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Integrated Impact Model on Critical Infrastructure Safety Related to Climate-Weather Change Process Including Extreme Weather Hazards

Keywords

Climate change process, weather hazards, impact, model, safety.

Abstract

In the paper a general safety analytical model of complex technical system related to the climate-weather change process in its operating area is defined. First, the system operation at climate-weather variable conditions is given. Additionally, the semi-Markov approach is used. Further, the safety model of the multistate system at climate-weather variable conditions is introduced. The notions of the conditional safety functions at the climate-weather particular states, the unconditional safety function and the risk function of the complex system at changing in time climate-weather conditions are presented. The other safety indices like mean lifetime up to the exceeding a critical safety state, the moment when the risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components and the coefficients of the climate-weather impact on the critical infrastructure and its components intensities of ageing are defined.

1. Introduction

The paper is devoted to the climate change influence on the safety of a critical infrastructure defined as a complex system in its operating environment that in the case of its degradation have significant destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas. A general safety analytical model of complex technical system related to the climate-weather change process in its operating area is proposed. It is the integrated model of complex technical system safety, linking its multistate safety model and the model of the climate-weather change process at its operating area, considering variable at the different climate-weather states impacted by them system components safety parameters. The notions of the conditional safety functions at the climate-weather particular states, the unconditional safety function and the risk function of the complex system at changing in time climate-weather conditions are defined. Other, practically significant, critical infrastructure safety indices introduced in the paper are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components and the coefficients of the climate-weather impact on the critical infrastructure and its components intensities of ageing. These safety indices are defined in general for any critical infrastructure and determined particularly for the port oil piping transportation system and the maritime ferry technical system considering varying in time their components safety parameters influenced by changing in time climate-weather conditions at their operating areas.

Most real complex technical systems are strongly influenced by changing in time the climate-weather conditions at their operating areas. The time dependent interactions between the climate-weather change process states varying at the system operating area and the system components safety states changing are evident features of most real technical systems including critical infrastructures. The common critical infrastructure safety and climate-weather change at its operating area analysis is of great value in the industrial practice because of negative impacts of extreme weather hazards on the critical infrastructure safety. The convenient tools for analyzing this problem are the multistate critical infrastructures safety modelling [Kołowrocki, Soszyńska-Budny, 2011; Xue, 1985; Xue, Yang, 1995a-b] commonly used with the semi-Markov modeling [Ferreira, Pacheco, 2007; Glynn, Hass, 2006; Grabski, 2014; Kołowrocki 2005; Limnios, Oprisan, 2005; Mercier 2008] of the climate-weather change processes at their operating areas, leading to the construction the joint general safety models of the critical infrastructures related to the climate-weather change processes at their operating areas.

In the case of critical infrastructure safety analysis, the determination of its safety function and its risk function which graph corresponds to the fragility curve are crucial indices for its operators and users. Other practically significant discussed in the report critical infrastructure safety indices are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components and the coefficients of the climate-weather impact on the critical infrastructure and its components intensities of ageing. These safety indices are defined in general and can be used to the port oil piping transportation system and the maritime ferry technical system, and other critical infrastructure.

2. System Operation at Climate-Weather Variable Conditions

2.1. States of Climate-Weather Change Process

To define the climate-weather states in the fixed area, we distinguish a, $a \in N$, parameters that describe the climate-weather states in this area and mark the values they can take by $w_1, w_2, ..., w_a$. Further, we assume that the possible values of the i-th parameter w_i, $i = 1,2,\ldots, a$, can belong to the interval $< b_i, d_i)$, $i = 1,2,\ldots, a$. We divide each of the intervals $< b_i, d_i)$, $i = 1,2,\ldots, a$, into n_i, $n_i \in N$, disjoint subintervals

$$< b_{i1}, d_{i1}), < b_{i2}, d_{i2}), \ldots, < b_{in_j}, d_{in_j}),$$

such that

$$< b_{i1}, d_{i1}) \cup < b_{i2}, d_{i2}) \cup \ldots \cup < b_{in_i}, d_{in_i})$$
$$= < b_i, d_i),\ d_{ij_i} = b_{ij_i+1},\ j_i = 1,2,...,n_i - 1,\ i = 1,2,..., a.$$

Thus, the vector $(w_1, w_2, ..., w_a)$ describing the climate-weather states can take values from the set of the a dimensional space points of the Descartes product

$$< b_1, d_1) \times < b_2, d_2) \times \ldots \times < b_a, d_a)$$

that is composed of the a dimensional space domains of the form

$$< b_{1j_1}, d_{1j_1}) \times < b_{2j_2}, d_{2j_2}) \times \ldots \times < b_{aj_a}, d_{aj_a}),$$

where $j_i = 1,2,\ldots n_i$, $i = 1,2,\ldots, a$.

The domains of the above form are called the climate-weather states of the climate-weather change process and numerated from 1 up to the value $w = n_1 \cdot n_2 \cdot \ldots \cdot n_a$ and mark by $c_1, c_2, \ldots, c_w$.

The interpretation of the states of the climate-weather change process in the case $a = 2$ is given in Figure 1. In this case, we have $w = n_1 \cdot n_2$ climate-weather states of the climate-weather change process represented in Figure 1 by the squares marked by $c_1, c_2, \ldots, c_w$.

