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Integrated Impact Model on Global Baltic Network of Critical Infrastructure Networks Safety Related to Its Operation Process

Keywords

Safety, Global Baltic Network, Critical Infrastructure Network, Network of Networks

Abstract

The purpose of the paper is to present the impact model for Global Baltic Network of Critical Infrastructure Networks (GBNCIN), related to its operation process. At first, basic issues regarding the GBNCIN operation at variable conditions have been described. Then, aspects regarding safety of the multistate GBNCIN at Variable operation conditions have been pointed. Finally, safety characteristics of multistate GBNCIN, consisting of exponential BCIN Networks at variable operation conditions are presented.

1. Introduction

Integrated impact model of Global Baltic Network of Critical Infrastructure Networks (GBNCIN) safety related to its operation process, is the continuation of works processed within the report [EU-CIRCLE Report D3.3-GMU11, 2016], that specified the GBNCIN operation process and safety model. The models were developed basing on outcomes of the report [EU-CIRCLE Report D1.2-GMU1, 2016], that analysed nature of some critical infrastructures operating within the Baltic Sea area, their interconnections and interdependencies, resulting with distinguishing certain critical infrastructure networks, defined as a set of interconnected and interdependent critical infrastructures, interacting directly and indirectly at various levels of their complexity and operating activity [EU-CIRCLE Report D1.1. EU-CIRCLE Taxonomy, 2015]. The networks have been abbreviated as the Baltic Critical Infrastructure Networks (BCIN). Consequently, distinguished networks, operating within the Baltic Sea area, interacting, and being also interconnected and interdependent, were classified as the Global Baltic Network of Critical Infrastructure Networks. The GBNCIN operation process model was defined basing on the operation process of critical infrastructure, critical infrastructure network, and

their parameters. The parameters of the GBNCIN operation process, specified in the report [EU-CIRCLE Report D3.3-GMU11, 2016], were: the vector of probabilities of the process staying at initial operation states, the matrix of probabilities of the process transitions between the operation states, and the matrices of conditional distribution and density functions of the process conditional sojourn times at the operation states.

The GBNCIN safety model was developed with the use of the multi-state approach [Amari, 1997], [Aven, 1985, 1999, 1993], [Barlow, Wu, 1978], [Brunelle, Kapur, 1999], [Hudson, Kapur, 1982, 1985], [Lisnianski, Levitin, 2003], [Natvig, 1982], [Ohio, Nishida, 1984], [Xue, 1985], [Xue, Yang, 1995a,b], [Yu et al, 1994], [Kołowrocki, Soszyńska-Budny, 2011], with the assumption that each particular network is composed of multi-state assets [EU-CIRCLE Report D3.1-GMU4], with safety states degrading in time [Guze, Kołowrocki, 2008], [Kołowrocki, 2004, 2014], [Kołowrocki, Soszyńska-Budny, 2011], [Xue, 1985], [Xue, Yang 1995 a, b], that gave the possibility to precise analysing of their safety and operational processes' effectiveness. This assumption allowed to distinguish a network safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Then, an

important network safety characteristic specified were: the time to the moment of exceeding its safety critical state and its distribution, which was called the network risk function. This distribution was strictly related to the safety function that are basic characteristics of the multi-state network. Then, the multistate asset and the multistate network main safety characteristics, i.e. their mean values of the lifetimes and in the safety state subsets and in the particular safety states and standard deviations and the moment when the network risk function exceeds a fixed permitted level, were determined.

The integrated impact model of Global Baltic Network of Critical Infrastructure Networks safety related to its operation process, is linking the GBNCIN safety and operation process models, taking into account its variable safety structure at different operation states, and particular BCIN safety parameters. The model introduces additional safety indices, typical for the critical infrastructure related to its varying in time safety structures and its components' safety parameters caused by its operation process, extending previous models with the set of safety indicators by the assets, BCIN and GBNCIN conditional intensities of ageing at particular operation states, and conditional and unconditional coefficients of the operation process impact on intensities of ageing.

2. The GBNCIN Operation at Variable Conditions

We assume that the Global Baltic Network of Critical Infrastructure Networks (*GBNCIN*), during its operation process is taking $t = \nu^{(1)} \cdot \nu^{(2)} \cdot \dots \cdot \nu^{(8)}$, different operation states z_1, z_2, \dots, z_i , of the form $[z^{(1)}, z^{(2)}, \dots, z^{(8)}]$, where

$$z^{(i)} \in \{z_1^{(i)}, z_2^{(i)}, \dots, z_{\nu^{(i)}}^{(i)}\}, i = 1, 2, \dots, 8,$$

are particular operation states of particular networks $BCIN^{(i)}, i = 1, 2, \dots, 8$, and $\nu^{(i)}, i = 1, 2, \dots, 8$, are the numbers of operation states of those particular networks respectively.

