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**Guze Sambor**

**Kołowrocki Krzysztof**
*Gdynia Maritime University, Gdynia, Poland*

# Integrated Impact Model on Baltic Port and Shipping Critical Infrastructure Network Safety Related to Its Operation Process

### Keywords

Port, shipping, critical infrastructure, safety, operation process, impact model, network.

### Abstract

The main aim of the paper is the adaptation of the known methods for safety modelling related to the operation process for Baltic Port and Shipping Critical Infrastructure Network. To realize this goal, the description of the four critical infrastructure networks, i.e. Baltic Port, Shipping, Ship traffic and port operation, and finally Baltic Port and Shipping are provided. The basic notations for operation process and multi-state CIN safety analysis for them are introduced. Furthermore, the Baltic Port and Shipping Critical Infrastructure Network safety function and its risk function are proposed. The practically significant critical infrastructure network safety indices like mean lifetime up to the exceeding a critical safety state, the moment when the risk function value exceeds the acceptable safety level, the component and critical infrastructure network intensities of ageing/degradation and the coefficients of operation impact on component and critical infrastructure network intensities of ageing are presented for four CIs considered in the paper.

## 1. Introduction

The paper is devoted to safety modelling and prediction of the Baltic port and shipping critical infrastructure joint network defined as a complex system with operation process changing in time. The multi-state approach in safety analysis with the semi-Markov modelling of the critical infrastructure network's operation process is used. It is the convenient tools for analysing this problem. There are many research and publications about the multistate system's safety modelling [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, 2014],[Xue, 1985], [Xue, Yang, 1995a-b] commonly used with the semi-Markov modelling [Ferreira, Pacheco, 2007], [Glynn, Hass, 2006], [Grabski, 2014], [Kołowrocki 2005], [Limnios, Oprisan, 2005], [Mercier, 2008], [Barbu, Limnios, 2006] of the systems' operation processes, leading to the construction the integrated general safety models of the complex technical systems related to their operation process [Kołowrocki, 2006], [Kołowrocki, Soszyńska, 2006], [Kołowrocki 2006], [Soszyńska, 2006], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, 2014] including critical infrastructures. Therefore, the methods, parameters, and indicators needed to model the safety of this critical infrastructure network related to its operation process are proposed. The basic notations for operation process and multi-state critical infrastructure network safety analysis are introduced. Furthermore, the Baltic Port and Shipping Critical Infrastructure Network safety function and its risk function are defined. The graph of the risk function corresponds to the fragility curve and other practically significant critical infrastructure network safety indices like its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the component and critical infrastructure network intensities of ageing/degradation and the coefficients of operation impact on component and critical infrastructure network intensities of ageing are defined. This is preceded by using the same methods to critical infrastructure networks treated separately, i.e. port critical infrastructure network, shipping critical infrastructure network, and ship traffic and port operation information critical infrastructure network.

The joint network of the port and shipping critical infrastructure networks is very complex and has hard to description operation process. Each of the critical infrastructure network has a number of elements important for safety aspects. Thus, the evaluation, prediction, and modelling of the safety of the joint network can be difficult to provide. The common safety and operation analysis of complex technical systems and critical infrastructure networks is of great value in the industrial practice. The main objective of this paper is to present recently developed, the general safety analytical models of complex multistate technical systems related to their operation processes [Kołowrocki, 2006], [Kołowrocki, Soszyńska, 2006], [Kołowrocki, Soszyńska-Budny, 2011] and to apply them practically to real industrial systems and processes [Kołowrocki 2006], [Kołowrocki, Soszyńska, 2006], [Kołowrocki 2006], [Soszyńska, 2006], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011] and critical infrastructure networks. Based on this approach the methods, parameters, and indicators needed to model the safety of critical infrastructure networks related to its operation process are proposed. Furthermore, the port and shipping critical infrastructure joint network is described. Each of the element of this combined network is analysed as a standalone critical infrastructure network. They are considered as the complex technical systems with operation process changing in time. The multi-state approach in safety analysis with the semi-Markov modelling of the CIN's operation process is used. The time-dependent interactions between the systems' operation processes operation states changing and the systems' structures, and their components safety states changing processes are obvious features of most critical infrastructure networks and real technical systems.

The basic notations for operation process and multi-state CIN safety analysis are introduced. Furthermore, the PSSTPOICIJN safety function and its risk function are defined in the report. They are the crucial indicators/indices from the safety practitioners point of view. The graph of the risk function corresponds to the fragility curve and other practically significant critical infrastructure network safety indices like its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the component and critical infrastructure network intensities of ageing/degradation and the coefficients of operation impact on component and critical infrastructure network intensities of ageing are defined.

## 2. Safety of Port Critical Infrastructure Network Related to Operation Process

### 2.1. Port Critical Infrastructure Network Description

We take into account the complex technical system $S_1$ composed of 18 Baltic core ports and called the Baltic Port Critical Infrastructure Network with the following subsystems:

- the subsystem $S_{11}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{11}^{(1)}$, $E_{12}^{(1)}$, $E_{13}^{(1)}$;
- the subsystem $S_{12}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{21}^{(1)}$, $E_{22}^{(1)}$, $E_{23}^{(1)}$;
- the subsystem $S_{13}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{31}^{(1)}$, $E_{32}^{(1)}$, $E_{33}^{(1)}$;
- the subsystem $S_{14}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{41}^{(1)}$, $E_{42}^{(1)}$, $E_{43}^{(1)}$;
- the subsystem $S_{15}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{51}^{(1)}$, $E_{52}^{(1)}$, $E_{53}^{(1)}$;
- the subsystem $S_{16}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{61}^{(1)}$, $E_{62}^{(1)}$, $E_{63}^{(1)}$;
- the subsystem $S_{17}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{71}^{(1)}$, $E_{72}^{(1)}$, $E_{73}^{(1)}$;
- the subsystem $S_{18}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{81}^{(1)}$, $E_{82}^{(1)}$, $E_{83}^{(1)}$;
- the subsystem $S_{19}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E_{91}^{(1)}$, $E_{92}^{(1)}$, $E_{93}^{(1)}$;

- the subsystem $S_{1,10}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{10,1}$, $E^{(1)}_{10,2}$, $E^{(1)}_{10,3}$;
- the subsystem $S_{1,11}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{11,1}$, $E^{(1)}_{11,2}$, $E^{(1)}_{11,3}$;
- the subsystem $S_{1,12}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{12,1}$, $E^{(1)}_{12,2}$, $E^{(1)}_{12,3}$;
- the subsystem $S_{1,13}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{13,1}$, $E^{(1)}_{13,2}$, $E^{(1)}_{13,3}$;
- the subsystem $S_{1,14}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{14,1}$, $E^{(1)}_{14,2}$, $E^{(1)}_{14,3}$;
- the subsystem $S_{1,15}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{15,1}$, $E^{(1)}_{15,2}$, $E^{(1)}_{15,3}$;
- the subsystem $S_{1,16}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{16,1}$, $E^{(1)}_{16,2}$, $E^{(1)}_{16,3}$;
- the subsystem $S_{1,17}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{17,1}$, $E^{(1)}_{17,2}$, $E^{(1)}_{17,3}$;
- the subsystem $S_{1,18}$ which consist of technical loading/unloading equipment, hydrotechnical infrastructure and transport infrastructure $E^{(1)}_{18,1}$, $E^{(1)}_{18,2}$, $E^{(1)}_{18,3}$.

## 2.2. Port Critical Infrastructure Network Operation Process

We assume that the critical infrastructure network $CIN^{(1)}$ during its operation process are taking numbers of different operation states defined as follows.

$$[Z^{(1)}]_{1\times\nu^{(1)}} = [z^{(1)}_{1},\ z^{(1)}_{2},\ \ldots,\ z^{(1)}_{\nu^{(1)}}], \tag{1}$$

where $z^{(1)}_a$ are the numbers of ships in the port $P_a$, $a = 1,2,\ldots,,\nu^{(1)}$, $\nu^{(1)} \in N$, is the number of ports under the consideration in the the Baltic Sea Region $D$ ($\nu^{(1)} = 18$ for whole Baltic Sea Region);

Further, we define the critical infrastructure network $CIN^{(1)}$, operation processes $Z^{(1)}(t)$, $t \in <0,+\infty)$, as follows:

$$Z^{(1)}(t) = [Z^{(1)}(t)]_{1\mathrm{x}\nu^{(1)}} = [z^{(1)}_1(t), z^{(1)}_2(t),\ldots, z^{(1)}_{\nu^{(1)}}(t)]. \tag{2}$$

with discrete operation states from the set defined by (2), where the operation subprocesses $z^{(1)}_a(t)$ assume the values equal to the numbers $z^{(1)}_a$ of ships in the ports $P_a$, $a = 1,2,\ldots,,\nu^{(1)}$, at the moment $t \in <0,+\infty)$;
In detailed definitions of the states and the operation process $Z^{(1)}(t)$ of the Baltic Port Critical Infrastructure Network $CIN^{(1)}$, consisted of all ports with their facilities, where the operation states are defined by the numbers of vessels in ports $P_a$, $a = 1,2,\ldots,,\nu^{(1)}$, either waiting for port services or being under port services, the impacts of those numbes of ships and their port operations interactions should be include.
Taking into account the above assumption to describe the operation process of the port critical infrastructure network we consider the $n^p = 18$ main Baltic Sea ports. In the port we consider the number of ships:

- entering to the port,
- outgoing into the port,
- waiting,
- handled (loaded/unloaded).

According to (1) – (2), we assume that the operation process for a single port $P_a$, $a = 1,2,\ldots,,18$, is given by the following vector with dimension $4$:

$$[Z^{(1)}_a(t)]_{1\mathrm{x}3} = [z^{(1)}_a(t)] = [n^{(1)}_a(t)], \tag{3}$$

where
$n^{(1)}_a(t) = n^{(1)}_{1,a}(t) + n^{(1)}_{2,a}(t) + n^{(1)}_{3,a}(t)$ - general number of ships in the $P_a$, $a = 1,2,\ldots,,18,$ at the moment $t \in <0,+\infty)$,
$n^{(1)}_1(t)$ - number of the entering ships in the port $P_a$, $a = 1,2,\ldots,,18,$ at the moment $t \in <0,+\infty)$,
$n^{(1)}_2(t)$ - number of the outgoing ships in the port $P_a$, $a = 1,2,\ldots,,18,$ at the moment $t \in <0,+\infty)$,

$n_3^{(1)}(t)$ - number of the waiting ships in the port $P_a$, $a = 1,2,...,,18,$ at the moment $t \in <0,+\infty)$,
$n_4^{(1)}(t)$ - number of the handled(loaded/unloaded) ships in the port $P_a$, $a = 1,2,...,,18,$ at the moment $t \in <0,+\infty)$.

Thus, the operation states for single port are defined as follows:

$$z_0^{(1)} = [0],\ z_1^{(1)} = [1],\ ...,\ z_{n_a^{(1)}}^{(1)} = [n_a^{(1)}].$$

It means, we consider the $n_a^{(1)}$ operation states for every single port $P_a$, $a = 1,2,...,,18$.
When take into account the all ports in critical infrastructure network, the operation states of the $CIN^{(1)}$ are given as the vector of dimension *18*, into the form:

$$[Z^{(1)}(t)]_{1\text{x}18} = [z_1^{(1)}(t), z_2^{(1)}(t), \ldots, z_{18}^{(1)}(t)]$$
$$= [n_1^{(1)}(t), n_2^{(1)}(t), ..., n_{18}^{(1)}(t)]. \quad (4)$$

Moreover, the operation states for $CIN^{(1)}$ are defined in following way:

$z_0^{(1)} = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$,
$z_1^{(1)} = [1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$,
..., $z_{n_1^{(1)}}^{(1)} = [n_1^{(1)},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$,
$z_{n_1^{(1)}+1}^{(1)} = [0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$,
$\ldots, z_{n_1^{(1)}+n_2^{(1)}}^{(1)} = [n_1^{(1)}, n_2^{(1)},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$,
$z_{n_1^{(1)}+n_2^{(1)}+1}^{(1)} = [n_1^{(1)}, n_2^{(1)},1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$,
$\ldots,$
$z_{n_1^{(1)}+n_2^{(1)}+n_3^{(1)}}^{(1)} = [n_1^{(1)}, n_2^{(1)}, n_3^{(1)},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$
,
$\ldots,$
$z_{n_1^{(1)}\cdot n_2^{(1)}\cdot n_3^{(1)}\cdot\ldots\cdot n_{18}^{(1)}}^{(1)} = [n_1^{(1)}, n_2^{(1)}, n_3^{(1)}, n_4^{(1)}, n_5^{(1)}, n_6^{(1)}, n_7^{(1)}, n_8^{(1)}, n_9^{(1)},$
$n_{10}^{(1)}, n_{11}^{(1)}, n_{12}^{(1)}, n_{13}^{(1)}, n_{14}^{(1)}, n_{15}^{(1)}, n_{16}^{(1)}, n_{17}^{(1)}, n_{18}^{(1)}].$

Thus, we consider the $\prod_{i=1}^{18} n_i^{(1)}$ operation states for $CIN^{(1)}$.