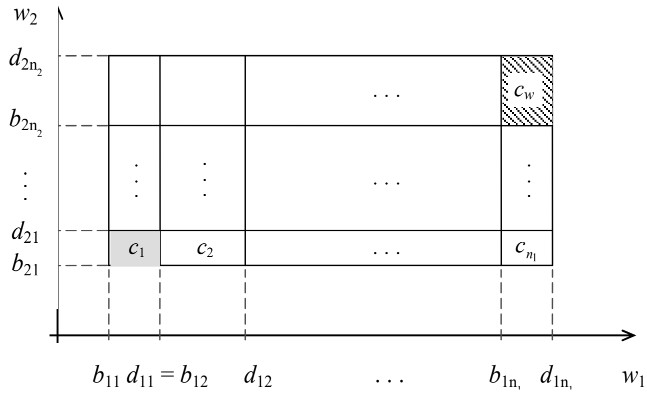

Figure 1. Interpretation of the climate-weather change process two dimensional climate-weather states

According to Chapter 2 in [EU-CIRCLE Report D2.1-GMU3, 2016], the climate-weather change process states are defined by the vectors

$$(w_1, w_2, ..., w_a)$$

and marked by

$$c_1, c_2, \ldots, c_w,$$

where

$$w = n_1 \cdot n_2 \cdot \ldots \cdot n_a.$$

Further, we can call each of the the climate-weather change process state c_j, $j = 1,2,\ldots, w$, of the vector form $(w_1, w_2, \ldots, w_a)$:

- the a^{th} category extreme weather hazard state of the climate-weather change process if all a weather parameters w_i, $i = 1,2,\ldots, a$, are at the 1^{st} category extreme weather hazard state;
- the $(a-1)^{th}$ category extreme weather hazard state of the climate-weather change process if a-1 of weather parameters w_i, $i = 1,2,\ldots, a$, are at the 1^{st} category extreme weather hazard state;
- the $(a-2)^{th}$ category extreme weather hazard state of the climate-weather change process if a-2 of weather parameters w_i, $i = 1,2,\ldots, a$, are at the 1^{st} category extreme weather hazard state;
- …
- the 1^{st} category extreme weather hazard state of the climate-weather change process if 1 of weather parameters w_i, $i = 1,2,\ldots, a$, are at the 1^{st} category extreme weather hazard state;
- the 0^{th} category extreme weather hazard state of the climate-weather change process if none of weather parameters w_i, $i = 1,2,\ldots, a$, are at the 1^{st} category extreme weather hazard state.

Thus, the a^{th} category extreme weather hazard state of the climate-weather change process is the most denderous for the critical infrastructure operation and safety.

2.2. Semi-Markov Model of Climate-Weather Change Process

To model the climate-weather change process for the critical infrastructure operating area we assume that the climate-weather in this area is taking w, $w \in N$, different climate-weather states $c_1, c_2,\ldots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{c_1, c_2,\ldots, c_w\}$. Assuming that the climate-weather change process $C(t)$ is a semi-Markov process it can be described by[EU-CIRCLE Report D2.1-GMU3, 2016]:

– the vector $[q_b(0)]_{1\mathrm{x}w}$ of the initial probabilities $q_b(0) = P(C(0) = c_b)$, $b = 1,2,\ldots, w$, of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b at the moment $t = 0$;

– the matrix $[q_{bl}]_{w\mathrm{x}w}$ of the probabilities of transitions q_{bl}, b, $l = 1,2,\ldots, w$, $b \neq l$, of the climate-weather change process $C(t)$ from the climate-weather states c_b to c_l;

– the matrix $[C_{bl}(t)]_{w\mathrm{x}w}$ of the conditional distribution functions $C_{bl}(t) = P(C_{bl} < t)$, b, $l = 1,2,\ldots, w$, of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, b, $l = 1,2,\ldots, w$, $b \neq l$,

Assuming that we have identified the above parameters of the climate-weather change process semi-Markov model, we can predict this process basic characteristics.

The mean values of the conditional sojourn times C_{bl}, are given by [EU-CIRCLE Report D2.1-GMU3, 2016],

$$N_{bl} = E[C_{bl}] = \int_0^\infty t dC_{bl}(t) = \int_0^\infty t c_{bl}(t) dt,$$
$$b, l = 1,2,\ldots, w. \tag{1}$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times C_b, $b = 1,2,\ldots, w$, of the climate-weather change process $C(t)$ at the climate-weather states c_b $b = 1,2,\ldots, w$, are given by [EU-CIRCLE Report D2.1-GMU3, 2016],

$$C_b(t) = \sum_{l=1}^{v} q_{bl} C_{bl}(t), \; b = 1,2,\ldots, w, \tag{2}$$

Hence, the mean values $E[C_b]$ of the climate-weather change process $C(t)$ unconditional sojourn times C_b, $b = 1,2,\ldots, w$, at the climate-weather states are given by

$$N_b = E[C_b] = \sum_{l=1}^{v} q_{bl} N_{bl}, \; b = 1,2,\ldots, w, \tag{3}$$

where N_{bl} are defined by the formula (4.1) in a case of any distribution of sojourn times C_{bl} and by the formulae (3.2)-(3.8) given in [EU-CIRCLE Report D2.1-GMU3, 2016], in the cases of particular defined respectively by (2.5)-(2.11) in [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.