Further, it was assumed in the above report that the *GBNCIN* operation process $Z_{GBNCIN}(t)$, $t \in \langle 0, +\infty \rangle$, has operation states from the set $\{z_1, z_2, \dots, z_i\}$.

Thus, we denote particular operation states of the *GBNCIN* operation process $Z_{GBNCIN}(t)$, $t \in \langle 0, +\infty \rangle$, by

$$z_{j_1 j_2 \dots j_8}, j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8,$$

and understand them according to the following relationship

$$(Z_{GBNCIN}(t) = z_{j_1 j_2 \dots j_8}) \Leftrightarrow$$

$$(Z^{(1)}(t) = z_{j_1}^{(1)} \wedge Z^{(2)}(t) = z_{j_2}^{(2)} \wedge \dots$$

$$\wedge Z^{(8)}(t) = z_{j_8}^{(8)}), j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8,$$

where $z_{j_1}^{(1)}, z_{j_2}^{(2)}, \dots, z_{j_8}^{(8)}, j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8$, are particular operation states of single operation processes

$$Z^{(1)}(t), Z^{(2)}(t), \dots, Z^{(8)}(t), t \in \langle 0, +\infty \rangle,$$

of particular *BCIN* networks.

Moreover, it was assumed, that the *GBNCIN* Network operation process $Z_{GBNCIN}(t)$ is a semi-Markov process [Grabski, 2014], [Habibullah et al, 2009], [Kołowrocki, Soszyńska-Budny, 2011] with the conditional sojourn times $\theta_{j_1 j_2 \dots j_8, k_1 k_2 \dots k_8}^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8$, at the operation states $z_{j_1 j_2 \dots j_8}$, when its next operation state is $z_{k_1 k_2 \dots k_8}$, $j_i, k_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8, j_1 j_2 \dots j_8 \neq k_1 k_2 \dots k_8$.

Under these assumptions, the *GBNCIN* operation process may be described by:

– the vector

$$[p_{j_1 j_2 \dots j_8}^{GBNCIN}(0)]_{1 \times \nu}, t = \nu^{(1)} \cdot \nu^{(2)} \cdot \dots \cdot \nu^{(8)},$$

of the initial probabilities

$$p_{j_1 j_2 \dots j_8}^{GBNCIN}(0) = P(Z_{GBNCIN}(0) = z_{j_1 j_2 \dots j_8}),$$

$$j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8,$$

of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ staying at particular operation states at the moment $t = 0$;

– the matrix

$$[P_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}]_{1 \times 1}, \quad t = \nu^{(1)} \cdot \nu^{(2)} \cdot \dots \cdot \nu^{(8)},$$

of probabilities

$$p_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}, \quad j_i, k_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \\ j_1 j_2 \dots j_8 \neq k_1 k_2 \dots k_8,$$

of the GBNCIN Network operation process $Z_{GBNCIN}(t)$ transitions between the operation states

$z_{j_1 j_2 \dots j_8}$, and $z_{k_1 k_2 \dots k_8}$;

– the matrix

$$[H_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}(t)]_{1 \times 1}, \quad t = \nu^{(1)} \cdot \nu^{(2)} \cdot \dots \cdot \nu^{(8)},$$

of conditional distribution functions

$$H_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}(t) = P(\theta_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN} < t), \quad t \geq 0, \\ j_i, k_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad j_1 j_2 \dots j_8 \neq k_1 k_2 \dots k_8,$$

of the GBNCIN operation process $Z_{GBNCIN}(t)$ conditional sojourn times $\theta_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}$ at the operation states.

It is well known that the mean values $E[\theta_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}]$ of the conditional sojourn times $\theta_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}$ are given by

$$M_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN} = E[\theta_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}] \\ = \int_0^{\infty} t dH_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}(t), \quad j_i k_i \in \{1, 2, \dots, \nu^{(i)}\}, \\ i = 1, 2, \dots, 8, \quad j_1 j_2 \dots j_8 \neq k_1 k_2 \dots k_8. \quad (1)$$

Then, from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $\theta_{j_1 j_2 \dots j_8}^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, of the GBNCIN operation process $Z_{GBNCIN}(t)$ at the operation states $z_{j_1 j_2 \dots j_8}$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$H_{j_1 j_2 \dots j_8}^{GBNCIN}(t) = \sum_{k_1 k_2 \dots k_8=1}^l p_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN} H_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}(t), \\ t \geq 0, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (2)$$

Hence and from (1), the mean values $E[\theta_{j_1 j_2 \dots j_8}^{GBNCIN}]$ of the GBNCIN operation process $Z_{GBNCIN}(t)$

unconditional sojourn times $\theta_{j_1 j_2 \dots j_8}^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, at the operation states are given by

$$M_{j_1 j_2 \dots j_8}^{GBNCIN} = E[\theta_{j_1 j_2 \dots j_8}^{GBNCIN}] = \\ \sum_{k_1 k_2 \dots k_8=1}^l p_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN} M_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}, \\ j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (3)$$

where $M_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}$ are defined by the formula (1).