Using semi-Markov model introduced in Section 2.1, we can define the port critical infrastructure network operation process $Z(t)$ by:

- the vector of the initial probabilities $p_b^{(1)}(0) = P(Z^{(1)}(0) = z_b^{(1)})$, $b = 1,2,...,\nu^{(1)}$, of the port critical infrastructure network $CIN^{(1)}$ operation processes $Z^{(1)}(t)$ staying at particular operation states at the moment $t = 0$

$$[p_b^{(1)}(0)]_{1\text{x}\nu^{(1)}} = [p_1^{(1)}(0), p_2^{(1)}(0), ..., p_{\nu^{(1)}}^{(1)}(0)]; \quad (5)$$

- the matrix of probabilities $p_{bl}^{(1)}$, $b,l = 1,2,...,\nu^{(1)}$, $b \neq l$, of the port critical infrastructure network $CIN^{(1)}$, operation processes $Z^{(1)}(t)$ transitions between the operation states $z_b^{(1)}$ and $z_l^{(1)}$

$$[p_{bl}^{(1)}]_{\nu^{(1)}\text{x}\nu^{(1)}} = \begin{bmatrix} p_{11}^{(1)} & p_{12}^{(1)} & \cdots & p_{1\nu^{(1)}}^{(1)} \\ p_{21}^{(1)} & p_{22}^{(1)} & \cdots & p_{2\nu^{(1)}}^{(1)} \\ \cdots \\ p_{\nu^{(1)}1}^{(1)} & p_{\nu^{(1)}2}^{(1)} & \cdots & p_{\nu^{(1)}\nu^{(1)}}^{(1)} \end{bmatrix}, \quad (6)$$

where by formal agreement

$$p_{bb}{}^{(1)} = 0 \text{ for } b = 1,2,...,\nu^{(1)};$$

- the matrix of conditional distribution functions $H_{bl}^{(1)}(t) = P(\theta_{bl}^{(1)} < t)$, $b,l = 1,2,...,\nu^{(1)}$, $b \neq l$, of the port critical infrastructure network $CIN^{(1)}$, operation processes $Z^{(1)}(t)$ conditional sojourn times $\theta_{bl}{}^{(1)}$ at the operation states

$$[H_{bl}^{(1)}(t)]_{\nu^{(1)}\text{x}\nu^{(1)}}$$
$$= \begin{bmatrix} H_{11}^{(1)}(t) & H_{12}^{(1)}(t) & \cdots & H_{1\nu^{(1)}}^{(1)}(t) \\ H_{21}^{(1)}(t) & H_{22}^{(1)}(t) & \cdots & H_{2\nu^{(1)}}^{(1)}(t) \\ \cdots \\ H_{\nu^{(1)}1}^{(1)}(t) & H_{\nu^{(1)}2}^{(1)}(t) & \cdots & H_{\nu^{(1)}\nu^{(1)}}^{(1)}(t) \end{bmatrix}, \quad (7)$$

where by formal agreement

$$H_{bb}{}^{(1)}(t) = 0 \text{ for } b = 1,2,...,\nu^{(1)};$$

We introduce the matrix of the conditional density functions of the port critical infrastructure network $CIN^{(1)}$,operation processes $Z^{(1)}(t)$ conditional sojourn times $\theta_{bl}$ at the operation states corresponding to the conditional distribution functions $H_{bl}{}^{(1)}(t)$

$$[h_{bl}^{(1)}(t)]_{\nu^{(1)} \times \nu^{(1)}}$$

$$= \begin{bmatrix} h_{11}^{(1)}(t) & h_{12}^{(1)}(t) & \cdots & h_{1\nu^{(1)}}^{0}(t) \\ h_{21}^{(1)}(t) & h_{22}^{(1)}(t) & \cdots & h_{2\nu^{(1)}}^{(1)}(t) \\ \cdots & & & \\ h_{\nu^{(1)}1}^{(1)}(t) & h_{\nu^{(1)}2}^{(1)}(t) & \cdots & h_{\nu^{(1)}\nu^{(1)}}^{(1)}(t) \end{bmatrix}, \quad (8)$$

where

$$h_{bl}^{(1)}(t) = \frac{d}{dt}[H_{bl}^{(1)}(t)] \text{ for } b, l = 1,2,\ldots,\nu^{(1)},\ b \neq l,$$

and by formal agreement

$$h_{bb}^{(1)}(t) = 0 \text{ for } b = 1,2,\ldots,\nu^{(1)}.$$

## 2.3. Port Critical Infrastructure Network Safety Parameters

According to the effectiveness and safety aspects of the operation of the Baltic Port Critical Infrastructure Network, we fix:

- the number of port critical infrastructure network safety states $(z = 4)$ and we distinguish the following five safety states:
  - a safety state 4 – BPCIN operations are fully safe,
  - a safety state 3 – BPCIN operations are less safe and more dangerous because of the possibility of damage of the land loading/unloading equipment without the environmental pollution,
  - a safety state 2 – BPCIN operations are less safe and more dangerous because of the possibility of collisions or groundings of ships in port area without the environmental pollution,
  - a safety state 1 – BPCIN operations are less safe and very dangerous because of the possibility of collisions or groundings in port area and environmental pollution,
  - a safety state 0 – BPCIN is destroyed,

  Moreover, by the expert opinions, we assume that there are possible the transitions between the components safety states only from better to worse ones;

- the safety structure of the system and subsystems

We consider the three cases of the port critical infrastructure network safety structures as follows:

**Case 1.** It is a complex series system composed of 18 series subsystems $S_{11}$, $S_{12}$, …, $S_{1,18}$, each containing four components as it was mentioned above.

**Case 2.** It is a complex "m out of n" system composed of 18 series subsystems $S_{11}$, $S_{12}$, …, $S_{1,18}$, each containing four components as it was mentioned above.

**Case 3.** It is a complex consecutive "m out of n:F" system composed of 18 series subsystems $S_{11}$, $S_{12}$, …, $S_{1,18}$, each containing four components as it was mentioned above.

The input necessary parameters of the port critical infrastructure network safety models are as follows [EU-CIRCLE Report D3.3-GMU1, 2016], [EU-CIRCLE Report D2.2-GMU1] :

- the number of safety states of the system and components $z$=4,

- the critical safety state of the system $r = 2$,

- the system risk permitted level $\delta = 0.05$,

- the parameters of a system safety structure:

  - Case 1 - series system
    - the number of components (subsystem) $n$=18
  - Case 2 – "m out of n" system
    - the number of components (subsystem) $n$=18
    - the threshold number of subsystems m = 3
  - Case 3 – consecutive "m out of n: F" system
    - the number of components (subsystem) $n$=18
    - the threshold number of subsystems m = 2
    - the parameters of the subsystems $S_{11}$, …, $S_{1,18}$ safety structures
      - series system:
        - the number of components $k$=3.

Considering this chapter assumptions and agreements, similarly to Section 3 in [EU-CIRCLE Report D3.3-GMU1, 2016], we assume that the components $E_{ij}^{(\upsilon)}$, $i = 1,2,3,\ldots,k$, $j = 1,2,\ldots,l_i$, $\upsilon = 1,2,3,\ldots,18$ at the system operation states $z_b$,

$b = 1,2,...,v^{(1)}$, have the exponential safety functions, i.e. the coordiantes of the vector

$$[S_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1,\ [S_{ij}^{(\upsilon)}(t,1)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,2)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,3)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,4)]^{(b)}],\ t \geq 0,\ i = 1,2,3,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,18,\ b = 1,2,...,v^{(1)}, \tag{9}$$

are given

$$[S_{ij}^{(\upsilon)}(t,u)]^{(b)} = P([T_{ij}^{(\upsilon)}]^{(b)}(u) > t \mid Z(t) = z_b) = \exp[-[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} t],\ t \geq 0,\ i = 1,2,3,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,18,\ b = 1,2,...,v^{(1)}. \tag{10}$$

Existing in the above formula the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,18$, (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,18$, departure from the safety state subset $\{u, u+1,...,4\}$) at the system operation process states $z_b$, $b = 1,2,...,v^{(1)}$, i.e. the coordinates of the vector of intensities

$$[\lambda_{ij}^{(\upsilon)}(\cdot)]^{(b)} = [1, [\lambda_{ij}^{(\upsilon)}(1)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(2)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(3)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(4)]^{(b)}],\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,18\ b = 1,2,...,v^{(1)}, \tag{11}$$

are given by

$$[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} = [\rho_{ij}^{(\upsilon)}(u)]^{(b)} \lambda_{ij}^{(\upsilon)}(u),\ u = 1,2,3,4,\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,18\ b = 1,2,...,v^{(1)}, \tag{12}$$

where $\lambda_{ij}^{(\upsilon)}(u)$, $u = 1,2,3,4$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, are the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,18$ (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,18$ departure from the safety state subset $\{u, u+1,...,4\}$) without of operation impact, i.e. the coordinate of the vector of intensities

$$\lambda_{ij}^{(\upsilon)}(\cdot) = [0,\ \lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3),\ \lambda_{ij}^{(\upsilon)}(4)],\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,...,18 \tag{13}$$

and

$$[\rho_{ij}^{(\upsilon)}(u)]^{(b)},\ u = 1,2,3,4,\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,18\ \ b = 1,2,...,v^{(1)}, \tag{14}$$

are the coefficients of the operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,18$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,18$ intensities of departure from the safety state subset $\{u, u+1,...,4\}$) at the system operation process states $z_b$, $b = 1,2,...,v^{(1)}$, i.e. the coordinate of the vector coefficients of impact

$$[\rho_{ij}^{(\upsilon)}(\cdot)]^{(b)} = [0, [\rho_{ij}^{(\upsilon)}(1)]^{(b)}, [\rho_{ij}^{(\upsilon)}(2)]^{(b)}, [\rho_{ij}^{(\upsilon)}(3)]^{(b)}, [\rho_{ij}^{(\upsilon)}(4)]^{(b)}],\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,18\ b = 1,2,...,v^{(1)}, \tag{15}$$

The intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, $\{u, u+1,...,4\}$) without of operation impact on their safety are as follows:
- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3)\ ,\ \lambda_{ij}^{(\upsilon)}(4)\ ,\ i = 1,2,...,k,\ j = 1,2,...,l_i\ \upsilon = 1,2,3,...,18. \tag{16}$$

According to expert opinions, changing the port critical infrastructure network operation process states have influence on changing this system safety structures only, without of the impact on its components' safety.

Thus, the coefficients of the operation process impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the port critical infrastructure network subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,18$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the port critical infrastructure network subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,18$ intensities of departure from the safety state subset

$\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, at the system operation process states $z_b$, $b = 1,2,...,v^{(1)}$, are as follows:
- for subsystems $S_\upsilon$:

$$[\rho_{ij}^{(\upsilon)}(1)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(3)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i,\ b = 1,2,...,v^{(1)},$$
$$\upsilon = 1,2,3,...,18 \tag{17}$$

Thus, by (12), (16) and (18), the new intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, related to the climate-weather influence on its safety are as follows:
- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3)\ ,\ \lambda_{ij}^{(\upsilon)}(4)\ ,$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i\ \upsilon = 1,2,3,...,18\,. \tag{18}$$

## 2.4. Port Critical Infrastructure Network Safety Characteristics

In [Kołowrocki, Soszyńska-Budny, 2011], it is fixed that the port critical infrastructure network safety structure and its subsystems depend on its changing in time operation states and its components safety are not changing at the particular operation states. The influence of the system operation states changing on the changes of the system safety structure and its components safety functions are as follows.

We assume that at the system operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, the system is composed of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,18$, each composed of 3 components $E_{ij}^{(1)}$, $i = 1,2,3,...,18$, $j = 1,2,3$ at the system operation states $z_b^{(1)}$ $b = 1,2,...,v^{(1)}$, with the exponential safety functions given below.

At the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, the port critical infrastructure network five-state conditional safety function is given by:

$$[\boldsymbol{S}(t,\cdot)]^{(b)} =$$
$$[1, [\boldsymbol{S}(t,1)]^{(b)}, [\boldsymbol{S}(t,2)]^{(b)}, [\boldsymbol{S}(t,3)]^{(b)}, [\boldsymbol{S}(t,4)]^{(b)}],$$
$$t \geq 0,\ b = 1,2,...,v^{(1)}, \tag{19}$$

where

**Case 1.** Series system with coordinates given by

$$[\boldsymbol{S}(t,u)]^{(b)} = [\ddot{\boldsymbol{S}}_{18}(t,u)]^{(b)} =$$
$$[\boldsymbol{S}^{(1)}(t,u)]^{(b)} \cdot [\boldsymbol{S}^{(2)}(t,u)]^{(b)} \cdot ... \cdot [\boldsymbol{S}^{(18)}(t,u)]^{(b)}$$
$$\text{for } u = 1,2,3,4,\ b = 1,2,...,v^{(1)}, \tag{20}$$

and particularly

$$[\boldsymbol{S}^{(1)}(t,1)]^{(b)} = \prod_{i=1}^{18} [\exp[-\lambda_{ij}^{(1)}(1)t]]^{(b)},\ j = 1,2,3,$$
$$\text{for } t \geq 0, \tag{21}$$

$$[\boldsymbol{S}^{(1)}(t,2)]^{(b)} = \prod_{i=1}^{18} [\exp[-\lambda_{ij}^{(1)}(2)]]^{(b)},\ j = 1,2,3,$$
$$\text{for } t \geq 0, \tag{22}$$

$$[\boldsymbol{S}^{(1)}(t,3)]^{(b)} = \prod_{i=1}^{18} [\exp[-\lambda_{ij}^{(1)}(3)]]^{(b)},\ j = 1,2,3,$$
$$\text{for } t \geq 0, \tag{23}$$

$$[\boldsymbol{S}^{(1)}(t,4)]^{(b)} = \prod_{i=1}^{18} [\exp[-\lambda_{ij}^{(1)}(4)]]^{(b)},\ j = 1,2,3,$$
$$\text{for } t \geq 0. \tag{24}$$

The expected values and standard deviations of the port critical infrastructure network conditional lifetimes in the safety state subsets $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, at the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, calculated according to (3.15)-(3.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1), \mu_b(2), \mu_b(3), \mu_b(4), \tag{25}$$

and

$$\sigma_b(1), \sigma_b(2)\ , \sigma_b(3)\ , \sigma_b(4), \tag{26}$$

and further, using (17) and (5.67), the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, respectively are:

$$\bar{\mu}_b(1),\ \bar{\mu}_b(2),\ \bar{\mu}_b(3),\ \bar{\mu}_b(4). \tag{27}$$

In the case when the operation time is large enough, the port critical infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],$$
$$t \geq 0, \tag{28}$$

where according to (3.13) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and considering the port critical infrastructure network operation process transient propabilities at the operation states given by (4), the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \ge 0,\ u = 1,2,...,z, \quad (29)$$

Since the critical safety state is $r$ =1, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \ge 0, \quad (30)$$

where $\boldsymbol{S}(t,2)$ is given by (29).

