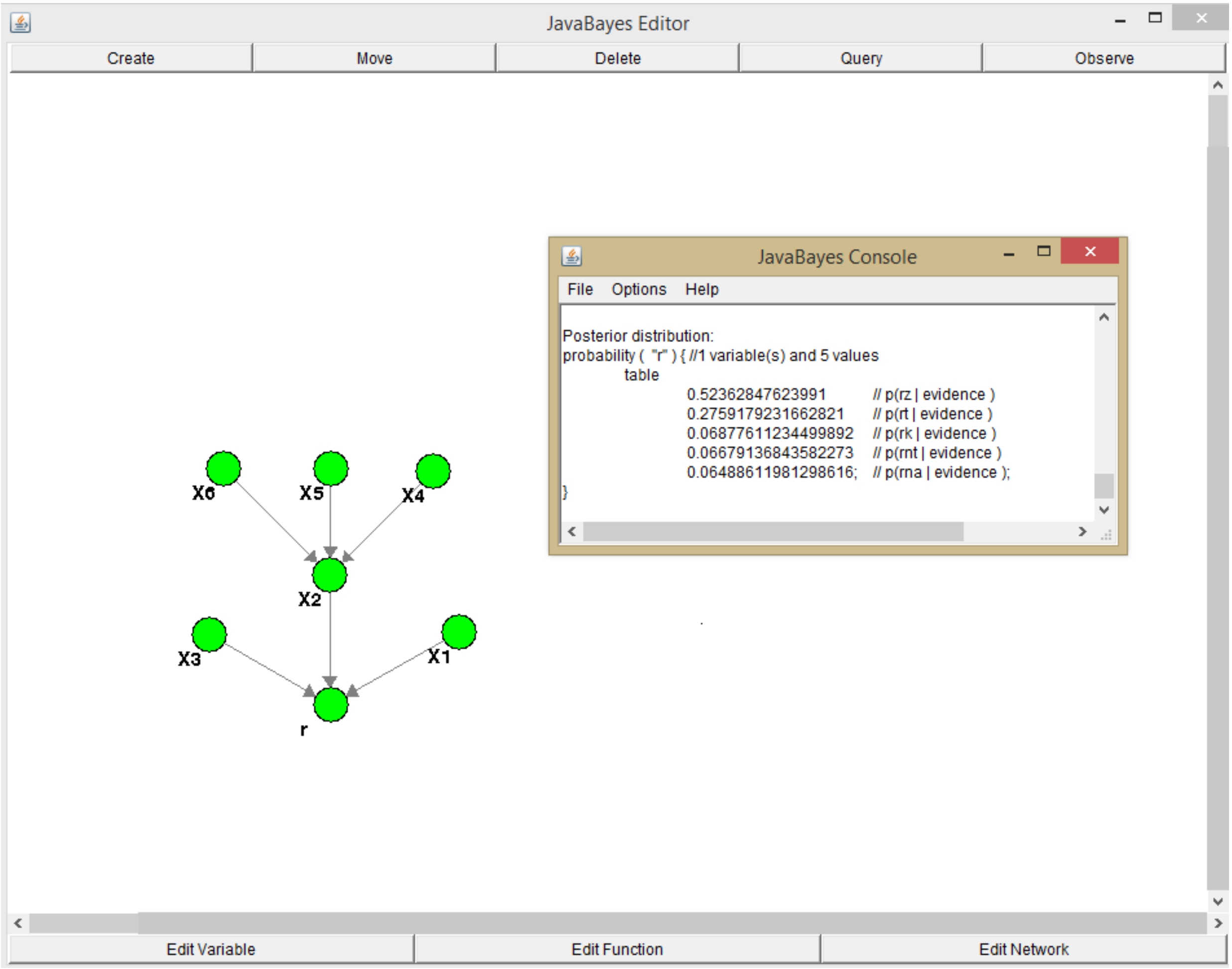
X_3	$P(X_3 = 1)$
x_{31}	0,3
x_{32}	0,4
x_{33}	0,24
x_{34}	0,05
x_{35}	0,01

Figure 3 shows the image generated from the program JavaBayes showing the results of analysis. In addition, the calculation results are shown in Table 5.

Table 5. Results of the analysis of disruption in the seaport functioning with the use of Bayesian networks

Risk level	Probability of given risk level	Risk assessment
Neglected	0,5236	Neglected
Tolerable	0,2759	
Controlled	0,0688	
Intolerable	0,0668	
Unacceptable	0,0649	

For the assumptions, the analysis of risk of disruption in the seaport functioning showed that the risk is at a negligible level The program also allows determining the probability of intermediate events in the Bayesian network.



```
Posterior distribution:
probability (  "r" ) { //1 variable(s) and 5 values
        table
                0.52362847623991        // p(rz | evidence )
                0.2759179231662821      // p(rt | evidence )
                0.06877611234499892     // p(rk | evidence )
                0.06679136843582273     // p(rnt | evidence )
                0.06488611981298616;    // p(rna | evidence );
}
```

Figure 3. The graphical environment of JavaBayes program - the results of the risk analysis

5. Conlusions

In the analysis and assessment of the risk of disturbance in the sea port functioning even the least likely events should be taken into account because they can cause disastrous consequences. For the safety of seaports the most important is the continuity of their operation.

Methods derived from Bayes' theorem are the statistical inference methods that allow to combine information from the generalized data with current information obtained from current research.

The proposed method of risk analysis using the Bayesian network is used primarily in the decision-making processes. The Bayesian network shows cause-and-effect dependencies between events.

Using the developed method one obtains the information as to what level of risk (in the adopted scale) occurs and with what probability. In this way, the proposed model can be an important element in the decision-making process for the operators of the sea port. The model can be modified for all the elements of the sea port. Its use should be part of the safety management and decision-making on exploitation and modernization.

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