The limit values of the climate-weather change process $C(t)$ transient probabilities at the particular operation states

$$q_b(t) = P(C(t) = c_b), \; t \in <0,+\infty), \; b = 1,2,\ldots, w, \tag{4}$$

are given by [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016],

$$q_b = \lim_{t \to \infty} q_b(t) = \frac{\pi_b N_b}{\sum_{l=1}^{v} \pi_l N_l}, \; b = 1,2,\ldots, w, \tag{5}$$

where N_b, $b = 1,2,...,w$, are given by (3), while the steady probabilities π_b of the vector $[\pi_b]_{1\times w}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][q_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \tag{6}$$

In the case of a periodic climate-weather change process, the limit transient probabilities q_b, $b = 1,2,...,w$, at the climate-weather states defined by (5), are the long term proportions of the climate-weather change process $C(t)$ sojourn times at the particular climate-weather states C_b, $b = 1,2,...,w$.

Other interesting characteristics of the system climate-weather change process $C(t)$ possible to obtain are its total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b, $b = 1,2,...,w$, during the fixed time. It is well known, [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016], that the climate-weather change process total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b for sufficiently large time C have approximately normal distributions with the expected value given by

$$\hat{N}[\hat{C}_b] = q_b C, \quad b = 1,2,...,w, \tag{7}$$

where q_b are given by (5).

2.3. Safety of Multistate Systems at Climate-Weather Variable Conditions

We assume that the changes of the climate-weather change process $C(t)$ states at the system operating area have an influence on the system multistate components E_i, $i = 1,2,...,n$, safety. Consequently, we denote the system multistate component E_i, $i = 1,2,...,n$, conditional lifetime in the safety state subset $\{u, u+1,...,z\}$ while the climate-weather change process $C(t)$ at the system operating area is at the state c_b, $b = 1,2,...,w$, by $T''^{(b)}_i(u)$ and its conditional safety function by the vector

$$[S''_i(t,\cdot)]^{(b)} = [1, [S''_i(t,1)]^{(b)}, ..., [S''_i(t,z)]^{(b)}],$$
$$t \in <0,\infty), \; b = 1,2,...,w, \; i = 1,2,...,n, \tag{8}$$

with the coordinates defined by

$$[S''_i(t,u)]^{(b)} = P(T''^{(b)}_i(u) > t \mid C(t) = c_b) \tag{9}$$

for $t \in <0,\infty)$, $u = 1,2,...,z$, $b = 1,2,...,w$.

The safety function $[S''_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T''^{(b)}_i(u)$ in the safety state subset $\{u, u+1,...,z\}$ is greater than t, while the climate-weather change process $C(t)$ at the system operating area is at the state c_b, $b = 1,2,...,w$.

In the case, the system components E_i, $i = 1,2,...,n$, at the climate-weather change process $C(t)$ at the system operating area states c_b, $b = 1,2,...,w$, have the exponential safety functions, the coordinates of the vector (8) are given by

$$[S''_i(t,u)]^{(b)} = P(T''^{(b)}_i(u) > t \mid C(t) = c_b)$$
$$= \exp[-[\lambda''_i(u)]^{(b)} t], \; t \in <0,\infty),$$
$$b = 1,2,...,w, \; i = 1,2,...,n. \tag{10}$$

Existing in (10) the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1,...,z\}$) at the climate-weather change process $C(t)$ at the system operating area states c_b, $b = 1,2,...,w$, i.e. the coordinates of the vector

$$[\lambda''_i(\cdot)]^{(b)} = [0, [\lambda''_i(1)]^{(b)}, \ldots, [\lambda''_i(z)]^{(b)}],$$
$$t \in <0,+\infty), \; b = 1,2,...,w, \; i = 1,2,...,n, \tag{11}$$

are given by

$$[\lambda''_i(u)]^{(b)} = \rho''^{(b)}_i(u) \cdot \lambda_i(u), \; u = 1,2,...,z,$$
$$b = 1,2,...,w, \; i = 1,2,...,n, \tag{12}$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1,...,z\}$) without climate-weather change impact, i.e. the coordinate of the vector

$$\lambda_i(\cdot) = [0, \lambda_i(1), \ldots, [\lambda_i(z)], \; i = 1,2,...,n, \tag{13}$$

and

$$[\rho''_i(u)]^{(b)}, \; u = 1,2,...,z, \; b = 1,2,...,w,$$
$$i = 1,2,...,n, \tag{14}$$

are the coefficients of climate-weather impact on the system components E_i, $i = 1,2,...,n$, intensities of ageing (the coefficients of climate-weather impact on critical infrastructure component E, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u, u+1,..., z\}$) at the climate-weather change process operating area states c_b, $b = 1,2,..., w$, i.e. the coordinate of the vector

$$[\rho_i^{"}(\cdot)]^{(b)} = [0, [\rho_i^{"}(1)]^{(b)}, \ldots, [\rho_i^{"}(z)]^{(b)}],$$
$$b = 1,2,..., w, \ i = 1,2,..., n. \tag{15}$$

The system component safety function (8), the system components intensities of ageing (11) and the coefficients of the climate-weather impact on the system components intensities of ageing (15) are main system component safety indices.

Similarly, we denote the system conditional lifetime in the safety state subset $\{u, u+1,..., z\}$ while the climate-weather change process $C(t)$ at the system operating area is at the state c_b, $b = 1,2,..., w$, by $T''^{(b)}(u)$ and the conditional safety function of the system by the vector

$$[\boldsymbol{S''}(t,\cdot)]^{(b)} = [1, [\boldsymbol{S''}(t,1)]^{(b)}, ..., [\boldsymbol{S''}(t,z)]^{(b)}], \tag{16}$$

with the coordinates defined by

$$[\boldsymbol{S''}(t,u)]^{(b)} = P(T''^{(b)}(u) > t | C(t) = c_b) \tag{17}$$

for $t \in \langle 0, \infty)$, $u = 1,2,..., z$, $b = 1,2,..., w$.