**Case 2.** series -“3 out of 18” system with coordinates given by

$$[\boldsymbol{S}(t,u)]^{(b)} = \ddot{\boldsymbol{S}}_{18}(t,u)$$
$$= 1 - \sum_{\substack{r_1,r_2,...,r_{18}=0 \\ r_1+r_2+...+r_{18} \le 2}}^{1} \prod_{i=1}^{18} [\prod_{j=1}^{3} [S_{ij}^{(1)}(t,u)]^{(b)}]^{r_i}$$
$$[1 - \prod_{j=1}^{3} [S_{ij}^{(1)}(t,u)]^{(b)}]^{1-r_i}, \quad (31)$$

for $t \in <0,\infty)$, $u$ = 1,2,...,4, where

$$S_{ij}^{(1)}(t,1) = \exp[-\lambda_{ij}^{(1)}(1)\,t],$$
$$S_{ij}^{(1)}(t,2) = \exp[-\lambda_{ij}^{(1)}(2)\,t],$$
$$S_{ij}^{(1)}(t,3) = \exp[-\lambda_{ij}^{(1)}(3)\,t],$$
$$S_{ij}^{(1)}(t,4) = \exp[-\lambda_{ij}^{(1)}(4)\,t],$$
$$i = 1,2,3,...,18,\ j = 1,2,3. \quad (32)$$

The expected values and standard deviations of the port critical infrastructure network conditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, at the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, calculated according to (3.15)-(3.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1), \mu_b(2), \mu_b(3), \mu_b(4), \quad (33)$$

$$\sigma_b(1), \sigma_b(2), \sigma_b(3), \sigma_b(4), \quad (34)$$

and further, using (3.17) and (5.67) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, respectively are:

$$\bar{\mu}_b(1),\ \bar{\mu}_b(2),\ \bar{\mu}_b(3),\ \bar{\mu}_b(4). \quad (35)$$

In the case when the operation time is large enough, the port critical infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],$$
$$t \ge 0, \quad (36)$$

where according to (3.13) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and considering the port critical infrastructure network operation process transient probabilities at the operation states given by (5.4), the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \ge 0,$$
$$u = 1,2,...,z, \quad (37)$$

Since the critical safety state is $r$ =1, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \ge 0, \quad (38)$$

where $\boldsymbol{S}(t,2)$ is given by (37).

**Case 3.** series-consecutive “2 out of 18:F” system with the coordinates given by the following recurrent formula

$$\boldsymbol{S}(t,u) = \ddot{\boldsymbol{S}}_{18}(t,u) = \boldsymbol{S}_{\boldsymbol{k}}(t,u)$$
$$= \begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k}[1 - \prod_{j=1}^{3} S_{ij}(t,u)] & \text{for } k = m, \\ [\prod_{j=1}^{3} S_{kj}(t,u)]\boldsymbol{S}_{k-1}(t,u) & \\ + \sum_{j=1}^{m-1}[\prod_{v=1}^{3-j} S_{k-j,v}(t,u)]\boldsymbol{S}_{k-j-1}(t,u) & \\ \cdot \prod_{i=k-j+1}^{k}[1 - \prod_{v=1}^{3} S_{iv}(t,u)] & \text{for } k > m, \end{cases} \quad (39)$$

for $t \ge 0$, $u$ = 1,2,3,4, where

$$S_{ij}^{(1)}(t,1) = \exp[-\lambda_{ij}^{(1)}(1)\,t],$$

$$S_{ij}^{(1)}(t,2) = \exp[-\lambda_{ij}^{(1)}(2)\,t],$$
$$S_{ij}^{(1)}(t,3) = \exp[-\lambda_{ij}^{(1)}(3)\,t],$$
$$S_{ij}^{(1)}(t,4) = \exp[-\lambda_{ij}^{(1)}(4)\,t],$$
$$i = 1,2,3,...,18,\ j = 1,2,3. \quad (40)$$

The expected values and standard deviations of the port critical infrastructure network conditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, at the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, calculated according to (3.15)-(3.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1), \mu_b(2), \mu_b(3), \mu_b(4), \quad (41)$$

$$\sigma_b(1), \sigma_b(2), \sigma_b(3), \sigma_b(4), \quad (42)$$

and further, using (3.17) and (5.67), the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(1)}$, $b = 1,2,...,v^{(1)}$, , respectively are:

$$\bar{\mu}_b(1),\ \bar{\mu}_b(2),\ \bar{\mu}_b(3),\ \bar{\mu}_b(4). \quad (43)$$

In the case when the operation time is large enough, the port critical infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],$$
$$t \ge 0, \quad (44)$$

where according to (3.13) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and considering the port critical infrastructure network operation process transient propabilities at the operation states given by (5.4), the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \ge 0,\ u = 1,2,...,z, \quad (45)$$

Since the critical safety state is $r = 1$, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \ge 0, \quad (46)$$

where $\boldsymbol{S}(t,2)$ is given by (44).

## 3. Safety of Shipping Critical Infrastructure Network Related to Operation Process

### 3.1. Shipping Critical Infrastructure Network Description

We take into account the complex Baltic Shipping Critical Infrastructure Network $S_2$ composed of numbers of ships ($n_{cd}$) into regions $D_{cd}$, $c = 1,2,..., m$, $d = 1,2,..., n$, $m,n \in N$ .

### 3.2. Shipping Critical Infrastructure Network Operation Process

We assume that the critical infrastructure network $CIN^{(2)}$ during its operation process are taking numbers of different operation states defined as follows.

$$[Z^{(2)}]_{m\times n} = \begin{bmatrix} z_{11}^{(2)} & z_{12}^{(2)} & \cdots & z_{1n}^{(2)} \\ z_{21}^{(2)} & z_{22}^{(2)} & \cdots & z_{2n}^{(2)} \\ \cdots \\ z_{m1}^{(2)} & z_{m2}^{(2)} & \cdots & z_{mn}^{(2)} \end{bmatrix}, \quad (47)$$

where $z_{cd}^{(2)}$ are the numbers of ships in the regions $D_{cd}$, $c = 1,2,..., m$, $d = 1,2,..., n$, $m,n \in N$;

Further, we define the critical infrastructure network $CIN^{(2)}$ operation processes $Z^{(2)}(t)$, $t \in <0,+\infty)$, as follows:

$$Z^{(2)}(t) = [Z^{(2)}(t)]_{m\times n}$$
$$= \begin{bmatrix} z_{11}^{(2)}(t) & z_{12}^{(2)}(t) & \cdots & z_{1n}^{(2)}(t) \\ z_{21}^{(2)}(t) & z_{22}^{(2)}(t) & \cdots & z_{2n}^{(2)}(t) \\ \cdots \\ z_{m1}^{(2)}(t) & z_{m2}^{(2)}(t) & \cdots & z_{mn}^{(2)}(t) \end{bmatrix}, \quad (48)$$

with discrete operation states from the set defined by (2.8) where the operation subprocesses $z_{cd}^{(2)}(t)$ is assumed as equal to the numbers $z_{cd}^{(2)}$ of ships in the rectangles $D_{cd}$ $c = 1,2,..., m$, $d = 1,2,..., n$, $m,n \in N$, at the moment $t \in <0,+\infty)$;

Considering interactions between ships creating the Baltic Shipping Critical Infrastructure Network $CIN^{(2)}$, we assume that there are strong inner and outer dependecies between the ships operating in a single fixed rectangle $D_{cd}$, $c = 1,2,..., m$, $d = 1,2,..., n$, $m,n \in N$, and that ships in each two adjacent rectangles influence each other as well. These influences that should be included in detailed definitions of this network operation process $Z^{(2)}(t)$ and its states strongly depend on the operation states of this network defined by the numbers of ships in these rectangles and those ships technical operations.

To describe the operation process of the shipping critical infrastructure network we divide the Baltic Sea area on the square matrix with dimension *m=5, n=14.* (see Fig. 2.1).


Figure 2.1 The exemplary division of the Baltic Sea region according to the geographical coordinates

Further, we assume that the operation process is given by the rectangular matrix with accordance to (47) – (48)

$$[Z^{(2)}(t)]_{5\times 14} = \begin{bmatrix} z_{11}^{(2)}(t) & z_{12}^{(2)}(t) & \cdots & z_{1,14}^{(2)}(t) \\ z_{21}^{(2)}(t) & z_{22}^{(2)}(t) & \cdots & z_{2,14}^{(2)}(t) \\ \cdots & & & \\ z_{51}^{(2)}(t) & z_{52}^{(2)}(t) & \cdots & z_{5,14}^{(2)}(t) \end{bmatrix}, \quad (49)$$

where the operation subprocesses $z_{cd}^{(2)}(t)$ is assumed as equal to the numbers $z_{cd}^{(2)}$ of ships in the rectangles $D_{cd}$ $c = 1,2,...,5,$ $d = 1,2,...,14,$ at the moment $t \in < 0,+\infty)$ ;

The operation states are defined as follows:

$$[z_0^{(2)}]_{5\times 14} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}}^{(2)}]_{5\times 14} = \begin{bmatrix} n_{11}^{(2)} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}+1}^{(2)}]_{5\times 14} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}+n_{12}^{(2)}}^{(2)}]_{5\times 14} = \begin{bmatrix} 0 & n_{12}^{(2)} & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}+n_{12}^{(2)}+...+n_{1,13}^{(2)}+1}^{(2)}]_{5\times 14} = \begin{bmatrix} 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}+n_{12}^{(2)}+n_{13}^{(2)}+...+n_{1,14}^{(2)}}^{(2)}]_{5\times 14} = \begin{bmatrix} 0 & 0 & \cdots & n_{1,14}^{(2)} \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}\cdot n_{12}^{(2)}\cdot n_{13}^{(2)}\cdot ...\cdot n_{1,14}^{(2)}}^{(2)}]_{5\times 14} = \begin{bmatrix} n_{11}^{(2)} & n_{12}^{(2)} & \cdots & n_{1,14}^{(2)} \\ 0 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}\cdot n_{12}^{(2)}\cdot n_{13}^{(2)}\cdot ...\cdot n_{1,14}^{(2)}+1}^{(2)}]_{5\times 14} = \begin{bmatrix} n_{11}^{(2)} & n_{12}^{(2)} & \cdots & n_{1,14}^{(2)} \\ 1 & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}\cdot n_{12}^{(2)}\cdot n_{13}^{(2)}\cdot ...\cdot n_{1,14}^{(2)}+n_{21}^{(2)}}^{(2)}]_{5\times 14} = \begin{bmatrix} n_{11}^{(2)} & n_{12}^{(2)} & \cdots & n_{1,14}^{(2)} \\ n_{21}^{(2)} & 0 & \cdots & 0 \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, ...,$$

$$[z_{n_{11}^{(2)}\cdot ...\cdot n_{1,14}^{(2)}\cdot n_{21}^{(2)}\cdot ...\cdot n_{2,14}^{(2)}}^{(2)}]_{5\times 14} = \begin{bmatrix} n_{11}^{(2)} & n_{12}^{(2)} & \cdots & n_{1,14}^{(2)} \\ n_{21}^{(2)} & n_{22}^{(2)} & \cdots & n_{2,14}^{(2)} \\ \cdots & & & \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$..., [z_{n_{11}^{(2)}\cdot ...\cdot n_{1,14}^{(2)}\cdot n_{21}^{(2)}\cdot ...\cdot n_{2,14}^{(2)}\cdot n_{31}^{(2)}\cdot ...\cdot n_{3,14}^{(2)}\cdot n_{41}^{(2)}\cdot ...\cdot n_{4,14}^{(2)}\cdot n_{51}^{(2)}\cdot ...\cdot n_{5,14}^{(2)}}^{(2)}]_{5\times 14}$$

$$= \begin{bmatrix} n_{11}^{(2)} & n_{12}^{(2)} & \cdots & n_{1,14}^{(2)} \\ n_{21}^{(2)} & n_{22}^{(2)} & \cdots & n_{2,14}^{(2)} \\ \cdots & & & \\ n_{51}^{(2)} & n_{52}^{(2)} & \cdots & n_{5,14}^{(2)} \end{bmatrix}.$$

Thus, we consider the $n_{11}^{(2)} \cdot ... \cdot n_{1,14}^{(2)} \cdot n_{21}^{(2)} \cdot ... \cdot n_{2,14}^{(2)} \cdot n_{31}^{(2)} \cdot ... \cdot n_{3,14}^{(2)} \cdot n_{41}^{(2)} \cdot ...$ $\cdot n_{4,14}^{(2)} \cdot n_{51}^{(2)} \cdot ... \cdot n_{5,14}^{(2)}$ operation states.