The limit values of the GBNCIN operation process $Z_{GBNCIN}(t)$ transient probabilities at the particular operation states

$$p_{j_1 j_2 \dots j_8}^{GBNCIN}(t) = P(Z_{GBNCIN}(t) = z_{j_1 j_2 \dots j_8}), \quad t \in (-\infty, +\infty), \\ j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8,$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p_{j_1 j_2 \dots j_8}^{GBNCIN} = \lim_{t \rightarrow \infty} p_{j_1 j_2 \dots j_8}^{GBNCIN}(t) = \frac{\pi_{j_1 j_2 \dots j_8}^{GBNCIN} M_{j_1 j_2 \dots j_8}^{GBNCIN}}{\sum_{k_1 k_2 \dots k_8=1}^l \pi_{k_1 k_2 \dots k_8}^{GBNCIN} M_{k_1 k_2 \dots k_8}^{GBNCIN}}, \\ j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (4)$$

where $M_{j_1 j_2 \dots j_8}^{GBNCIN}$ are given by (3), while the steady probabilities $\pi_{j_1 j_2 \dots j_8}^{GBNCIN}$ of the vector $[\pi_{j_1 j_2 \dots j_8}^{GBNCIN}]_{1 \times 1}$ satisfy the system of equations

$$\begin{cases} [\pi_{j_1 j_2 \dots j_8}^{GBNCIN}] = [\pi_{j_1 j_2 \dots j_8}^{GBNCIN}] [p_{j_1 j_2 \dots j_8 \quad k_1 k_2 \dots k_8}^{GBNCIN}] \\ \sum_{k_1 k_2 \dots k_8=1}^l \pi_{k_1 k_2 \dots k_8}^{GBNCIN} = 1 \end{cases} \quad (5)$$

In the case of a periodic GBNCIN operation process, the limit transient probabilities $p_{j_1 j_2 \dots j_8}^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, at the operation states given by (4), are the long term proportions of the GBNCIN operation process $Z_{GBNCIN}(t)$ sojourn times at the particular operation states $z_{j_1 j_2 \dots j_8}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$.

Other interesting characteristics of the GBNCIN operation process $Z_{GBNCIN}(t)$ possible to obtain are its total sojourn times $\hat{\theta}_{j_1 j_2 \dots j_8}^{GBNCIN}$ at the particular operation states $z_{j_1 j_2 \dots j_8}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, during

the fixed *GBNCIN* operation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the *GBNCIN* operation process total sojourn times $\hat{\theta}_{j_1 j_2 \dots j_8}^{GBNCIN}$ at the particular operation states $z_{j_1 j_2 \dots j_8}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, for sufficiently large operation time θ have approximately normal distributions with the expected value given by

$$\hat{M}_{j_1 j_2 \dots j_8}^{GBNCIN} = E[\hat{\theta}_{j_1 j_2 \dots j_8}^{GBNCIN}] = p_{j_1 j_2 \dots j_8}^{GBNCIN} \theta, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (6)$$

where $p_{j_1 j_2 \dots j_8}^{GBNCIN}$ are given by (4).

3. Safety of Multistate GBNCIN at Variable Operation Conditions

It is assumed that the changes of the operation states of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ have an influence on particular *BCIN* networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, safety and the *GBNCIN* safety structure as well. Consequently, we denote the *BCIN* network E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, conditional lifetime in the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$, $u = 1, 2, \dots, z^{GBNCIN}$, while the *GBNCIN* is at the operation state $z_{j_1 j_2 \dots j_8}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, by

$$[T_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u), \quad u = 1, 2, \dots, z^{GBNCIN}, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8,$$

and its conditional safety function by the vector

$$[S_i^{GBNCIN}(t, \cdot)]^{(j_1 j_2 \dots j_8)} = [1, [S_i^{GBNCIN}(t, 1)]^{(j_1 j_2 \dots j_8)}, \dots, [S_i^{GBNCIN}(t, z^{GBNCIN})]^{(j_1 j_2 \dots j_8)}], \quad t \in \langle 0, \infty \rangle, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (7)$$

with the coordinates defined by

$$[S_i^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)} = P([T_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u) > t | Z^{GBNCIN}(t) = z_{j_1 j_2 \dots j_8}), \quad (8)$$

for $t \in \langle 0, \infty \rangle$, $u = 1, 2, \dots, z^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$,

The safety function $[S_i^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)}$ is the conditional probability that the *BCIN* network

E_i^{GBNCIN} lifetime $[T_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u)$ in the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$, is greater than t , while the *GBNCIN* operation process $Z_{GBNCIN}(t)$ is at the operation state $z_{j_1 j_2 \dots j_8}$.