Further, we denote the system unconditional lifetime in the safety state subset $\{u, u+1,..., z\}$ by $T''(u)$ and the unconditional safety function of the system by the vector

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)], \tag{18}$$

with the coordinates defined by

$$\boldsymbol{S''}(t,u) = P(T''(u) > t) \tag{19}$$

for $t \in \langle 0, \infty)$, $u = 1,2,..., z$.

In the case when the system operation time C is large enough, the coordinates (19) of the unconditional safety function of the system defined by (18) are given by

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b [\boldsymbol{S''}(t,u)]^{(b)}$$
$$\text{for } t \ge 0, \ u = 1,2,..., z, \tag{20}$$

where $[\boldsymbol{S''}(t,u)]^{(b)}$, $u = 1,2,..., z$, $b = 1,2,..., w$, are the coordinates of the system conditional safety functions defined by (16)-(17) and $q_b, b = 1,2,..., \nu$, are the climate-weather change process $C(t)$ at the system operating area limit transient probabilities at the state c_b, $b = 1,2,..., w$, given by (5).

The exemplary graph of a five-state (z = 4) critical infrastructure safety function

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), \boldsymbol{S''}(t,2), \boldsymbol{S''}(t,3), \boldsymbol{S''}(t,4)],$$
$$t \in \langle 0, \infty),$$

is shown in Figure 2.

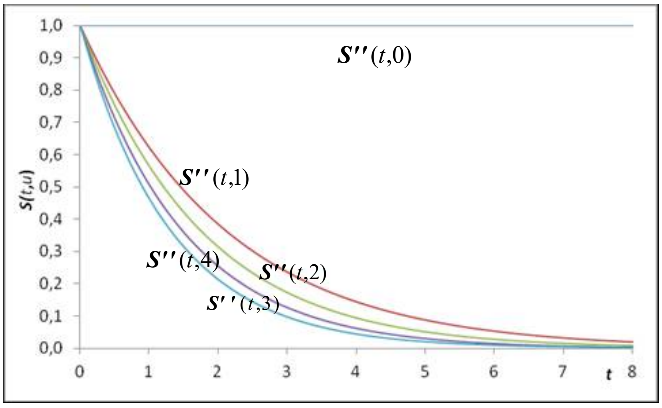

Figure 2 The graphs of a five-state critical infrastructure safety function $\boldsymbol{S''}(t,\cdot)$ coordinates

The mean value of the system unconditional lifetime $T''(u)$ in the safety state subset $\{u, u+1,..., z\}$ is given by [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016],

$$\mu''(u) \cong \sum_{b=1}^{w} q_b \mu''_b(u), \ u = 1,2,..., z, \tag{21}$$

where $\mu''_b(u)$ are the mean values of the system conditional lifetimes $T''^{(b)}(u)$ in the safety state subset $\{u, u+1,..., z\}$ at the climate-weather change process $C(t)$ at the system operating area state c_b, $b = 1,2,..., w$, given by

$$\mu''_b(u) = \int_0^\infty [\boldsymbol{S''}(t,u)]^{(b)} dt, \ u = 1,2,..., z, \tag{22}$$

$[\boldsymbol{S''}(t,u)]^{(b)}$, $u = 1,2,..., z$, $b = 1,2,..., w$, are defined by (16)-(17) and $q_b, b = 1,2,..., \nu$, are given by (4). Whereas, the variance of the system unconditional lifetime $T''(u)$ is given by

$$\sigma''^2(u) = 2\int_0^\infty t\boldsymbol{S''}(t,u)dt - [\mu''(u)]^2,$$
$$u = 1,2,...,z, \quad (23)$$

where $\boldsymbol{S''}(t,u)$, $u = 1,2,...,z$. are given by (19)-(20) and $\mu''(u)$ $u = 1,2,...,z$. are given by (21)-(22).

Hence, according to (1.19) [Kołowrocki, Soszyńska-Budny, 2011], we get the following formulae for the mean values of the unconditional lifetimes of the system in particular safety states

$$\bar{\mu}''(u) = \mu''(u) - \mu''(u+1), \; u = 0,1,..., z-1,$$
$$\bar{\mu}''(z) = \mu''(z), \quad (24)$$

where $\mu''(u)$, $u = 0,1,...,z$, are given by (4.21).

Moreover, according (1.20)-(1.21) [Kołowrocki, Soszyńska-Budny, 2011], if r is the system critical safety state, then the system risk function

$$\boldsymbol{r''}(t) = P(S''(t) < r \mid S''(0) = z) = P(T''(r) \le t),$$
$$t \in <0,\infty), \quad (25)$$

defined as a probability that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment t = 0 [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011] is given by

$$\boldsymbol{r''}(t) = 1 - \boldsymbol{S''}(t,r), \; t \in <0,\infty),$$
(4.26)

where $\boldsymbol{S''}(t,r)$ is the coordinate of the system unconditional safety function given by (20) for $u = r$.

The graph of the system risk function presented in Figure 3 is called the fragility curve of the system.