Using semi-Markov model introduced in Section 2.1, we can define the shipping critical infrastructure network operation process $Z(t)$ by:

- the vector of the initial probabilities $p_b^{(2)}(0) = P(Z^{(2)}(0) = z_b^{(2)})$, $b = 1,2,...,\nu^{(2)}$, of the shipping critical infrastructure network $CIN^{(2)}$ operation processes $Z^{(2)}(t)$ staying at particular operation states at the moment $t = 0$

$$[p_b^{(2)}(0)]_{1 \times \nu^{(2)}} = [p_1^{(2)}(0), p_2^{(2)}(0), ..., p_{\nu^{(2)}}^{(2)}(0)]; \qquad (50)$$

- the matrix of probabilities $p_{bl}^{(2)}$, $b, l = 1,2,...,\nu^{(2)}$, $b \neq l$, of the shipping critical infrastructure network $CIN^{(2)}$, operation processes $Z^{(2)}(t)$ transitions between the operation states $z_b^{(2)}$ and $z_l^{(2)}$

$$[p_{bl}^{(2)}]_{\nu^{(2)} \times \nu^{(2)}} = \begin{bmatrix} p_{11}^{(2)} & p_{12}^{(2)} & \cdots & p_{1\nu^{(2)}}^{(2)} \\ p_{21}^{(2)} & p_{22}^{(2)} & \cdots & p_{2\nu^{(2)}}^{(2)} \\ \cdots \\ p_{\nu^{(2)}1}^{(2)} & p_{\nu^{(2)}2}^{(2)} & \cdots & p_{\nu^{(2)}\nu^{(2)}}^{(2)} \end{bmatrix}, \qquad (51)$$

where by formal agreement

$$p_{bb}^{(2)} = 0 \text{ for } b = 1,2,...,\nu^{(2)};$$

- the matrix of conditional distribution functions $H_{bl}^{(2)}(t) = P(\theta_{bl}^{(2)} < t)$, $b, l = 1,2,...,\nu^{(2)}$, $b \neq l$, of the shipping critical infrastructure network $CIN^{(2)}$, operation processes $Z^{(2)}(t)$ conditional sojourn times $\theta_{bl}^{(2)}$ at the operation states

$$[H_{bl}^{(2)}(t)]_{\nu^{(2)} \times \nu^{(2)}} = \begin{bmatrix} H_{11}^{(2)}(t) & H_{12}^{(2)}(t) & \cdots & H_{1\nu^{(2)}}^{(2)}(t) \\ H_{21}^{(2)}(t) & H_{22}^{(2)}(t) & \cdots & H_{2\nu^{(2)}}^{(2)}(t) \\ \cdots \\ H_{\nu^{(1)}1}^{(2)}(t) & H_{\nu^{(2)}2}^{(2)}(t) & \cdots & H_{\nu^{(2)}\nu^{(2)}}^{(2)}(t) \end{bmatrix}, \qquad (52)$$

where by formal agreement

$$H_{bb}^{(2)}(t) = 0 \text{ for } b = 1,2,...,\nu^{(2)};$$

We introduce the matrix of the conditional density functions of the shipping critical infrastructure network $CIN^{(2)}$, operation processes $Z^{(2)}(t)$ conditional sojourn times $\theta_{bl}$ at the operation states corresponding to the conditional distribution functions $H_{bl}^{(2)}(t)$

$$[h_{bl}^{(2)}(t)]_{\nu^{(2)} \times \nu^{(2)}} = \begin{bmatrix} h_{11}^{(2)}(t) & h_{12}^{(2)}(t) & \cdots & h_{1\nu^{(1)}}^{(2)}(t) \\ h_{21}^{(2)}(t) & h_{22}^{(2)}(t) & \cdots & h_{2\nu^{(2)}}^{(2)}(t) \\ \cdots \\ h_{\nu^{(2)}1}^{(2)}(t) & h_{\nu^{(2)}2}^{(2)}(t) & \cdots & h_{\nu^{(2)}\nu^{(2)}}^{(2)}(t) \end{bmatrix}, \qquad (53)$$

where

$$h_{bl}^{(2)}(t) = \frac{d}{dt}[H_{bl}^{(2)}(t)] \text{ for } b, l = 1,2,...,\nu^{(2)}, \ b \neq l,$$

and by formal agreement

$$h_{bb}^{(2)}(t) = 0 \text{ for } b = 1,2,...,\nu^{(2)}.$$

## 3.3. Shipping Critical Infrastructure Network Safety Model

According to the effectiveness and safety aspects of the operation of the Baltic Shipping Critical Infrastructure Network, we fix:

- the number of shipping critical infrastructure network safety states $(z = 4)$ and we distinguish the following five safety states:
  - a safety state 4 – BSCIN operations are fully safe,
  - a safety state 3 – BSCIN operations are less safe and more dangerous because of the possibility of damage of the ships without the environmental pollution in regions area,
  - a safety state 2 – BSCIN operations are less safe and more dangerous because of the possibility of collisions or groundings of ships without the environmental pollution in regions area,
  - a safety state 1 – BSCIN operations are less safe and very dangerous because of the possibility of collisions or groundings and environmental pollution in regions area,
  - a safety state 0 – BSCIN is destroyed,

  Moreover, by the expert opinions, we assume that there are possible the transitions between the components safety states only from better to worse ones;

– the safety structure of the system and subsystems

The shipping critical infrastructure network is a complex series system composed of $c \cdot d$ subsystems $S_{2i}$, $i = 1,...,d, d+1,...,2d, 2d+1,...,3d,...,(c-1)d,(c-1)d+1,...,cd$ each containing numbers of ships as the components.

The input necessary parameters of the shipping critical infrastructure network safety models are as follows [EU-CIRCLE Report D3.3-GMU1, 2016], [EU-CIRCLE Report D2.2-GMU1] :

- the number of safety states of the system and components $z$=4,

- the critical safety state of the system $r = 2$,

- the system risk permitted level $\delta = 0.05$,

- the parameters of a system safety structure:

- Case 1 - series system
  - the number of components (subsystem) $n = c \cdot d$
- Case 2 – “l out of k” system
  - the number of components (subsystem) $k = c \cdot d$
  - *the tresholds* number of subsystems $l = 0.5 \cdot c \cdot d$
- Case 3 – consecutive “l out of k:F” system
  - the number of components (subsystem) $k = c \cdot d$
  - *the tresholds* number of subsystems $l = 0.25 \cdot c \cdot d$
  - the parameters of the subsystems $S_{2i}$, $i = 1,...,d, d+1,...,(c-1)d,(c-1)d+1,...,cd$ safety structures
    - series system:
      – the number of components $k = \sum_{D_{cd}} n_{cd}$, where $n_{cd}$ is the number of ships in area $D_{cd}$;

Considering this chapter assumptions and agreements, similiary to Section 3 in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], we assume that the components $E_{ij}^{(\upsilon)}$, $i = 1,...,d, d+1,...,(c-1)d,(c-1)d+1,...,cd$ $j = 1,2,...,l_i$, $\upsilon = 1,2,3,...,c \cdot d$ at the system operation states $z_b$, $b = 1,2,...,\nu^{(2)}$, have the exponential safety functions, i.e. the coordiantes of the vector

$$[S_{ij}^{(\upsilon)}(t,\cdot)]^{(b)}$$
$$= [1, [S_{ij}^{(\upsilon)}(t,1)]^{(b)}, [S_{ij}^{(\upsilon)}(t,2)]^{(b)}, [S_{ij}^{(\upsilon)}(t,3)]^{(b)},$$
$$[S_{ij}^{(\upsilon)}(t,4)]^{(b)}], t \geq 0,$$
$$i = 1,...,d, d+1,...,(c-1)d,...,cd \;\; j = 1,2,...,l_i,$$
$$\upsilon = 1,2,3,...,c \cdot d, \; b = 1,2,...,\nu^{(2)}, \tag{54}$$

are given

$$[S_{ij}^{(\upsilon)}(t,u)]^{(b)}$$
$$= P([T_{ij}^{(\upsilon)}]^{(b)}(u) > t \mid Z(t) = z_b) = \exp[-[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} t],$$
$$t \geq 0, \; i = 1,2,3,...,k, \; \upsilon = 1,2,3,...,c \cdot d,$$
$$b = 1,2,...,\nu^{(2)}. \tag{55}$$

Existing in the above formula the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ departure from the safety state subset $\{u, u+1,...,4\}$) at the system operation process states $z_b$, $b = 1,2,...,\nu^{(2)}$, i.e. the coordinates of the vector of intensities

$$[\lambda_{ij}^{(\upsilon)}(\cdot)]^{(b)}$$
$$= [1, [\lambda_{ij}^{(\upsilon)}(1)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(2)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(3)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(4)]^{(b)}],$$
$$i = 1,2,...,k, \; j = 1,2,...,l_i, \; \upsilon = 1,2,3,...,c \cdot d$$
$$b = 1,2,...,\nu^{(2)}, \tag{56}$$

are given by

$$[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} = [\rho_{ij}^{(\upsilon)}(u)]^{(b)} \lambda_{ij}^{(\upsilon)}(u), \; u = 1,2,3,4,$$
$$i = 1,2,...,k, \; j = 1,2,...,l_i, \; \upsilon = 1,2,3,...,c \cdot d$$
$$b = 1,2,...,\nu^{(2)}, \tag{57}$$

where $\lambda_{ij}^{(\upsilon)}(u)$, $u = 1,2,3,4$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, are the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ departure from the safety state subset $\{u, u+1,...,4\}$) without of operation impact, i.e. the coordinate of the vector of intensities

$$\lambda_{ij}^{(\upsilon)}(\cdot) = [0,\ \lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3),\ \lambda_{ij}^{(\upsilon)}(4)],$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,c \cdot d \qquad (58)$$

and

$$[\rho_{ij}^{(\upsilon)}(u)]^{(b)},\ u = 1,2,3,4,\ i = 1,2,...,k,\ j = 1,2,...,l_i,$$
$$\upsilon = 1,2,3,...,c \cdot d \quad b = 1,2,...,\nu^{(2)}, \qquad (59)$$

are the coefficients of the operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ intensities of departure from the safety state subset $\{u, u+1,...,4\}$) at the system operation process states $z_b$, $b = 1,2,...,\nu^{(2)}$, i.e. the coordinate of the vector coefficients of impact

$$[\rho_{ij}^{(\upsilon)}(\cdot)]^{(b)}$$
$$= [0, [\rho_{ij}^{(\upsilon)}(1)]^{(b)}, [\rho_{ij}^{(\upsilon)}(2)]^{(b)}, [\rho_{ij}^{(\upsilon)}(3)]^{(b)},$$
$$[\rho_{ij}^{(\upsilon)}(4)]^{(b)}],\ i = 1,2,...,k,\ j = 1,2,...,l_i,$$
$$\upsilon = 1,2,3,...,c \cdot d \quad b = 1,2,...,\nu^{(2)}, \qquad (60)$$

The intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, $\{u, u+1,...,4\}$) without of operation impact on their safety are as follows:
- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3)\ ,\ \lambda_{ij}^{(\upsilon)}(4)\ ,$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i\ \upsilon = 1,2,3,...,c \cdot d\ . \qquad (61)$$

According to expert opinions, changing the port critical infrastructure network operation process states have influence on changing this system safety structures only, without of the impact on its components‘ safety.

Thus, the coefficients of the operation process impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the shipping critical infrastructure network subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the shipping critical infrastructure network subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$ intensities of departure from the safety state subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, at the system operation process states $z_b$, $b = 1,2,...,\nu^{(2)}$, are as follows:
- for subsystems $S_\upsilon$:

$$[\rho_{ij}^{(\upsilon)}(1)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(3)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i,\ b = 1,2,...,\nu^{(2)},$$
$$\upsilon = 1,2,3,...,c \cdot d \qquad (62)$$

Thus, by (11), (15), the new intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, related to the climate-weather influence on its safety are as follows:
- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3)\ ,\ \lambda_{ij}^{(\upsilon)}(4)\ ,$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i\ \upsilon = 1,2,3,...,c \cdot d\ . \qquad (63)$$

### 3.4. Shipping Critical Infrastructure Network Safety Characteristics

In [Kołowrocki, Soszyńska-Budny, 2011], it is fixed that the shipping critical infrastructure network safety structure and its subsystems depend on its changing in time operation states and its components safety are not changing at the particular operation states. The influence of the system operation states changing on the changes of the system safety structure and its components safety functions are as follows.

We assume that at the system operation state $z_b^{(2)}$, $b = 1,2,...,\nu^{(2)}$, the system is composed of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,c \cdot d$, each composed of 3 components $E_{ij}^{(1)}$, $i = 1,...,d, d+1,...,c \cdot d$, $j = 1,2,..,n_{cd}$, at the system operation states $z_b^{(2)}$ $b = 1,2,...,\nu^{(2)}$, with the exponential safety functions given below.

At the operation state $z_b^{(2)}$, $b = 1,2,...,\nu^{(2)}$, the shipping critical infrastructure network five-state conditional safety function is given by:

$$[\boldsymbol{S}(t,\cdot)]^{(b)} =$$
$$[1, [\boldsymbol{S}(t,1)]^{(b)}, [\boldsymbol{S}(t,2)]^{(b)}, [\boldsymbol{S}(t,3)]^{(b)}, [\boldsymbol{S}(t,4)]^{(b)}],$$
$$t \geq 0,\ b = 1,2,...,\nu^{(2)}, \qquad (64)$$

where

**Case 1.** Series system with coordinates given by

$$[\boldsymbol{S}(t,u)]^{(b)} = [\ddot{\boldsymbol{S}}_{18}(t,u)]^{(b)} = [\boldsymbol{S}^{(1)}(t,u)]^{(b)} \cdot [\boldsymbol{S}^{(2)}(t,u)]^{(b)} \cdot \ldots \cdot [\boldsymbol{S}^{(c\cdot d)}(t,u)]^{(b)}$$

$$\text{for } u = 1,2,3,4,\ b = 1,2,\ldots,v^{(2)}. \qquad (65)$$

The expected values and standard deviations of the shipping critical infrastructure network conditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, at the operation state $z_b^{(2)}$, $b = 1,2,\ldots,v^{(2)}$, , calculated according to (3.15)-(3.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1), \mu_b(2), \mu_b(3), \mu_b(4), \qquad (66)$$

$$\sigma_b(1), \sigma_b(2), \sigma_b(3), \sigma_b(4), \qquad (67)$$

and further, using (3.17) and (5.67) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(2)}$, $b = 1,2,\ldots,v^{(2)}$, , respectively are:

$$\bar{\mu}_b(1),\ \bar{\mu}_b(2),\ \bar{\mu}_b(3),\ \bar{\mu}_b(4). \qquad (68)$$

In the case when the operation time is large enough, the shipping critical infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \ge 0, \qquad (69)$$

where according to (3.13) in […] and considering shipping critical infrastructure network operation process transient probabilities at the operation states given by (5.4), the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \ge 0,\ u = 1,2,\ldots,4, \qquad (70)$$

Since the critical safety state is $r$ =1, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \ge 0, \qquad (71)$$

where $\boldsymbol{S}(t,2)$ is given by (70).