In the case, when the *BCIN* networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, at the *GBNCIN* operation process $Z_{GBNCIN}(t)$ states $z_{j_1 j_2 \dots j_8}$ have the exponential safety functions, the coordinates of the vector (7) are given by

$$[S_i^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)} = P([T_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u) > t | Z^{GBNCIN}(t) = z_{j_1 j_2 \dots j_8}) = \exp[-[\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 \dots j_8)} t], \quad t \in \langle 0, \infty \rangle, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (9)$$

The intensities of ageing/degradation of the *BCIN* networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, given by (9) - the intensities of the *BCIN* networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, departure from the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$, at the *GBNCIN* operation states $z_{j_1 j_2 \dots j_8}$, i.e. the coordinates of the vector

$$[\lambda_i^{GBNCIN}(\cdot)]^{(j_1 j_2 \dots j_8)} = [0, [\lambda_i^{GBNCIN}(1)]^{(j_1 j_2 \dots j_8)}, \dots, [\lambda_i^{GBNCIN}(z^{GBNCIN})]^{(j_1 j_2 \dots j_8)}], \quad t \in \langle 0, +\infty \rangle, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (10)$$

are given by

$$[\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 \dots j_8)} = [\rho_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u) \cdot \lambda_i^{GBNCIN}(u), \quad u = 1, 2, \dots, z^{GBNCIN}, \quad j_i \in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \quad (11)$$

where $\lambda_i^{GBNCIN}(u)$ are the intensities of ageing of the *BCIN* networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, (the intensities of the *BCIN* networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, departure from the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$), without operation process impact, i.e. the coordinate of the vector

$$[\lambda_i^{GBNCIN}(\cdot)]^{(j_1 j_2 \dots j_8)} = [0, \lambda_i^{GBNCIN}(1), \dots, \lambda_i^{GBNCIN}(z^{GBNCIN})], \quad i = 1, 2, \dots, 8, \quad (12)$$

and

$$[\rho_i^{GBNCIN}(u)]^{(j_1 j_2 \dots j_8)}, u = 1, 2, \dots, z^{GBNCIN},$$

$$j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8, \quad (13)$$

are the coefficients of operation impact on the BCIN networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, intensities of ageing (the coefficients of operation impact on BCIN networks E_i^{GBNCIN} , $i = 1, 2, \dots, 8$, intensities of departure from the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$), at the GBNCIN operation states $z_{j_1 j_2 \dots j_8}$, i.e. the coordinate of the vector

$$[\rho_i^{GBNCIN}(\cdot)]^{(j_1 j_2 \dots j_8)} =$$

$$[0, [\rho_i^{GBNCIN}(1)]^{(j_1 j_2 \dots j_8)}, \dots, [\rho_i^{GBNCIN}(z^{GBNCIN})]^{(j_1 j_2 \dots j_8)}],$$

$$j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8, \quad (14)$$

The BCIN network safety function (7), the BCIN network intensities of ageing (10) and the coefficients of the operation impact on the BCIN network intensities of ageing (14), are main BCIN network safety indices.

Similarly, we denote the GBNCIN conditional lifetime in the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$, while the GBNCIN is at the operation state $z_{j_1 j_2 \dots j_8}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, by $T_{GBNCIN}^{(j_1 j_2 \dots j_8)}(u)$, and the conditional safety function of the GBNCIN Network by the vector

$$[S^{GBNCIN}(t, \cdot)]^{(j_1 j_2 \dots j_8)} = [1, [S^{GBNCIN}(t, 1)]^{(j_1 j_2 \dots j_8)}, \dots,$$

$$[S^{GBNCIN}(t, z^{GBNCIN})]^{(j_1 j_2 \dots j_8)}], j_i \in \{1, 2, \dots, \nu^{(i)}\},$$

$$i = 1, 2, \dots, 8, \quad (15)$$

with the coordinates defined by

$$[S^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)} =$$

$$= P(T_{GBNCIN}^{(j_1 j_2 \dots j_8)}(u) > t | Z^{GBNCIN}(t) = z_{j_1 j_2 \dots j_8}) \quad (16)$$

for $t \in \langle 0, \infty \rangle$, $u = 1, 2, \dots, z^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$,

The safety function $[S^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)}$ is the conditional probability that the GBNCIN lifetime $T_{GBNCIN}^{(j_1 j_2 \dots j_8)}(u)$ in the safety state subset

$\{u, u + 1, \dots, z^{GBNCIN}\}$, is greater than t , while the GBNCIN operation process $Z_{GBNCIN}(t)$ is at the operation state $z_{j_1 j_2 \dots j_8}$.