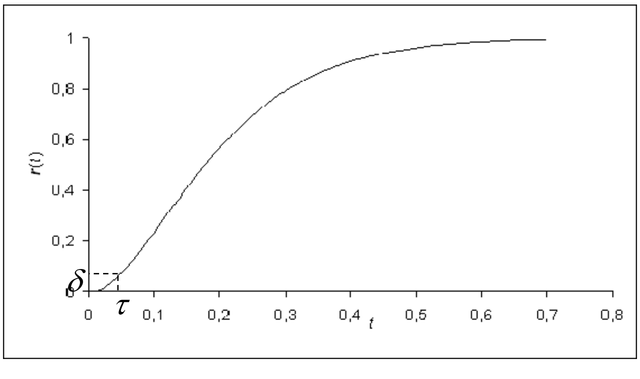

Figure 3 The graph (The fragility curve) of a system risk function $\boldsymbol{r''}(t)$

The system safety function, the system risk function and the system fragility curve are main system safety indices. Other practically useful system safety induices are:

- the mean value of the unconditional system lifetime $T''(r)$ up to the exceeding the critical safety state r given by

$$\mu''(r) \cong \sum_{b=1}^{w} q_b \mu''_b(r), \quad (27)$$

where $\mu''_b(r)$ are the mean values of the system conditional lifetimes $T''^{(b)}(r)$ in the safety state subset $\{r, r+1,...,z\}$ at the climate-weather change process $C(t)$ at the system operating area state c_b, $b = 1,2,...,w$,
given by

$$\mu''_b(r) = \int_0^\infty [\boldsymbol{S''}(t,r)]^{(b)} dt, \; b = 1,2,...,w, \quad (28)$$

$[\boldsymbol{S''}(t,r)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,w$, are defined by (21)-(22) and q_b are given by (5);

- the standard deviation of the system lifetime $T(r)$ up to the exceeding the critical safety state r given by

$$\sigma''(r) = \sqrt{n''(r) - [\mu''(r)]^2}, \quad (29)$$

where

$$n''(r) = 2\int_0^\infty t\,\boldsymbol{S''}(t,r)dt, \quad (30)$$

where $\boldsymbol{S''}(t,r)$ is given by (20) and $\mu''(r)$ is given by (27) for $u = r$.

- the moment τ the system risk function exceeds a permitted level δ given by

$$\tau = \boldsymbol{r''}^{-1}(\delta), \quad (31)$$

and illustrated in Figure 3, where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r''}(t)$ given by (26).

Other critical infrastructure safety indices are:

- the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$) related to the climate-weather change impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}''(t,\cdot) = [0, \boldsymbol{\lambda}''(t,1), \ldots, \boldsymbol{\lambda}''(t,z)],$$
$$t \in <0,+\infty), \tag{32}$$

where

$$\boldsymbol{\lambda}''(t,u) = \frac{-\frac{d\boldsymbol{S}''(t,u)}{dt}}{\boldsymbol{S}''(t,u)}, \quad t \in <0,+\infty),$$
$$u = 1,2,\ldots,z; \tag{33}$$

- the coefficients of the climate-weather impact on the critical infrastructure intensities of ageing (the coefficients of the climate-weather impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, \ldots, z\}$), i.e. the coordinates of the vector

$$\boldsymbol{\rho}''(t,\cdot) = [0, \boldsymbol{\rho}''(t,1), \ldots, \boldsymbol{\rho}''(t,z)],$$
$$t \in <0,+\infty), \tag{34}$$

where

$$\boldsymbol{\lambda}''(t,u) = \boldsymbol{\rho}''(t,u) \cdot \lambda(t,u),$$
$$t \in <0,+\infty), \quad u = 1,2,\ldots,z. \tag{35}$$

and $\lambda(t,u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$) without of climate-weather impact, i.e. the coordinate of the vector

$$\lambda(t,\cdot) = [0, \lambda(t,1), \ldots, \lambda(t,z)], \quad t \in <0,+\infty). \tag{36}$$

In the case, the critical infrastructure have the exponential safety functions, i.e.

$$\boldsymbol{S}''(t,\cdot) = [0, \boldsymbol{S}''(t,1), \ldots, \boldsymbol{S}''(t,z)],$$
$$t \in <0,+\infty), \tag{37}$$

where

$$\boldsymbol{S}''(t,r) = \exp[-\boldsymbol{\lambda}''(u)t], \quad t \in <0,+\infty),$$
$$\boldsymbol{\lambda}''(u) \geq 0, \quad u = 1,2,\ldots,z, \tag{38}$$

the critical infrastructure safety indices defined by (32)-(36) take forms:
- the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$) related to climate-weather change impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}''(\cdot) = [0, \boldsymbol{\lambda}''(1), \ldots, \boldsymbol{\lambda}''(z)], \tag{39}$$

- the coefficients of the climate-weather impact on the scritical infrastructure intensities of ageing (the coefficients of the climate-weatcher impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, \ldots, z\}$), i.e. the coordinate of the vector

$$\boldsymbol{\rho}''(\cdot) = [0, \boldsymbol{\rho}''(1), \ldots, \boldsymbol{\rho}''(z)], \tag{40}$$

where

$$\boldsymbol{\lambda}''(u) = \boldsymbol{\rho}''(u) \cdot \boldsymbol{\lambda}(u), \quad u = 1,2,\ldots,z. \tag{41}$$

and $\boldsymbol{\lambda}(u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u, u+1, \ldots, z\}$) without of climate-weather impact, i.e. the coordinate of the vector

$$\boldsymbol{\lambda}(\cdot) = [0, \boldsymbol{\lambda}(1), \ldots, \boldsymbol{\lambda}(z)]. \tag{42}$$

3. Safety of Multistate Exponential Systems at Climate-Weather Variable Conditions

We assume that the system components at the climate-weather change process $C(t)$ at the system operating area states have the exponential safety functions. This assumption and the results given in Chapter 1 [Kołowrocki, Soszyńska-Budny, 2011] yield the following results formulated in the form of the following proposition.