**Case 2.** series -"$\lceil 0.5 \cdot c \cdot d \rceil$ out of $c \cdot d$" system with coordinates given by

$$[\boldsymbol{S}(t,u)]^{(b)} = [\ddot{\boldsymbol{S}}_{a\cdot b}(t,u)]^{(b)} = 1 - \sum_{\substack{r_1,r_2,\ldots,r_{a\cdot b}=0 \\ r_1+r_2+\ldots+r_{a\cdot b} \le \lceil 0.5\cdot a\cdot b \rceil - 1}}^{1} \prod_{i=1}^{ab} [\prod_{j=1}^{l_i} [S_{ij}(t,u)]^{(b)}]^{r_i} [1 - \prod_{j=1}^{l_i} [S_{ij}(t,u)]^{(b)}]^{1-r_i}, \qquad (72)$$

for $t \in <0,\infty)$, $u = 1,2,3,4, l_1 = n_{11}, l_2 = n_{12}, \ldots, l_b = n_{1d}$, $l_{d+1} = n_{21}, \ldots, l_{c\cdot d} = n_{cd}$

where

$$S_{ij}^{(1)}(t,1) = \exp[-\lambda_{ij}^{(1)}(1)\, t],$$
$$S_{ij}^{(1)}(t,2) = \exp[-\lambda_{ij}^{(1)}(2)\, t],$$
$$S_{ij}^{(1)}(t,3) = \exp[-\lambda_{ij}^{(1)}(3)\, t],$$
$$S_{ij}^{(1)}(t,4) = \exp[-\lambda_{ij}^{(1)}(4)\, t],$$
$$i = 1,2,3,\ldots,c \cdot d,\ j = 1,2,3. \qquad (73)$$

The expected values and standard deviations of the shipping critical infrastructure network conditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, at the operation state $z_b^{(2)}$, $b = 1,2,\ldots,v^{(2)}$, calculated according to (3.15)-(3.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1), \mu_b(2), \mu_b(3), \mu_b(4), \qquad (74)$$

$$\sigma_b(1), \sigma_b(2), \sigma_b(3), \sigma_b(4), \qquad (75)$$

and further, using (3.17) and (3.67) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(2)}$, $b = 1,2,\ldots,v^{(2)}$, respectively are:

$$\bar{\mu}'_b(1),\ \bar{\mu}'_b(2),\ \bar{\mu}'_b(3),\ \bar{\mu}'_b(4). \qquad (76)$$

In the case when the operation time is large enough, the shipping critical infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \ge 0, \qquad (77)$$

where according to (3.13) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and considering the shipping critical infrastructure network operation process transient propabilities at the operation states given by (5.4) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \geq 0,\ u = 1,2,3,4, \quad (78)$$

Since the critical safety state is $r$ =1, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \geq 0, \quad (79)$$

where $\boldsymbol{S}(t,2)$ is given by (78).

**Case 3.** series-consecutive "$\lfloor 0.25 \cdot c \cdot d \rfloor$ out of $c \cdot d$ : F" system with the coordinates given by the following recurrent formula

$$[\boldsymbol{S}(t,u)]^{(b)} = [\ddot{\boldsymbol{S}}_{c \cdot d}(t,u)]^{(b)} = [\boldsymbol{S}_k(t,u)]^{(b)} =$$

$$\begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k}[1 - \prod_{j=1}^{l_i}[S_{ij}(t,u)]^{(b)}] & \text{for } k = m, \\ [\prod_{j=1}^{l_k}[S_{kj}(t,u)]^{(b)}][\boldsymbol{S}_{k-1}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m-1}[\prod_{v=1}^{l_{k-j}}[S_{k-j,v}(t,u)]^{(b)}][\boldsymbol{S}_{k-j-1}(t,u)]^{(b)} & \\ \cdot \prod_{i=k-j+1}^{k}[1 - \prod_{v=1}^{l_i}[S_{iv}(t,u)]^{(b)}], & \text{for } k > m, \end{cases} \quad (80)$$

for $t \geq 0$, $k = 1,...,d,d+1,...,2d,...,3d,...,cd$, $u$ = 1,2,3,4, $l_1 = n_{11}, l_2 = n_{12},..., l_d = n_{1d}, l_{d+1} = n_{21},...,$ $l_{c \cdot d} = n_{cd}$

where

$$\begin{aligned} S_{ij}^{(2)}(t,1) &= \exp[-\lambda_{ij}^{(2)}(1)\,t], \\ S_{ij}^{(2)}(t,2) &= \exp[-\lambda_{ij}^{(2)}(2)\,t], \\ S_{ij}^{(2)}(t,3) &= \exp[-\lambda_{ij}^{(2)}(3)\,t], \\ S_{ij}^{(2)}(t,4) &= \exp[-\lambda_{ij}^{(2)}(4)\,t], \\ i &= 1,2,3,...,cd,\ j = 1,2,3. \end{aligned} \quad (81)$$

The expected values and standard deviations of the shipping critical infrastructure network conditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, at the operation state $z_b^{(2)}$, $b = 1,2,...,v^{(2)}$, calculated according to (3.15)-(3.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1), \mu_b(2), \mu_b(3), \mu_b(4), \quad (82)$$

$$\sigma_b(1), \sigma_b(2), \sigma_b(3), \sigma_b(4), \quad (83)$$

and further, using (3.17) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and (82), the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(2)}$, $b = 1,2,...,v^{(2)}$, respectively are:

$$\bar{\mu}_b(1), \bar{\mu}_b(2), \bar{\mu}_b(3), \bar{\mu}_b(4). \quad (84)$$

In the case when the operation time is large enough, the shipping critical infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],$$
$$t \geq 0, \quad (85)$$

where according to (3.13) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and considering the shipping critical infrastructure network operation process transient propabilities at the operation states given by (5.4) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \geq 0,$$
$$u = 1,2,...,z, \quad (86)$$

Since the critical safety state is $r$ =1, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \geq 0, \quad (87)$$

where $\boldsymbol{S}(t,2)$ is given by (86).

## 4. Safety of Ship Traffic and Port Operation Information Critical Infrastructure Network Related to Operation Process

### 4.1. Ship Traffic and Port Operation Information Critical Infrastructure Network Description

We take into account the complex technical ship traffic and port operation information critical infrastructure network $S_3$ composed of :

- the subsystem $S_{31}$ which consist of 121 AIS base stations and 25 DGPS stations $E_{11}^{(3)}, E_{21}^{(3)}, \ldots, E_{146,1}^{(3)}$;
- the subsystem $S_{32}$ which consist of at least 18 port operation information systems $E_{21}^{(3)}, \ldots, E_{2,18}^{(3)}$.

### 4.2. Ship Traffic and Port Operation Information Critical Infrastructure Network Operation Process

We assume that the critical infrastructure network $CIN^{(3)}$ during its operation states process are taking numbers of different operation states defined as follows.

$$[Z^{(3)}]_{1\times\nu^{(3)}} = [z_1^{(3)},\ z_2^{(3)},\ \ldots,\ z_{\nu^{(3)}}^{(3)}], \tag{88}$$

where $z_a^{(3)}$ are the numbers of ships in the range of the information systems $I_a$, $a = 1,2,\ldots,,\nu^{(3)}$, $\nu^{(3)} \in N$, is the number of information systems under the consideration in the the Baltic Sea Region $D$ (for general case $\nu^{(3)} = 146$).
Further, we define the critical infrastructure networks $CIN^{(3)}$, operation processes $Z^{(3)}(t)$, $t \in <0,+\infty)$, as follows:

$$Z^{(3)}(t) = [Z^{(3)}(t)]_{1\times\nu^{(3)}} = [z_1^{(3)}(t), z_2^{(3)}(t), \ldots, z_{\nu^{(3)}}^{(3)}(t)]. \tag{89}$$

with discrete operation states from the set defined by (4.11), where the operation subprocesses $z_a^{(3)}(t)$ assume the values equal to the numbers $z_a^{(3)}$ of ships in the range of the information systems $I_a$, $a = 1,2,\ldots,,\nu^{(3)}$, at the moment $t \in <0,+\infty)$.

The Shipping, Ship Traffic and Operation Information Critical Infrastructure Network $CIN^{(3)}$ is a platform to exchange the information about ships' operations and and their cargo. Due to the fact that all informations are given mainly in electronic form, this network is very sensitive for any disruption, especially cyber attacs, but also for natural hazards. Thus, in detailed defining this network operation process and its states those features should be taken into account.

According to Section 2.3 in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and formulae (88) – (89), the operation process of BSTPOICIN is given by the vector

$$[Z^{(3)}(t)]_{1\text{x}146} = [z_1^{(3)}(t), z_2^{(3)}(t), \ldots, z_{146}^{(3)}(t)], \tag{90}$$

where the operation subprocesses $z_a^{(3)}(t)$ assume the values equal to the numbers $z_a^{(3)}$ of ships in the range of the information systems $TPOIS_a$, $a = 1,2,\ldots,,146$, at the moment $t \in <0,+\infty)$.

The following operation states are defined:

$$z_0^{(3)} = [0,0,0,\ldots,0],\ z_1^{(3)} = [1,0,0,\ldots,0],\ \ldots,\ z_{n_1}^{(3)} = [n_1,0,0,\ldots,0],$$

$$z_{n_1+1}^{(3)} = [0,1,0,\ldots,0],\ z_{n_1+2}^{(3)} = [0,2,0,\ldots,0],\ \ldots,\ z_{n_1+n_2}^{(3)} = [0,n_2,0,\ldots,0],$$

$$z_{n_1+n_2+1}^{(3)} = [0,0,1,\ldots,0],\ z_{n_1+n_2+2}^{(3)} = [0,0,2,\ldots,0],\ \ldots,\ z_{n_1+n_2+n_3}^{(3)} = [0,0,n_3,\ldots,0],\ \ldots,\ z_{n_1+\ldots+n_{146}}^{(3)} = [0,0,0,\ldots,n_{146}],$$

$$z_{n_1+\ldots+n_{146}+1}^{(3)} = [1,1,\ldots,0],\ z_{n_1+\ldots+n_{146}+2}^{(3)} = [1,2,\ldots,0],\ \ldots,\ z_{n_1+2n_2+\ldots+146}^{(3)} = [1,n_2,\ldots,0],\ \ldots,$$

$$z_{n_1 n_2}^{(3)} = [n_1,n_2,\ldots,0],\ \ldots,\ z_{n_1\cdot n_2\cdot n_3\cdot n_4}^{(3)} = [n_1,n_2,\ldots,n_{146}].$$

It means, we consider the $n_1 \cdot n_2 \cdot \ldots \cdot n_{146}$ operation states for Baltic Ship Traffic and Port Operation Information Critical Infrastructure Network.
Using semi-Markov model introduced in Section 2.1, we can define the ship traffic and port operation information critical infrastructure network operation process $Z(t)$ by:

- the vector of the initial probabilities $p_b^{(3)}(0) = P(Z^{(3)}(0) = z_b^{(3)})$, $b = 1,2,\ldots,\nu^{(3)}$, of the ship traffic and port operation information critical infrastructure network $CIN^{(3)}$ operation processes $Z^{(3)}(t)$ staying at particular operation states at the moment $t = 0$

$$[p_b^{(3)}(0)]_{1 \times \nu^{(3)}} = [p_1^{(3)}(0), p_2^{(3)}(0), ..., p_{\nu^{(3)}}^{(3)}(0)]; \quad (91)$$