Further, we denote the GBNCIN unconditional lifetime in the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$, by $T_{GBNCIN}(u)$ and the unconditional safety function of the GBNCIN by the vector

$$[S^{GBNCIN}(t, \cdot)]$$

$$= [1, S^{GBNCIN}(t, 1), \dots, S^{GBNCIN}(t, z^{GBNCIN})], \quad (17)$$

with the coordinates defined by

$$S^{GBNCIN}(t, u) = P(T_{GBNCIN}(u) > t) \quad (18)$$

for $t \in \langle 0, \infty \rangle$, $u = 1, 2, \dots, z^{GBNCIN}$.

In the case when the GBNCIN operation time θ^{GBNCIN} is large enough, the coordinates (18) of the unconditional safety function of the GBNCIN defined by (17) are given by

$$S^{GBNCIN}(t, u)$$

$$\cong \sum_{j_1=1}^{\nu^{(1)}} \sum_{j_2=1}^{\nu^{(2)}} \dots \sum_{j_8=1}^{\nu^{(8)}} p_{j_1 j_2 \dots j_8} [S^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)} \quad (19)$$

for $t \geq 0$, $u = 1, 2, \dots, z^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$,

where $[S^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)}$, $u = 1, 2, \dots, z^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, are the coordinates of the GBNCIN conditional safety functions defined by (15)-(16) and $p_{j_1 j_2 \dots j_8}^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, are the GBNCIN operation process limit transient probabilities given by (4).

The mean value of the GBNCIN Network unconditional lifetime $T_{GBNCIN}(u)$ in the safety state subset $\{u, u + 1, \dots, z^{GBNCIN}\}$ is given by [Kołowrocki, Soszyńska-Budny, 2011]

$$\mu^{GBNCIN}(u) \cong \sum_{j_1 j_2 \dots j_8=1}^l p_{j_1 j_2 \dots j_8}^{GBNCIN} \mu_{j_1 j_2 \dots j_8}^{GBNCIN}(u),$$

$$u = 1, 2, \dots, z^{GBNCIN}, j_i \in \{1, 2, \dots, \nu^{(i)}\}, i = 1, 2, \dots, 8, \quad (20)$$

where $\mu_{j_1 j_2 \dots j_8}^{GBNCIN}(u)$ are the mean values of the *GBNCIN* conditional lifetimes $T_{GBNCIN}^{j_1 j_2 \dots j_8}(u)$ in the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$, at the operation state $z_{j_1 j_2 \dots j_8}$, given by

$$\mu_{j_1 j_2 \dots j_8}^{GBNCIN}(u) = \int_0^{\infty} [S^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)} dt, \quad u = 1, 2, \dots, z^{GBNCIN}, \quad (21)$$

$[S^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)}$, $u = 1, 2, \dots, z^{GBNCIN}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, are defined by (15)-(16) and $p_{j_1 j_2 \dots j_8}^{GBNCIN}$ are given by (4). Whereas, the variance of the *GBNCIN* unconditional lifetime $T_{GBNCIN}(u)$ is given by

$$\sigma_{GBNCIN}^2(u) = 2 \int_0^{\infty} t S^{GBNCIN}(t, u) dt - [\mu^{GBNCIN}(u)]^2, \quad u = 1, 2, \dots, z^{GBNCIN}, \quad (22)$$

where $S^{GBNCIN}(t, u)$, $u = 1, 2, \dots, z^{GBNCIN}$, are given by (17)-(18) and $\mu^{GBNCIN}(u)$, $u = 0, 1, \dots, z^{GBNCIN}$, are given by (20).

According to [Kołowrocki, Soszyńska-Budny, 2011], we get the following formulae for the mean values of the unconditional lifetimes of the *GBNCIN* in particular safety states

$$\begin{aligned} \bar{\mu}^{GBNCIN}(u) &= \mu^{GBNCIN}(u) - \mu^{GBNCIN}(u+1), \\ u &= 0, 1, \dots, z^{GBNCIN} - 1, \\ \bar{\mu}^{GBNCIN}(z^{GBNCIN}) &= \mu^{GBNCIN}(z^{GBNCIN}), \end{aligned} \quad (23)$$

where $\mu^{GBNCIN}(u)$, $u = 0, 1, \dots, z^{GBNCIN}$, are given by (20).