Proposition 1
If components of the multi-state system at the climate-weather change process $C(t)$ at the system operating area states c_b, $b = 1,2,\ldots,w,$ have the exponential safety functions given by

$$[S''_i(t,\cdot)]^{(b)} = [1, [S''_i(t,1)]^{(b)}, \ldots, [S''_i(t,z)]^{(b)}],$$
$$t \in <0,\infty), \quad b = 1,2,\ldots,w, \quad i = 1,2,\ldots,n, \tag{43}$$

with the coordinates

$$[S''_i(t,u)]^{(b)} = P(T''^{(b)}_i(u) > t \mid C(t) = c_b)$$
$$= \exp[-[\lambda''_i(u)]^{(b)} t],$$
$$t \in <0,\infty), \quad b = 1,2,\ldots,w, \quad i = 1,2,\ldots,n, \tag{44}$$

and the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1,...,z\}$) related to climate-weather change impact, existing in (44), are given by

$$[\lambda''_i(u)]^{(b)} = \rho''^{(b)}_i(u) \cdot \lambda_i(u),\ u = 1,2,...,z,\ b = 1,2,...,w,\ i = 1,2,...,n, \tag{45}$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1,...,z\}$) without climate-weather change impact and

$$[\rho''_i(u)]^{(b)},\ u = 1,2,...,z,\ b = 1,2,...,w,\ i = 1,2,...,n, \tag{46}$$

are the coefficients of the climate-weather impact on the system components E_i, $i = 1,2,...,n$, intensities E_i, $i = 1,2,...,n$, of ageing (the coefficients of operation impact on critical infrastructure component E, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u, u+1,...,z\}$) without climate-weather change impact, in the case of series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F" systems and respectively by

$$[S''_{ij}(t,\cdot)]^{(b)} = [1, [S''_{ij}(t,1)]^{(b)}, \ldots, [S''_{ij}(t,z)]^{(b)}],\ t \in <0,\infty),\ b = 1,2,...,w,\ i = 1,2,...,k,\ j = 1,2,...,l_i, \tag{47}$$

with the coordinates

$$[S''_{ij}(t,u)]^{(b)} = P(T''^{(b)}_{ij}(u) > t \mid C(t) = c_b) = \exp[-[\lambda''_{ij}(u)]^{(b)} t],\ t \in <0,\infty),\ b = 1,2,...,w,\ i = 1,2,...,k,\ j = 1,2,...,l_i, \tag{48}$$

and the intensities of ageing of the system components E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, (the intensities of the system components E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, departure from the safety state subset $\{u, u+1,...,z\}$) related to climate-weather change impact, existing in (48), are given by

$$[\lambda''_{ij}(u)]^{(b)} = \rho''^{(b)}_{ij}(u) \cdot \lambda_{ij}(u),\ u = 1,2,...,z,\ b = 1,2,...,w,\ i = 1,2,...,k,\ j = 1,2,...,l_i, \tag{49}$$

where $\lambda_{ij}(u)$ are the intensities of ageing of the system components E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, (the intensities of the system components E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, departure from the safety state subset $\{u, u+1,...,z\}$) without climate-weather change impact and

$$[\rho''_{ij}(u)]^{(b)},\ u = 1,2,...,z,\ b = 1,2,...,w,\ i = 1,2,...,k,\ j = 1,2,...,l_i, \tag{50}$$

are the coefficients of the climate-weather impact on the system components E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, intensities E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, of ageing (the coefficients of operation impact on critical infrastructure component E, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u, u+1,...,z\}$) without climate-weather change impact, in the case of series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series systems and the system operation time C is large enough, then its multistate unconditional safety function is given by the vector:

i) for a series system

$$\boldsymbol{S''}(t,\cdot) = [1,\ \boldsymbol{S''}(t,1),\ ...,\ \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \tag{51}$$

where

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b \exp[-\sum_{i=1}^{n} [\lambda''_i(u)]^{(b)} t] \text{ for } t \geq 0,\ u = 1,2,...,z; \tag{52}$$

ii) for a parallel system

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \tag{53}$$

where

$$\boldsymbol{S''}(t,u) \cong 1 - \sum_{b=1}^{w} q_b \prod_{i=1}^{n} [1 - \exp[-[\lambda''_i(u)]^{(b)} t]] \text{ for } t \geq 0,\ u = 1,2,...,z; \tag{54}$$

iii) for a "*m* out of *n*" system

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S'}(t,z)] \text{ for } t \geq 0, \quad (55)$$

where

$$\boldsymbol{S''}(t,u) \cong 1 - \sum_{b=1}^{w} q_b \sum_{\substack{r_1,r_2,...,r_n=0 \\ r_1+r_2+...+r_n \leq m-1}}^{1} \prod_{i=1}^{n} \exp[-r_i[\lambda''_i(u)]^{(b)} t] [1-\exp[-[\lambda''_i(u)]^{(b)} t]]^{1-n}$$
$$\text{for } t \geq 0, \; u = 1,2,...,z, \quad (56)$$