- the matrix of probabilities $p_{bl}^{(3)}$, $b, l = 1,2,...,\nu^{(3)}$, $b \neq l$, of the ship traffic and port operation information critical infrastructure network $CIN^{(3)}$, operation processes $Z^{(3)}(t)$ transitions between the operation states $z_b^{(3)}$ and $z_l^{(3)}$

$$[p_{bl}^{(3)}]_{\nu^{(3)} \times \nu^{(3)}} = \begin{bmatrix} p_{11}^{(3)} & p_{12}^{(3)} & \cdots & p_{1\nu^{(3)}}^{(3)} \\ p_{21}^{(3)} & p_{22}^{(3)} & \cdots & p_{2\nu^{(3)}}^{(3)} \\ \cdots & & & \\ p_{\nu^{(3)}1}^{(3)} & p_{\nu^{(3)}2}^{(3)} & \cdots & p_{\nu^{(3)}\nu^{(3)}}^{(3)} \end{bmatrix}, \quad (92)$$

where by formal agreement

$$p_{bb}^{(3)} = 0 \text{ for } b = 1,2,...,\nu^{(3)};$$

- the matrix of conditional distribution functions $H_{bl}^{(3)}(t) = P(\theta_{bl}^{(3)} < t)$, $b, l = 1,2,...,\nu^{(3)}$, $b \neq l$, of the ship traffic and port operation information critical infrastructure network $CIN^{(3)}$, operation processes $Z^{(3)}(t)$ conditional sojourn times $\theta_{bl}^{(3)}$ at the operation states

$$[H_{bl}^{(3)}(t)]_{\nu^{(3)} \times \nu^{(3)}} = \begin{bmatrix} H_{11}^{(3)}(t) & H_{12}^{(3)}(t) & \cdots & H_{1\nu^{(3)}}^{(3)}(t) \\ H_{21}^{(3)}(t) & H_{22}^{(3)}(t) & \cdots & H_{2\nu^{(3)}}^{(3)}(t) \\ \cdots & & & \\ H_{\nu^{(3)}1}^{(3)}(t) & H_{\nu^{(3)}2}^{(3)}(t) & \cdots & H_{\nu^{(3)}\nu^{(3)}}^{(3)}(t) \end{bmatrix}, \quad (93)$$

where by formal agreement

$$H_{bb}^{(3)}(t) = 0 \text{ for } b = 1,2,...,\nu^{(3)};$$

We introduce the matrix of the conditional density functions of the ship traffic and port operation information critical infrastructure network $CIN^{(3)}$, operation processes $Z^{(3)}(t)$ conditional sojourn times $\theta_{bl}$ at the operation states corresponding to the conditional distribution functions $H_{bl}^{(3)}(t)$

$$[h_{bl}^{(3)}(t)]_{\nu^{(3)} \times \nu^{(3)}} = \begin{bmatrix} h_{11}^{(3)}(t) & h_{12}^{(3)}(t) & \cdots & h_{1\nu^{(3)}}^{(3)}(t) \\ h_{21}^{(3)}(t) & h_{22}^{(3)}(t) & \cdots & h_{2\nu^{(3)}}^{(3)}(t) \\ \cdots & & & \\ h_{\nu^{(3)}1}^{(3)}(t) & h_{\nu^{(3)}2}^{(3)}(t) & \cdots & h_{\nu^{(3)}\nu^{(3)}}^{(3)}(t) \end{bmatrix}, \quad (94)$$

where

$$h_{bl}^{(3)}(t) = \frac{d}{dt}[H_{bl}^{(3)}(t)] \text{ for } b, l = 1,2,...,\nu^{(3)}, b \neq l,$$

and by formal agreement

$$h_{bb}^{(3)}(t) = 0 \text{ for } b = 1,2,...,\nu^{(3)}.$$

## 4.3. Ship Traffic and Port Operation Information Critical Infrastructure Network Safety Parameters

According to the effectiveness and safety aspects of the operation of the Baltic Port Critical Infrastructure Network, we fix:

- the number of port critical infrastructure network safety states $(z = 4)$ and we distinguish the following five safety states:
  - a safety state 4 – port operation information subsystem is less safe and more dangerous because of the possibility of environment pollution and and causing small accidents,
  - a safety state 3 – ship traffic information subsystem is less safe and more dangerous because of the possibility of environment pollution and causing big accidents,
  - a safety state 2 – port operation information subsystem is less safe and more dangerous because of the possibility of environment pollution and causing big accidents,
  - a safety state 1 – both subsystems are less safe and more dangerous because of the possibility of environment pollution and causing accidents,
  - a safety state 0 – STPOICIN is destroyed,

Moreover, by the expert opinions, we assume that there are possible the transitions between the components safety states only from better to worse ones;

- the safety structure of the system and subsystems

We consider the ship traffic and port operation information critical infrastructure network as a series safety structures.

The input necessary parameters of the port critical infrastructure network safety models are as follows

[EU-CIRCLE Report D3.3-GMU1, 2016], [EU-CIRCLE Report D2.2-GMU1]:

- the number of safety states of the system and components *z*=5,

- the critical safety state of the system $r = 2$,

- the system risk permitted level $\delta = 0.05$,

- the parameters of a system safety structure:

- series system
  - the number of components (subsystem) *n*=2,
  - the parameters of the subsystem $S_{31}$ safety structures
    Case 1 - series system
    - the number of components (subsystem) *n*=146,

    Case 2 – "m out of n" system
    - the number of components (subsystem) *n, n*=146
    - the threshold number of subsystems m, m = 73

    Case 3 – consecutive "m out of n: F" system
    - the number of components (subsystem) *n, n*=146
    - the threshold number of subsystems m, m = 2.
  - the parameters of the subsystem $S_{32}$ safety structures
    Case 1 - series system
    - the number of components (subsystem) *n, n*=18

    Case 2 – "m out of n" system
    - the number of components (subsystem) *n, n*=18
    - the threshold number of subsystems m, m = 3

    Case 3 – consecutive "m out of n: F" system
    - the number of components (subsystem) *n, n*=18
    - the threshold number of subsystems m, m = 2.

Considering this chapter assumptions and agreements, similarly to Section 3 in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], we assume that the components $E_{ij}^{(\upsilon)}$, $i = 1,2,3,...,k$, $j = 1,2,...,l_i$, $\upsilon = 1,2,3,...,158$ at the system operation states $z_b$, $b = 1,2,...,\nu^{(3)}$, have the exponential safety functions, i.e. the coordiantes of the vector

$$[S_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1,\ [S_{ij}^{(\upsilon)}(t,1)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,2)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,3)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,4)]^{(b)}],\ t \ge 0,\ i = 1,2,3,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,158,\ b = 1,2,...,\nu^{(3)}, \quad (95)$$

are given

$$[S_{ij}^{(\upsilon)}(t,u)]^{(b)} = P([T_{ij}^{(\upsilon)}]^{(b)}(u) > t \mid Z(t) = z_b) = \exp[-[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} t],\ t \ge 0,\ i = 1,2,3,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,158,\ b = 1,2,...,\nu^{(3)}. \quad (96)$$

Existing in the above formula the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,158$, (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,158$, departure from the safety state subset $\{u, u+1,...,4\}$ ) at the system operation process states $z_b$, $b = 1,2,...,\nu^{(3)}$, i.e. the coordinates of the vector of intensities

$$[\lambda_{ij}^{(\upsilon)}(\cdot)]^{(b)} = [1, [\lambda_{ij}^{(\upsilon)}(1)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(2)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(3)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(4)]^{(b)}],\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,158\ \ b = 1,2,...,\nu^{(3)}, \quad (97)$$

are given by

$$[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} = [\rho_{ij}^{(\upsilon)}(u)]^{(b)} \lambda_{ij}^{(\upsilon)}(u),\ u = 1,2,3,4,\ i = 1,2,...,k,\ j = 1,2,...,l_i,\ \upsilon = 1,2,3,...,158\ b = 1,2,...,\nu^{(3)}, \quad (98)$$

where $\lambda_{ij}^{(\upsilon)}(u)$, $u = 1,2,3,4$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, are the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,158$ (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystem $S_\upsilon$, $\upsilon = 1,2,3,...,158$ departure from the safety state subset

$\{u, u+1,...,4\}$) without of operation impact, i.e. the coordinate of the vector of intensities

$$\lambda_{ij}^{(\upsilon)}(\cdot) = [0, \lambda_{ij}^{(\upsilon)}(1), \lambda_{ij}^{(\upsilon)}(2), \lambda_{ij}^{(\upsilon)}(3), \lambda_{ij}^{(\upsilon)}(4)],$$
$$i = 1,2,...,k, \; j = 1,2,...,l_i, \; \upsilon = 1,2,...,158 \quad (99)$$

and

$$[\rho_{ij}^{(\upsilon)}(u)]^{(b)}, \; u = 1,2,3,4, \; i = 1,2,...,k, \; j = 1,2,...,l_i,$$
$$\upsilon = 1,2,3,...,158 \;\; b = 1,2,...,\nu^{(3)}, \quad (100)$$

are the coefficients of the operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,158$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,158$ intensities of departure from the safety state subset $\{u, u+1,...,4\}$) at the system operation process states $z_b$, $b = 1,2,...,\nu^{(3)}$, i.e. the coordinate of the vector coefficients of impact

$$[\rho_{ij}^{(\upsilon)}(\cdot)]^{(b)} = [0, [\rho_{ij}^{(\upsilon)}(1)]^{(b)}, [\rho_{ij}^{(\upsilon)}(2)]^{(b)},$$
$$[\rho_{ij}^{(\upsilon)}(3)]^{(b)}, [\rho_{ij}^{(\upsilon)}(4)]^{(b)}], \; i = 1,2,...,k,$$
$$j = 1,2,...,l_i, \; \upsilon = 1,2,3,...,158 \;\; b = 1,2,...,\nu^{(3)}, \quad (101)$$

The intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, $\{u, u+1,...,4\}$) without of operation impact on their safety are as follows:
- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1), \lambda_{ij}^{(\upsilon)}(2), \lambda_{ij}^{(\upsilon)}(3), \lambda_{ij}^{(\upsilon)}(4), \quad i = 1,2,...,k,$$
$$j = 1,2,...,l_i \; \upsilon = 1,2,3,...,158. \quad (102)$$

According to expert opinions, changing the port critical infrastructure network operation process states have influence on changing this system safety structures only, without of the impact on its components' safety.
Thus, the coefficients of the operation process impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the port critical infrastructure network subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,158$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of the port critical infrastructure network subsystems $S_\upsilon$, $\upsilon = 1,2,3,...,158$ intensities of departure from the safety state subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, at the system operation process states $z_b$, $b = 1,2,...,\nu^{(3)}$, are as follows:
- for subsystems $S_\upsilon$:

$$[\rho_{ij}^{(\upsilon)}(1)]^{(b)}, [\rho_{ij}^{(\upsilon)}(2)]^{(b)}, [\rho_{ij}^{(\upsilon)}(3)]^{(b)}, [\rho_{ij}^{(\upsilon)}(2)]^{(b)},$$
$$i = 1,2,...,k, \; j = 1,2,...,l_i, \; b = 1,2,...,\nu^{(3)},$$
$$\upsilon = 1,2,3,...,158 \quad (103)$$

Thus, by (2.11), (2.15) and (2.18)-(2.20) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the new intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, related to the climate-weather influence on its safety are as follows:
- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1), \lambda_{ij}^{(\upsilon)}(2), \lambda_{ij}^{(\upsilon)}(3), \lambda_{ij}^{(\upsilon)}(4),$$
$$i = 1,2,...,k, \; j = 1,2,...,l_i \; \upsilon = 1,2,3,...,158. \quad (104)$$

## 4.4. Ship Traffic and Port Operation Information Critical Infrastructure Network Safety Characteristics

In [Kołowrocki, Soszyńska-Budny, 2011], [Guze, Kołowrocki, 2017], it is fixed that the shipping critical infrastructure network safety structure and its subsystems depend on its changing in time operation states and its components safety are not changing at the particular operation states. The influence of the system operation states changing on the changes of the system safety structure and its components safety functions are as follows.
The subsystem $S_3$ consist of $k = 2$ subsystems, each composed of $n(i)$ components $E_{ij}^{(3)}$, $i = 1,2$, $j = 1,2,...,n(i)$ i.e. $l_1 = 146$, $l_2 = 18$ with the exponential safety functions given below.
We assume that at the system operation state $z_b^{(3)}$, $b = 1,2,...,\nu^{(3)}$, the system is composed of the subsystems $S_\upsilon$, $\upsilon = 1,2,$, each composed of of $l_i$ components $E_{ij}^{(3)}$, $i = 1,2$, $j = 1,2,...,l_i$ i.e. $l_1 = 146$, $l_2 = 18$ at the system operation states $z_b^{(3)}$ $b = 1,2,...,\nu^{(3)}$, with the exponential safety functions given below.

At the operation state $z_b^{(3)}$, $b = 1,2,...,\nu^{(3)}$, the ship traffic and port operation information critical infrastructure network series five-state conditional safety function is given by:

$$[\boldsymbol{S}(t,\cdot)]^{(b)} = [1, [\boldsymbol{S}(t,1)]^{(b)}, [\boldsymbol{S}(t,2)]^{(b)}, [\boldsymbol{S}(t,3)]^{(b)}, [\boldsymbol{S}(t,4)]^{(b)}],$$
$$t \ge 0,\ b = 1,2,\dots,v^{(3)}, \tag{105}$$

where

$$[\boldsymbol{S}(t,u)]^{(b)} = [\ddot{\boldsymbol{S}}_{18}(t,u)]^{(b)} = [\boldsymbol{S}^{(1)}(t,u)]^{(b)} \cdot [\boldsymbol{S}^{(2)}(t,u)]^{(b)} \cdot \ldots \cdot [\boldsymbol{S}^{(158)}(t,u)]^{(b)}$$
$$\text{for } u = 1,2,3,4,\ b = 1,2,\dots,v^{(3)}. \tag{106}$$

The expected values and standard deviations of the ship traffic and port operation information critical infrastructure network conditional lifetimes in the safety state subsets $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, at the operation state $z_b^{(3)}$, $b = 1,2,\dots,v^{(3)}$, , calculated according to (4.15)-(4.16) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1),\ \mu_b(2),\ \mu_b(3),\ \mu_b(4), \tag{107}$$

$$\sigma_b(1),\ \sigma_b(2),\ \sigma_b(3),\ \sigma_b(4), \tag{108}$$

and further, using (3.17) and (5.67) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(3)}$, $b = 1,2,\dots,v^{(3)}$, , respectively are:

$$\bar{\mu}_b(1),\ \bar{\mu}_b(2),\ \bar{\mu}_b(3),\ \bar{\mu}_b(4). \tag{109}$$

In the case when the operation time is large enough, the ship traffic and port operation information crititcal infrastructure network unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],$$
$$t \ge 0, \tag{110}$$

where according to (3.13) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3] and considering shipping critical infrastructure network operation process transient probabilities at the operation states given by (5.4) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \ge 0,$$
$$u = 1,2,\dots,4, \tag{111}$$

Since the critical safety state is $r = 1$, then according to (3.19) in [EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \ge 0, \tag{112}$$

where $\boldsymbol{S}(t,2)$ is given by (111).

## 5. Safety of Joint Network of Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks

### 5.1. Joint Network of Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks Description

The Joint Network of Baltic Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks (JNBPSSTPOICIN) is operating at the Baltic Sea Region. We assume that this system is composed of a number of main subsystems having an essential influence on its safety.
There are distinguished following subsystems:

- $S_1$ - the Port Critical Infrastructure Network subsystem,
- $S_2$ - the Shipping Critical Infrastructure Network subsystem,
- $S_3$ - the Ship Traffic and Port Operation Information Critical Infrastructure subsystem.