Moreover, according to [Kołowrocki, Soszyńska-Budny, 2011], if r_{GBNCIN} is the *GBNCIN* critical safety state, then the *GBNCIN* risk function

$$\begin{aligned} r_{GBNCIN}(t) &= P(S^{GBNCIN}(t) < r_{GBNCIN} | S^{GBNCIN}(0) = z^{GBNCIN}) \\ &= P(T_{GBNCIN}(r_{GBNCIN}) \leq t), \quad t \in \langle 0, \infty \rangle, \end{aligned} \quad (24)$$

defined as a probability that the *GBNCIN* is in the subset of safety states worse than the critical safety state r_{GBNCIN} , $r_{GBNCIN} \in \{1, 2, \dots, z^{GBNCIN}\}$, while it was in

the safety state z^{GBNCIN} at the moment $t=0$ [Kołowrocki 2004, 2014], [Kołowrocki, Soszyńska-Budny, 2011] is given by

$$r_{GBNCIN}(t) = 1 - S^{GBNCIN}(t, r_{GBNCIN}), \quad t \in \langle 0, \infty \rangle, \quad (25)$$

where $S^{GBNCIN}(t, r_{GBNCIN})$ is the coordinate of the *GBNCIN* unconditional safety function given by (19) for $u = r_{GBNCIN}$.

The *GBNCIN* safety function, the *GBNCIN* risk function and the *GBNCIN* fragility curve are main *GBNCIN* safety factors. Other practically useful *GBNCIN* safety factors are:

- the mean value of the unconditional *GBNCIN* lifetime $T_{GBNCIN}(r_{GBNCIN})$ up to the exceeding the critical safety state r_{GBNCIN} given by

$$\mu^{GBNCIN}(r_{GBNCIN}) \cong \sum_{j_1 j_2 \dots j_8=1}^l p_{j_1 j_2 \dots j_8}^{GBNCIN} \mu_{j_1 j_2 \dots j_8}^{GBNCIN}(r_{GBNCIN}), \quad (26)$$

where $\mu_{j_1 j_2 \dots j_8}^{GBNCIN}(r_{GBNCIN})$ are the mean values of the *GBNCIN* conditional lifetimes $T_{GBNCIN}^{j_1 j_2 \dots j_8}(r_{GBNCIN})$ in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN} + 1, \dots, z^{GBNCIN}\}$ at the operation state $z_{j_1 j_2 \dots j_8}$, given by

$$\begin{aligned} \mu_{j_1 j_2 \dots j_8}^{GBNCIN}(r_{GBNCIN}) &= \int_0^{\infty} [S^{GBNCIN}(t, r_{GBNCIN})]^{(j_1 j_2 \dots j_8)} dt, \\ j_i &\in \{1, 2, \dots, \nu^{(i)}\}, \quad i = 1, 2, \dots, 8, \end{aligned} \quad (27)$$

$[S^{GBNCIN}(t, r_{GBNCIN})]^{(j_1 j_2 \dots j_8)}$, $j_i \in \{1, 2, \dots, \nu^{(i)}\}$, $i = 1, 2, \dots, 8$, are defined by (3.9)-(3.10) and $p_{j_1 j_2 \dots j_8}^{GBNCIN}$ are given by (4);

- the standard deviation of the *GBNCIN* lifetime $T_{GBNCIN}(r_{GBNCIN})$ up to the exceeding the critical safety state r_{GBNCIN} given by

$$\begin{aligned} \sigma_{GBNCIN}(r_{GBNCIN}) &= \sqrt{n(r_{GBNCIN}) - [\mu^{GBNCIN}(r_{GBNCIN})]^2} \end{aligned} \quad (28)$$

where

$$n(r_{GBNCIN}) = 2 \int_0^{\infty} t S^{GBNCIN}(t, r_{GBNCIN}) dt, \quad (29)$$

and $S^{GBNCIN}(t, r_{GBNCIN})$ is given by (19) and $\mu^{GBNCIN}(r_{GBNCIN})$ is given by (20) for $u = r_{GBNCIN}$.

- the moment τ_{GBNCIN} the *GBNCIN* risk function exceeds a permitted level δ_{GBNCIN} given by

$$\tau_{GBNCIN} = r_{GBNCIN}^{-1}(\delta_{GBNCIN}), \quad (30)$$

where $r_{GBNCIN}^{-1}(t)$, if exists, is the inverse function of the risk function $r_{GBNCIN}(t)$, given by (25).