or

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \quad (57)$$

where

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b \sum_{\substack{r_1,r_2,...,r_n=0 \\ r_1+r_2+...+r_n \leq \overline{m}}}^{1} \prod_{i=1}^{n} [1-\exp[-[\lambda''_i(u)]^{(b)} t]]^{r_i} \exp[-(1-r_i)[\lambda''_i(u)]^{(b)} t]$$
$$\text{for } t \geq 0, \; u = 1,2,...,z, \text{ and } \overline{m} = n-m; \quad (58)$$

iv) for a consecutive "m out of n: F" system

$$\boldsymbol{CS''}(t,\cdot) = [1, \boldsymbol{CS''}(t,1), ..., \boldsymbol{CS''}(t,z)] \text{ for } t \geq 0, \quad (59)$$

where

$$\boldsymbol{CS''}(t,u) \cong \begin{cases} 1 & \text{for } n < m, \\ 1 - \sum_{b=1}^{w} q_b \prod_{i=1}^{n} [1-\exp[-[\lambda''_i(u)]^{(b)} t]] & \text{for } n = m, \\ \sum_{b=1}^{w} q_b [\exp[-[\lambda''_n(u)]^{(b)} t][\boldsymbol{CS''}_{n-1}(t,u)]^{(b)} \\ + \sum_{i=1}^{m-1} \exp[-[\lambda''_{n-i}(u)]^{(b)} t][\boldsymbol{CS''}_{n-i-1}(t,u)]^{(b)} \\ \prod_{j=n-i+1}^{n} [1-\exp[-[\lambda''_j(u)]^{(b)} t]] & \text{for } n > m, \end{cases}$$
$$\text{for } t \geq 0, \; u = 1,2,...,z; \quad (60)$$

v) for a series-parallel system

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \quad (61)$$

where

$$\boldsymbol{S''}(t,u) \cong 1 - \sum_{b=1}^{w} q_b \prod_{i=1}^{k} [1-\exp[-\sum_{j=1}^{l_i} [\lambda''_{ij}(u)]^{(b)} t]]$$
$$\text{for } t \geq 0, \; u = 1,2,...,z; \quad (62)$$

vi) for a parallel-series system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \quad (62)$$

where

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b \prod_{i=1}^{k} [1-\prod_{j=1}^{l_i} [1-\exp[-[\lambda''_{ij}(u)]^{(b)} t]]]$$
$$\text{for } t \geq 0, \; u = 1,2,...,z; \quad (63)$$

vii) for a series-"m out of k" system

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \quad (64)$$

where

$$\boldsymbol{S''}(t,u) \cong 1 - \sum_{b=1}^{w} q_b \sum_{\substack{r_1,r_2,...,r_k=0 \\ r_1+r_2+...+r_k \leq m-1}}^{1} \prod_{i=1}^{k} [\prod_{j=1}^{l_i} \exp[-[\lambda''_{ij}(u)]^{(b)} t]]^{r_i} \cdot [1-\prod_{j=1}^{l_i} \exp[-[\lambda''_{ij}(u)]^{(b)} t]]^{1-r_i}$$
$$\text{for } t \geq 0, \; u = 1,2,...,z, \quad (65)$$

or

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \quad (66)$$

where

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b \sum_{\substack{r_1,r_2,...,r_k=0 \\ r_1,r_2,...+r_k \leq \overline{m}}}^{1} \prod_{i=1}^{k} [1-[\prod_{i=1}^{l_i} \exp[-[\lambda''_{ij}(u)]^{(b)} t]]^{r_i} \cdot [\prod_{j=1}^{l_i} \exp[-[\lambda''_{ij}(u)]^{(b)} t]]^{1-r_i}$$
$$\text{for } t \geq 0, \; \overline{m} = k-m, \; u = 1,2,...,z; \quad (67)$$

viii) for a "m_i out of l_i"-series system

$$\boldsymbol{S''}(t,\cdot) = [1, \boldsymbol{S''}(t,1), ..., \boldsymbol{S''}(t,z)] \text{ for } t \geq 0, \quad (68)$$

where

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b \prod_{i=1}^{k} [1 - \sum_{\substack{r_1,r_2,...,r_{l_i}=0 \\ r_1+r_2+...+r_{l_i} \le m_i - 1}}^{1} \prod_{j=1}^{l_i} \exp[-r_j [\lambda''_{ij}(u)]^{(b)} t]] \cdot [1 - \exp[-[\lambda''_{ij}(u)]^{(b)} t]]^{1-r_j}]$$

for $t \ge 0,\ u = 1,2,...,z,$ (69)

or

$\boldsymbol{S''}(t,\cdot) = [1,\ \boldsymbol{S''}(t,1),...,\boldsymbol{S''}(t,z)]$ for $t \ge 0,$ (70)

where

$$\boldsymbol{S''}(t,u) \cong \sum_{b=1}^{w} q_b \prod_{i=1}^{k} [\sum_{\substack{r_1,r_2,...,r_{l_i}=0 \\ r_1+r_2+...+r_{l_i} \le \overline{m}_i}}^{1} \prod_{j=1}^{l_i} [1 - \exp[-[\lambda''_{ij}(u)]^{(b)} t]]^{r_j} \cdot \exp[-(1-r_j)[\lambda''_{ij}(u)]^{(b)} t]]$$

for $t \ge 0,\ \overline{m}_i = l_i - m_i,\ i = 1,2,...,k,$ $u = 1,2,...,z;$ (71)