### 5.2. Joint Network of Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks Operation Process

We assume that the joint network of the critical infrastructure networks $CIN^{(1)}$, $CIN^{(2)}$, $CIN^{(3)}$ during its operation states process are taking numbers of different operation states defined as the vector [Guze, Kołowrocki, 2017]:

$$Z(t) = [Z^{(1)}(t),\ Z^{(2)}(t),\ Z^{(3)}(t)],\ t \in <0,+\infty). \tag{114}$$

where $Z^{(1)}(t)$ defines the port critical infrastructure network operation process with $\prod_{i=1}^{18} n_i^{(1)}$ operation states for the eighteen core ports, $Z^{(2)}(t)$ defines the shipping critical infrastructure network operation process with

$$n_{11}^{(2)}\cdot\ldots\cdot n_{1,14}^{(2)}\cdot n_{21}^{(2)}\cdot\ldots\cdot n_{2,14}^{(2)}\cdot n_{31}^{(2)}\cdot\ldots\cdot n_{3,14}^{(2)}\cdot n_{41}^{(2)}\cdot\ldots$$
$$\cdot n_{4,14}^{(2)}\cdot n_{51}^{(2)}\cdot\ldots\cdot n_{5,14}^{(2)}$$

operation states and $Z^{(3)}(t)$ defines the ship traffic and port operation information critical infrastructure network with $n_1^{(3)}\cdot n_2^{(3)}\cdot\ldots\cdot n_{146}^{(3)}$ operation states.

Furthermore, the maximum possible operation states for JNPSSTPOICIN is given by:

$$N=\left(\prod_{a=1}^{18} n_a^{(1)}\right)\cdot n_{11}^{(2)}\cdot\ldots\cdot n_{1,14}^{(2)}\cdot n_{21}^{(2)}\cdot\ldots\cdot n_{2,14}^{(2)}$$
$$\cdot n_{31}^{(2)}\cdot\ldots\cdot n_{3,14}^{(2)}\cdot n_{41}^{(2)}\cdot\ldots\cdot n_{4,14}^{(2)}\cdot n_{51}^{(2)}\cdot\ldots\cdot n_{5,14}^{(2)}\cdot n_1^{(3)}$$
$$\cdot n_2^{(3)}\cdot\ldots\cdot n_{146}^{(3)},$$

i.e. we consider the following operation states $z_0,\ldots,z_{N-1}$ described by paticular vector with accordance to (114).

According to Section 2.1 the critical infrastructure network joint operation process may be described by:

- the vector of the initial probabilities $p_b(0)=P(Z\ (0)=z_b)$, $b=1,2,\ldots,\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)}$, of the joint critical infrastructure network operation processes $Z(t)$ staying at particular operation states at the moment $t=0$

$$[p_b(0)]_{1\times\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)}}$$
$$=[p_{b_1}^{(1)}(0);p_{b_2}^{(2)}(0);p_{b_3}^{(3)}(0)]=[p_{b_1b_2b_3}(0)]$$
$$=[p_{111}(0),p_{112}(0),\ldots,p_{11\nu^{(3)}}(0);p_{121}(0),p_{131}(0),\ldots,$$
$$p_{1\nu^{(2)}1}(0);p_{211}(0),p_{212}(0),\ldots,p_{21\nu^{(3)}}(0);p_{221}(0),$$
$$p_{231}(0),p_{2\nu^{(2)}1}(0);\ldots;p_{\nu^{(1)}11}(0),p_{\nu^{(1)}12}(0),\ldots,$$
$$p_{\nu^{(1)}1\nu^{(3)}}(0);\ldots,p_{\nu^{(1)}\nu^{(2)}\nu^{(3)}}(0)] \quad (113)$$

where $b_i=1,2,\ldots,\nu^{(i)}$, $i=1,2,3$;

- the matrix of the probabilities $p_{bl}$, $b,l=1,2,\ldots,\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)}$, $b\neq l$, of the joint critical infrastructure network operation process $Z(t)$ transitions between the operation states $z_b$ and $z_l$

$$[p_{bl}]_{(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})\times(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})}=\left[p_{i_1j_1;i_2j_2;i_3j_3}\right]$$

$$=\begin{bmatrix} p_{11;11;11} & p_{11;11;12} & \cdots & p_{1\nu^{(1)};1\nu^{(2)};1\nu^{(3)}} \\ p_{11;11;21} & p_{11;11;22} & \cdots & p_{2\nu^{(1)};1\nu^{(2)};\nu^{(3)}\nu^{(3)}} \\ \cdots & \cdots & \cdots & \cdots \\ p_{\nu^{(1)}1;\nu^{(2)}1;\nu^{(3)}1} & p_{\nu^{(1)}1;\nu^{(2)}1;\nu^{(3)}2} & \cdots & p_{\nu^{(1)}\nu^{(1)};\nu^{(2)}\nu^{(2)};\nu^{(3)}\nu^{(3)}}\end{bmatrix}, \quad (114)$$

where $[p_{i_1j_1;i_2j_2;i_3j_3}]_{(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})\times(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})}$ is given by

a) $p_{i_1j_1}\cdot p_{i_2j_2}\cdot p_{i_3j_3}$, when critical infrastructutre networks are independent,
b) $p_{i_1j_1}\cdot p_{i_2j_2|i_1j_1}\cdot p_{i_3j_3|i_1j_1\cap i_2j_2}$, when critical infrastructutre networks are dependent,

and by formal agreement

$$p_{bb;bb;bb}=0 \text{ for } b=1,2,\ldots,\nu^{(i)},\ i=1,2,3;$$

- the matrix of conditional distribution functions $H_{bl}(t)=P(\theta_{bl}<t)$, $b,l=1,2,\ldots,\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)}$, $b\neq l$, of the joint critical infrastructure network operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ at the operation states

$$[H_{bl}(t)]_{(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})\times(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})}=\left[H_{i_1j_1;i_2j_2;i_3j_3}(t)\right]=$$
$$\begin{bmatrix} H_{11;11;11}(t) & H_{11;11;12}(t) & \cdots & H_{1\nu^{(1)};1\nu^{(2)};1\nu^{(3)}}(t) \\ H_{11;11;21}(t) & H_{11;11;22}(t) & \cdots & H_{2\nu^{(1)};1\nu^{(2)};\nu^{(3)}\nu^{(3)}}(t) \\ \cdots & \cdots & \cdots & \cdots \\ H_{\nu^{(1)}1;\nu^{(2)}1;\nu^{(3)}1}(t) & H_{\nu^{(1)}1;\nu^{(2)}1;\nu^{(3)}2}(t) & \cdots & H_{\nu^{(1)}\nu^{(1)};\nu^{(2)}\nu^{(2)};\nu^{(3)}\nu^{(3)}}(t)\end{bmatrix}, \quad (115)$$

and by formal agreement

$$H_{bb;bb;bb}(t)=0 \text{ for } b=1,2,\ldots,\nu^{(i)},\ i=1,2,3.$$

We introduce the matrix of the conditional density functions of the joint critical infrastructure network operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ at the operation states corresponding to the conditional distribution functions $H_{bl}(t)$

$$[h_{bl}(t)]_{(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})\times(\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)})}=\left[h_{i_1j_1;i_2j_2;i_3j_3}(t)\right]=$$
$$\begin{bmatrix} h_{11;11;11}(t) & h_{11;11;12}(t) & \cdots & h_{1\nu^{(1)};1\nu^{(2)};1\nu^{(3)}}(t) \\ h_{11;11;21}(t) & h_{11;11;22}(t) & \cdots & h_{2\nu^{(1)};1\nu^{(2)};\nu^{(3)}\nu^{(3)}}(t) \\ \cdots & \cdots & \cdots & \cdots \\ h_{\nu^{(1)}1;\nu^{(2)}1;\nu^{(3)}1}(t) & h_{\nu^{(1)}1;\nu^{(2)}1;\nu^{(3)}2}(t) & \cdots & h_{\nu^{(1)}\nu^{(1)};\nu^{(2)}\nu^{(2)};\nu^{(3)}\nu^{(3)}}(t)\end{bmatrix}, \quad (116)$$

and

$$h_{bl}(t)=\frac{d}{dt}[H_{bl}(t)]$$

for $b,l=1,2,\ldots,\nu^{(1)}\cdot\nu^{(2)}\cdot\nu^{(3)}$ $b\neq l$, and by formal agreement

$h_{bb;bb;bb}\ (t) = 0$ for $b = 1,2,...,v^{(i)}$, $i = 1,2,3$.

## 5.3. Joint Network of Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks Safety Parameters

According to the effectiveness and safety aspects of the operation of the Joint Network of Baltic Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks, we fix [Guze, Kołowrocki, 2017]:

- the number of JNBPSSTPOICIN safety states $(z = 4)$ and we distinguish the following five safety states:
  - a safety state 4 – JNBPSSTPOICIN operations are fully safe,
  - a safety state 3 – JNBPSSTPOICIN operations are less safe and more dangerous, because of fact that one of the three CINs is less safe,
  - a safety state 2 – JNBPSSTPOICIN operations are less safe and more dangerous, , because of fact that two of the three CINs are less safe,
  - a safety state 1 – JNBPSSTPOICIN operations are less safe and very dangerous, three CINs are less safe,
  - a safety state 0 – JNBPSSTPOICIN is destroyed, three CINs are dangerous for users and environment.

Moreover, by the expert opinions, we assume that there are possible the transitions between the components safety states only from better to worse ones;

- the safety structure of the system and subsystems

The JNBPSSTPOICIN is a complex series system composed of

**Case 1.** Three series subsystems $S_1$, $S_2$ $S_3$.

**Case 2.** Two "m out of n" subsystems $S_1$, $S_2$ and one series $S_3$.

**Case 3.** Two consecutive "k out of n:F" subsystems $S_1$, $S_2$ and one series $S_3$.

Each of them containing fixed number of components as it was mentioned above in Sections 2-4.

The necessary parameters of the Joint Network of Baltic Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks safety models are as follows [EU-CIRCLE Report D3.3-GMU1, 2016], [EU-CIRCLE Report D2.2-GMU1] :

- the number of safety states of the system and components $z$=4,

- the critical safety state of the system $r$ = 2,

- the system risk permitted level $\delta$ = 0.05,

- the parameters of a system safety structure:

- series system
  - the number of components (subsystem) $n$=3
  - the parameters of the subsystems $S^{(i)}$, $i = 1,2,3$ safety structures

    Case 1 - series system
    - the number of components (subsystem) $n_i$, $i = 1,2,3$,

      $n_1 = 18$

      $n_2 = c \cdot d$

      $n_3 = 164$;

    Case 2 – "$m_i$ out of $n_i$" system
    - the number of components (subsystem) $n_i$, $i = 1,2$,

      $n_1 = 18$

      $n_2 = c \cdot d$
    - the threshold number of subsystems $m_i$, $i = 1,2$,

      $m_1 = 3$

      $m_2 = \lceil 0.5 \cdot c \cdot d \rceil$

    Case 3 – consecutive "$m_i$ out of $n_i$: F" system
    - the number of components (subsystem) $n_i$, $i = 1,2$,

      $n_1 = 18$

      $n_2 = c \cdot d$
    - the threshold number of subsystems $m_i$, $i = 1,2$,

      $m_1 = 2$

      $m_2 = \lfloor 0.25 \cdot c \cdot d \rfloor$

Considering this chapter assumptions and agreements, similar to Section 3 in [EU-CIRCLE Report D3.3-GMU3], we assume that the components $E_{ij}^{(\upsilon)}$, $i = 1,2,3,...,k$, $j = 1,2,...,l_i$, $\upsilon = 1,2,3$, at the system operation states $z_b$, $b = 1,2,...,v^{\upsilon}$, have the exponential safety functions, i.e. the coordiantes of the vector

$$[S_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1,\ [S_{ij}^{(\upsilon)}(t,1)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,2)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,3)]^{(b)},\ [S_{ij}^{(\upsilon)}(t,4)]^{(b)}],\ t \geq 0,$$
$$i = 1,2,3,\dots,k,\ j = 1,2,\dots,l_i,\ \upsilon = 1,2,3,\ b = 1,2,\dots,v^{\upsilon}, \quad (117)$$

are given

$$[S_{ij}^{(\upsilon)}(t,u)]^{(b)} = P([T_{ij}^{(\upsilon)}]^{(b)}(u) > t \mid Z(t) = z_b) = \exp[-[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} t],$$
$$t \geq 0,\ i = 1,2,3,\dots,k,\ j = 1,2,\dots,l_i,\ \upsilon = 1,2,3,\ b = 1,2,\dots,v^{\upsilon}, \quad (118)$$