Other *GBNCIN* safety indices are:

- the intensities of ageing/degradation of the *GBNCIN* (the intensities of *GBNCIN* departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$) related to the operation process impact, i.e. the coordinates of the vector

$$\lambda^{GBNCIN}(t, \cdot) = [0, \lambda^{GBNCIN}(t, 1), \dots, \lambda^{GBNCIN}(t, z^{GBNCIN})], \quad t \in < 0, +\infty), \quad (31)$$

where

$$\lambda^{GBNCIN}(t, u) = \frac{dS^{GBNCIN}(t, u)}{S^{GBNCIN}(t, u)} dt, \quad t \in < 0, +\infty), \quad u = 1, 2, \dots, z^{GBNCIN}, \quad (32)$$

- the coefficients of operation process impact on the *GBNCIN* intensities of ageing (the coefficients of operation process impact on *GBNCIN* intensities of departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$), i.e. the coordinates of the vector

$$\rho^{GBNCIN}(t, \cdot) = [0, \rho^{GBNCIN}(t, 1), \dots, \rho^{GBNCIN}(t, z^{GBNCIN})], \quad t \in < 0, +\infty), \quad (33)$$

where

$$\lambda^{GBNCIN}(t, u) = \rho^{GBNCIN}(t, u) \cdot \lambda^{GBNCIN}(t, u), \quad t \in < 0, +\infty), \quad u = 1, 2, \dots, z^{GBNCIN}, \quad (34)$$

and $\lambda^{GBNCIN}(t, u)$ are the intensities of ageing of the *GBNCIN* (the intensities of the *GBNCIN* departure

from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$) without climate-weather impact, i.e. the coordinate of the vector

$$\lambda^{GBNCIN}(t, \cdot) = [0, \lambda^{GBNCIN}(t, 1), \dots, \lambda^{GBNCIN}(t, z^{GBNCIN})], \quad t \in < 0, +\infty), \quad (35)$$

In the case, when the *GBNCIN* have the exponential safety functions, i.e.

$$S^{GBNCIN}(t, \cdot) = [0, S^{GBNCIN}(t, 1), \dots, S^{GBNCIN}(t, z^{GBNCIN})], \quad t \in < 0, +\infty), \quad (36)$$

where

$$S^{GBNCIN}(t, u) = \exp[-\lambda^{GBNCIN}(u)t], \quad t \in < 0, +\infty), \quad u = 1, 2, \dots, z^{GBNCIN}, \quad (37)$$

the *GBNCIN* safety indices defined by (31)-(35) take forms:

- the intensities of ageing of the *GBNCIN* (the intensities of *GBNCIN* departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$) related to the operation impact, i.e. the coordinates of the vector

$$\lambda^{GBNCIN}(\cdot) = [0, \lambda^{GBNCIN}(1), \dots, \lambda^{GBNCIN}(z^{GBNCIN})], \quad (38)$$

- the coefficients of the operation impact on the *GBNCIN* intensities of ageing (the coefficients of the climate-weather impact on *GBNCIN* intensities of departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$), i.e. the coordinate of the vector

$$\rho^{GBNCIN}(\cdot) = [0, \rho^{GBNCIN}(1), \dots, \rho^{GBNCIN}(z^{GBNCIN})], \quad (39)$$

where

$$\lambda^{GBNCIN}(u) = \rho^{GBNCIN}(u) \cdot \lambda^{GBNCIN}(u), \quad u = 1, 2, \dots, z^{GBNCIN}, \quad (40)$$

and $\lambda^{GBNCIN}(u)$ are the intensities of ageing of the *GBNCIN* (the intensities of the *GBNCIN* departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$)

without operation impact, i.e. the coordinate of the vector

$$\lambda^{GBNCIN}(\cdot) = [0, \lambda^{GBNCIN}(1), \dots, \lambda^{GBNCIN}(z^{GBNCIN})]. \quad (41)$$

4. Safety of Multistate GBNCIN consisting of exponential BCIN Networks at Variable Operation Conditions

We assume that the BCIN networks E_i^{GBNCIN} , $i=1,2,\dots,8$, at the GBNCIN operation states have the exponential safety functions.

This assumption and the results given in [Kołowrocki, Soszyńska-Budny, 2011] yield the following results.