Existing in the above formula the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the subsystem $S_\upsilon,\ \upsilon = 1,2,3,$ (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the subsystem $S_\upsilon,\ \upsilon = 1,2,3,$ departure from the safety state subset $\{u, u+1,\dots,4\}$) at the system operation process states $z_b,\ b = 1,2,\dots,v$ , i.e. the coordinates of the vector of intensities

$$[\lambda_{ij}^{(\upsilon)}(\cdot)]^{(b)} = [1, [\lambda_{ij}^{(\upsilon)}(1)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(2)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(3)]^{(b)}, [\lambda_{ij}^{(\upsilon)}(4)]^{(b)}],\ i = 1,2,\dots,k,$$
$$j = 1,2,\dots,l_i,\ \upsilon = 1,2,3,\ b = 1,2,\dots,v^{\upsilon}, \quad (119)$$

are given by

$$[\lambda_{ij}^{(\upsilon)}(u)]^{(b)} = [\rho_{ij}^{(\upsilon)}(u)]^{(b)} \lambda_{ij}^{(\upsilon)}(u),\ u = 1,2,3,4,$$
$$i = 1,2,\dots,k,\ j = 1,2,\dots,l_i,\ \upsilon = 1,2,3,\ b = 1,2,\dots,v^{\upsilon}, \quad (120)$$

where $\lambda_{ij}^{(\upsilon)}(u),\ u = 1,2,3,4,\ i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, are the intensities of ageing of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the subsystems $S_\upsilon,\ \upsilon = 1,2,3$ (the intensities of the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the subsystem $S_\upsilon,\ \upsilon = 1,2,3,$ departure from the safety state subset $\{u, u+1,\dots,4\}$) without of operation impact, i.e. the coordinate of the vector of intensities

$$\lambda_{ij}^{(\upsilon)}(\cdot) = [0,\ \lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3),\ \lambda_{ij}^{(\upsilon)}(4)],$$
$$i = 1,2,\dots,k,\ j = 1,2,\dots,l_i,\ \upsilon = 1,2,3 \quad (121)$$

and

$$[\rho_{ij}^{(\upsilon)}(u)]^{(b)},\ u = 1,2,3,4,\ i = 1,2,\dots,k,\ j = 1,2,\dots,l_i,$$
$$\upsilon = 1,2,3,\ b = 1,2,\dots,v^{\upsilon}, \quad (122)$$

are the coefficients of the operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the subsystems $S_\upsilon,\ \upsilon = 1,2,3,$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the subsystems $S_\upsilon,\ \upsilon = 1,2,3,$ intensities of departure from the safety state subset $\{u, u+1,\dots,4\}$) at the system operation process states $z_b,\ b = 1,2,\dots,v^{\upsilon}$, , i.e. the coordinate of the vector coefficients of impact

$$[\rho_{ij}^{(\upsilon)}(\cdot)]^{(b)} = [0,\ [\rho_{ij}^{(\upsilon)}(1)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(3)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(4)]^{(b)}\ ],\ i = 1,2,\dots,k,\ j = 1,2,\dots,l_i,$$
$$\upsilon = 1,2,3,\ b = 1,2,\dots,v^{\upsilon}. \quad (123)$$

The intensities of components departure from the safety states subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, $\{u, u+1,\dots,4\}$) without of operation impact on their safety are as follows:

- for subsystems $S_\upsilon$:

$$\lambda_{ij}^{(\upsilon)}(1),\ \lambda_{ij}^{(\upsilon)}(2),\ \lambda_{ij}^{(\upsilon)}(3)\ ,\ \lambda_{ij}^{(\upsilon)}(4)\ ,$$
$$i = 1,2,\dots,k,\ j = 1,2,\dots,l_i\ \upsilon = 1,2,3. \quad (124)$$

According to expert opinions, changing the Joint Network of Baltic Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks operation process states have influence on changing this system safety structures only, without of the impact on its components‘ safety.

Thus, the coefficients of the operation process impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the port critical infrastructure network subsystems $S_\upsilon,\ \upsilon = 1,2,3,$ intensities of ageing (the coefficients of operation impact on the components $E_{ij}^{(\upsilon)}$, $i = 1,2,\dots,k,\ j = 1,2,\dots,l_i$, of the Joint Network of Baltic Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks subsystems $S_\upsilon,\ \upsilon = 1,2,3,$ intensities of departure from the safety state subset $\{1,2,3,4\}$, $\{2,3,4\}$, $\{3,4\}$, $\{4\}$, at the system operation process states $z_b,\ b = 1,2,\dots,v^{\upsilon}$, are as follows:

- for subsystems $S_\upsilon$:

$$[\rho_{ij}^{(\upsilon)}(1)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(3)]^{(b)},\ [\rho_{ij}^{(\upsilon)}(2)]^{(b)},$$
$$i=1,2,...,k,\ j=1,2,...,l_i,\ b=1,2,...,v^\upsilon,$$
$$\upsilon=1,2,3. \quad (125)$$

Thus, by (11), (15) and (18)-(20), the new intensities of components departure from the safety states subset {1,2,3,4}, {2,3,4}, {3,4}, {4}, related to the climate-weather influence on its safety are as follows:

- for subsystems $S_1$:

$$[\lambda_{ij}^{(1)}(1)],\ [\lambda_{ij}^{(1)}(2)],\ [\lambda_{ij}^{(1)}(3)],\ [\lambda_{ij}^{(1)}(4)],$$
$$i=1,2,...,18\quad j=1,2,3; \quad (126)$$

- for subsystems $S_2$:

$$[\lambda_{ij}^{(2)}(1)],\ [\lambda_{ij}^{(2)}(2)],\ [\lambda_{ij}^{(2)}(3)],\ [\lambda_{ij}^{(2)}(4)],$$
$$i=1,2,...,a\cdot b\quad j=1,2,3,...,l_i; \quad (127)$$

- for subsystems $S_3$:

$$[\lambda_{ij}^{(3)}(1)],\ [\lambda_{ij}^{(3)}(2)],\ [\lambda_{ij}^{(3)}(3)],\ [\lambda_{ij}^{(3)}(4)],$$
$$i=1,2,\ j=1,2,3,...,l_i. \quad (128)$$

## 5.4. Joint Network of Port, Shipping and Ship Traffic and Port Operation Information Critical Infrastructure Networks Safety Characteristics

In [Kołowrocki, Soszyńska-Budny, 2011], [Guze, Kołowrocki, 2017], it is fixed that the joint network of the port, shipping and ship traffic and port operation information critical networks safety structure and its subsystems depend on its changing in time operation states and its components safety are not changing at the particular operation states. The influence of the system operation states changing on the changes of the system safety structure and its components safety functions are as follows.

We assume that at the system operation state $z_b$, $b=1,2,...,v^\upsilon$, the system is composed of the subsystems $S_\upsilon$, $\upsilon=1,2,3,$, each composed of components $E_{ij}$, $i=1,2,...,k$, $j=1,2,...,l_i$ at the system operation states $z_b$ $b=1,2,...,v^\upsilon$, with the exponential safety functions given below [Guze, Kołowrocki, 2017].

At the operation state $z_b$, $b=1,2,...,v^\upsilon$, the joint network of the port, shipping and ship traffic and port operation information critical infrastructure networks series five-state conditional safety function is given by [Guze, Kołowrocki, 2017]:

$$[\boldsymbol{S}(t,\cdot)]^{(b)} =$$
$$[1, [\boldsymbol{S}(t,1)]^{(b)}, [\boldsymbol{S}(t,2)]^{(b)}, [\boldsymbol{S}(t,3)]^{(b)}, [\boldsymbol{S}(t,4)]^{(b)}],$$
$$t\ge 0,\ b=1,2,...,v^\upsilon, \quad (129)$$

with cooridantes given by

$$[\boldsymbol{S}(t,u)]^{(b)}=[\ddot{\boldsymbol{S}}_{18}(t,u)]^{(b)} =$$
$$[\boldsymbol{S}^{(1)}(t,u)]^{(b)}\cdot[\boldsymbol{S}^{(2)}(t,u)]^{(b)}\cdot...\cdot[\boldsymbol{S}^{(18)}(t,u)]^{(b)}$$
$$\text{for } u=1,2,3,4,\ b=1,2,...,v^\upsilon, \quad (130)$$

The expected values and standard deviations of the joint network conditional lifetimes in the safety state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, at the operation state $z_b^{(1)}$, $b=1,2,...,v^\upsilon$,, calculated according to (3.15)-(3.16) in [EU-CIRCLE Report D3.3-GMU3], respectively are:

$$\mu_b(1),\ \mu_b(2),\ \mu_b(3),\ \mu_b(4), \quad (131)$$

$$\sigma_b(1),\ \sigma_b(2),\ \sigma_b(3),\ \sigma_b(4), \quad (132)$$

and further, using (3.17) and (5.67) in [EU-CIRCLE Report D3.3-GMU3], the mean values of the conditional lifetimes in the particular safety states 1, 2, 3, 4 at the operation state $z_b^{(1)}$, $b=1,2,...,v^\upsilon$,, respectively are:

$$\bar{\mu}_b(1),\ \bar{\mu}_b(2),\ \bar{\mu}_b(3),\ \bar{\mu}_b(4). \quad (133)$$

In the case when the operation time is large enough, the joint network of the port, shipping and ship traffic and port operation information critical networks unconditional safety function is given by the vector

$$\boldsymbol{S}(t,\cdot) = [1,\ \boldsymbol{S}(t,1),\ \boldsymbol{S}(t,2),\ \boldsymbol{S}(t,3),\ \boldsymbol{S}(t,4)],$$
$$t\ge 0, \quad (134)$$

where according to (3.13) in [EU-CIRCLE Report D3.3-GMU3] and considering the joint network operation process transient probabilities at the operation states given by (5.4), the vector co-ordinates are given respectively by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b[\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \geq 0,$$
$$u = 1,2,...,z, \tag{135}$$

Since the critical safety state is $r$ =1, then according to (3.19) in [EU-CIRCLE Report D3.3-GMU3], the system risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \geq 0, \tag{136}$$

where $\boldsymbol{S}(t,2)$ is given by (135).

## 6. Conclusions

The material given in this paper delivers the basis for procedures and algorithms that allow finding the main and practically important safety characteristics of the joint network of port, shipping and ship traffic and port operation information critical infrastructure networks defined as complex technical systems. The predicted safety characteristics of these exemplary critical infrastructure networks will be applied to the assets operating at the variable conditions. It is important because they are different from those determined for this system operating at constant conditions [Kołowrocki, Soszyńska-Budny, 2011]. The multi-state approach in safety analysis with the semi-Markov modeling of the CIN's operation process has been used. This way of modeling is chosen with regard to the importance of the considered CIN's safety and operating process effectiveness. To realize this assumption, the safety function, and its risk function are defined as the crucial indicators/indices from the safety practitioners point of view. The graph of the risk function corresponds to the fragility curve. Other practically significant critical infrastructure network safety indices like its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level has been defined. Moreover, the component and critical infrastructure network intensities of ageing/degradation and the coefficients of operation impact on component and critical infrastructure network intensities of ageing has been introduced. These safety indicators have been identified and determined for the joint network of three critical infrastructure networks.

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## References

Barbu V., Limnios N., Empirical estimation for discrete-time semi-Markov processes with applications in reliability. Journal of Nonparametric Statistics, Vol. 18, No. 7-8, 483-498, 2006

EU-CIRCLE Report D1.1, EU-CIRCLE Taxonomy, 2015

Ferreira F., Pacheco A., Comparison of level-crossing times for Markov and semi-Markov processes. Statistics and Probability Letters, Vol. 7, No 2, 151-157, 2007

Glynn P.W., Haas P.J., Laws of large numbers and functional central limit theorems for generalized semi-Markov processes. Stochastic Models,Vol. 22, No 2, 201-231, 2006

Grabski F., Semi-Markov Processes: Application in System Reliability and Maintenance, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014

Guze S., Kołowrocki, K., Safety modelling of port, shipping and ship traffic and port operation information critical infrastructure join network related to its operation process, Proc. European Safety and Reliability Conference – ESREL 2017, 2017 (to appear)

Kołowrocki K., Soszyńska J., A general model of industrial systems operation processes related to their environment and infrastructure. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 2, Vol. 2, 223-226, 2008

Kolowrocki K., Soszynska J. Modeling environment and infrastructure influence on reliability and operation process of port oil transportation system. Electronic Journal Reliability & Risk Analisys: Theory & Applications, Vol. 2, No 3, 131-142, 2009a

Kolowrocki K., Soszynska J., Safety and risk evaluation of Stena Baltica ferry in variable operation conditions.ElectronicJournal Reliability & Risk Analisys: Theory & Applications, Vol. 2, No 4, 168-180, 2009b

Kołowrocki K., Soszyńska J., Statistical identification and prediction of the port oil pipeline system's operation process and its reliability and risk evaluation. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 3, Vol. 2, 241-250, 2009c

Kołowrocki K., Soszyńska J., Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 3, Vol. 1, 2, 211-222, 2009d

Kołowrocki K., Soszyńska J., Methods and algorithms for evaluating unknown parameters of components reliability of complex technical systems. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 2, 223-230, 2009e

Kołowrocki K., Soszyńska J., Statistical identification and prediction of the port oil pipeline system's operation process and its reliability and risk evaluation. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 2, 241-250, 2009f

Kolowrocki K., Soszynska J., Reliability modeling of a port oil transportation system's operation processes. International Journal of Performability Engineering, Vol. 6, No 1, 77-87, 2010a

Kolowrocki K., Soszynska J., Reliability, availability and safety of complex technical systems: modelling –identification – prediction – optimization.Journal of Polish Safety and Reliability Association, Summer Safetyand Reliability Seminars,Vol. 4, No 1, 133-158, 2010b

Limnios N., Oprisan G., Semi-Markov Processes and Reliability. Birkhauser, Boston, 2005

Mercier S., Numerical bounds for semi-Markovian quantities and application to reliability. Methodologyand Computing in Applied Probability, Vol. 10, No 2, 179-198, 2008

Soszyńska J., Systems reliability analysis in variable operation conditions. PhD Thesis, Gdynia MaritimeUniversity-System Research Institute Warsaw, 2007 (in Polish)

Soszyńska J., Reliability and risk evaluation of a port oil pipeline transportation system in variable operation conditions Int J of Press Vessel and Pip 87(2-3):81-87, 2010

Xue J., On multi-state system analysis, *IEEE Trans on Reliab.* 34, 329-337, 1985

Xue J., Yang K., Dynamic reliability analysis of coherent multi-state systems, *IEEE Trans on Reliab.* 4(44), 683-688, 1995a

Xue J., Yang K., Symmetric relations in multi-state systems, *IEEE Trans on Reliab* 4(44), 689-693, 1995