If the BCIN networks E_i^{GBNCIN} of the GBNCIN at the operation states $z_{j_1 j_2 \dots j_8}$, $j_i \in \{1,2,\dots,\nu^{(i)}\}$, $i=1,2,\dots,8$, have the exponential safety functions given by

$$\begin{aligned} [S_i^{GBNCIN}(t, \cdot)]^{(j_1 j_2 \dots j_8)} &= [1, [S_i^{GBNCIN}(t, 1)]^{(j_1 j_2 \dots j_8)}, \dots, \\ [S_i^{GBNCIN}(t, z^{GBNCIN})]^{(j_1 j_2 \dots j_8)}], \quad t \in \langle 0, \infty \rangle, \\ j_i &\in \{1,2,\dots,\nu^{(i)}\}, \quad i=1,2,\dots,8, \end{aligned} \quad (42)$$

with the coordinates

$$\begin{aligned} [S_i^{GBNCIN}(t, u)]^{(j_1 j_2 \dots j_8)} &= \\ &= P([T_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u) > t | Z^{GBNCIN}(t) = z_{j_1 j_2 \dots j_8}) \\ &= \exp[-\lambda_i^{GBNCIN}(u)^{(j_1 j_2 \dots j_8)} t], \quad t \in \langle 0, \infty \rangle, \\ j_i &\in \{1,2,\dots,\nu^{(i)}\}, \quad i=1,2,\dots,8, \end{aligned} \quad (43)$$

and the intensities of ageing of the BCIN networks E_i^{GBNCIN} (the intensities of the BCIN networks E_i^{GBNCIN} departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$) related to operation impact, existing in (4.2), are given by

$$\begin{aligned} [\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 \dots j_8)} &= \\ &= [\rho_i^{GBNCIN}]^{(j_1 j_2 \dots j_8)}(u) \cdot \lambda_i^{GBNCIN}(u), \\ u &= 1,2,\dots, z^{GBNCIN}, \quad j_i \in \{1,2,\dots,\nu^{(i)}\}, \\ i &= 1,2,\dots,8, \end{aligned} \quad (44)$$

where $\lambda_i^{GBNCIN}(u)$ are the intensities of ageing of the BCIN networks E_i^{GBNCIN} (the intensities of the

BCIN networks E_i^{GBNCIN} departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$) without operation impact and

$$\begin{aligned} [\rho_i^{GBNCIN}(u)]^{(j_1 j_2 \dots j_8)}, \quad u = 1,2,\dots, z^{GBNCIN}, \\ j_i \in \{1,2,\dots,\nu^{(i)}\}, \quad i = 1,2,\dots,8, \end{aligned} \quad (45)$$

are the coefficients of operation impact on the BCIN networks E_i^{GBNCIN} intensities of ageing (the coefficients of operation impact on BCIN networks E_i^{GBNCIN} intensities of departure from the safety state subset $\{u, u+1, \dots, z^{GBNCIN}\}$), without operation impact, then in the case of series structure, the GBNCIN unconditional safety function is given by the vector:

$$\begin{aligned} \bar{S}_n^{GBNCIN}(t, \cdot) &= [1, \bar{S}_n^{GBNCIN}(t, 1), \dots, \bar{S}_n^{GBNCIN}(t, z^{GBNCIN})], \\ \text{for } t &\geq 0, \end{aligned} \quad (46)$$

where

$$\begin{aligned} \bar{S}_n^{GBNCIN}(t, u) &\cong \\ &\cong \sum_{j_1 j_2 \dots j_8=1}^i p_{j_1 j_2 \dots j_8}^{GBNCIN} \exp[-\sum_{i=1}^n [\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 \dots j_8)} t], \\ \text{for } t &\geq 0, \quad u = 1,2,\dots, z^{GBNCIN}. \end{aligned} \quad (47)$$

5. Conclusions

Integrated Impact Model of Global Baltic Network of Critical Infrastructure Networks Safety Related to Its Operation Process, proposed in this paper, is basic background for considerations in further Tasks of the EU-CIRCLE Project. The model, together with the probabilistic model of the network of critical infrastructure networks operation process, related to the Global Baltic Network of Critical Infrastructure Networks, and the Global Baltic Network of Critical Infrastructure Networks safety model, will be the base to work on climate-weather change influence on critical infrastructures, by evolving them further to include Operating Environment Threats (OET), and Extreme Weather Hazards (EWE) impact. The impact of OET will base on analysis of GBNCIN and BCIN networks intensities of degradation and the coefficients of operation process including OET influence on the

GBNCIN and BCIN intensities of degradation. Next, a general safety analytical model of the GBNCIN safety related to the climate-weather change process in its operating area will be developed. The integrated model of GBNCIN safety, linking its multistate safety model and the model of the climate-weather change process at its operating area, considering variable at the different climate-weather states and impacted by them BCIN networks safety parameters. Finally, conditional safety functions at the climate-weather particular states, the unconditional safety function and the risk function of the GBNCIN at changing in time climate-weather conditions will be defined.

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