ix) for a series-consecutive "*m* out of *k*: F" system

$\boldsymbol{CS''}(t,\cdot) = [1,\ \boldsymbol{CS''}(t,1),\ ...,\ \boldsymbol{CS''}(t,z)]$ for $t \ge 0,$ (72)

where

$\boldsymbol{CS}(t,u) \cong \sum_{b=1}^{v} p_b [\ \boldsymbol{CS}(t,u)]^{(b)}$ for $t \ge 0,\ u = 1,2,...,z,$ (73)

and $[\boldsymbol{CS'}\ '(t,u)]^{(b)},\ t \ge 0,\ b = 1,2,...,w,$ are given by

$$[\boldsymbol{CS''}(t,u)]^{(b)} = \begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k} [1 - \exp[-\sum_{j=1}^{l_i} \lambda''_{ij}(u) t]] & \text{for } k = m, \\ \exp[-\sum_{j=1}^{l_k} [\lambda''_{kj}(u)]^{(b)} t][\boldsymbol{CS''}_{k-1;l_1,l_2,...,l_k}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m-1} [\exp[-\sum_{v=1}^{l_{k-j}} [\lambda''_{k-jv}(u)]^{(b)} t]] & \\ [\boldsymbol{CS}_{k-j-1;l_1,l_2,...,l_k}(t,u)]^{(b)} & \\ \cdot \prod_{i=k-j+1}^{k} [1 - \exp[-\sum_{v=1}^{l_i} [\lambda''_{iv}(u)]^{(b)} t]] & \text{for } k > m, \end{cases}$$

for $t \ge 0,\ u = 1,2,...,z;$ (74)

x) for a consecutive "m_i out of l_i: F"-series system

$\boldsymbol{CS''}(t,\cdot) = [1,\ \boldsymbol{CS''}(t,1),\ ...,\ \boldsymbol{CS''}(t,z)]$ for $t \ge 0,$ (75)

Where

$\boldsymbol{CS''}(t,u) \cong \sum_{b=1}^{w} q_b \prod_{i=1}^{k} [\boldsymbol{CS''}_{i,l_i}(t,u)]^{(b)}$ for $t \ge 0,$ $u = 1,2,...,z,$ (76)

and $[\boldsymbol{CS''}_{i,l_i}(t,u)]^{(b)}$, $t \ge 0,\ I = 1, 2,\dots, k,$ $b = 1,2,...,w,$ are given by

$$[\boldsymbol{CS''}^{m_i}_{i,l_i}(t,u)]^{(b)} = \begin{cases} 1 & \text{for } l_i < m_i, \\ 1 - \prod_{j=1}^{l_i} [1 - \exp[-[\lambda''_{ij}(u)]^{(b)} t]] & \text{for } l_i = m_i, \\ \exp[-[\lambda''_{il_i}(u)]^{(b)} t][\boldsymbol{CS''}^{m_i}_{i,l_i-1}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m_i-1} \exp[-[\lambda''_{il_i-j}(u)]^{(b)} t][\boldsymbol{CS''}^{m_i}_{i,l_i\text{-}j\text{-}1}(t,u)]^{(b)} & \\ \cdot \prod_{v=l_i-j+1}^{l_i} [1 - \exp[-[\lambda''_{iv}(u)]^{(b)} t]] & \text{for } l_i > m_i, \end{cases}$$

for $t \ge 0,\ u = 1,2,...,z.$ (77)

Remark 1

The formulae for the safety functions stated in *Proposition 1* are valid for the considered systems under the assumption that they do not change their structures at different climate-weather change process *C*(*t*) at the system operating area states c_b, $b = 1,2,...,w.$ This limitation can be simply omitted by the replacement in these formulae the system's structure shape constant parameters n, m, k, m_i, l_i, respectively by their changing at different operation states c_b, $b = 1,2,...,w,$ equivalent structure shape parameters $n^{(b)}, m^{(b)}, k^{(b)}, m_i^{(b)}, l_i^{(b)}, b = 1,2,...,w$

For the exponential complex technical systems, considered in *Proposition 1*, we determine the mean values $\mu''(u)$ and the standard deviations $\sigma''(u)$ of the unconditional lifetimes of the system in the safety state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ the mean values $\overline{\mu}''(u)$ of the unconditional lifetimes of the system in the particular safety states u, $u = 1,2,...,z,$ the system risk function ***r''***(*t*) and the moment τ'' when the system risk function exceeds a permitted level δ respectively defined by (21)-(26), after substituting for $\boldsymbol{S''}(t,u)$, $u = 1,2,...,z,$ the coordinates of the unconditional safety functions given respectively by (51)-(78).

4. Conclusions

The integrated general model of complex system safety, linking its safety model and its operating area climate-weather change process model and considering the climate-weather influence on its components' safety parameters is constructed. The report delivers the procedures and algorithms that allow to find the main an practically important safety characteristics of the complex technical systems impacted by changing in time climate-weather states. The results are applied to the safety evaluation of the port oil piping transportation system and the maritime ferry technical system impacted by the climate-weather change process at their operating areas. The predicted safety characteristics of these exemplary critical infrastructures operating at the variable climate-weather conditions are different from those determined for these systems operating at constant conditions without considering climate-weather influence [Kołowrocki, Soszyńska-Budny, 2011]. This fact justifies the sensibility of considering real systems' safety at the variable climate-weather conditions that is appearing out in a natural way from practice. This approach, upon the sufficient accuracy of the critical infrastructure operating area climate-weather change process and the critical infrastructure components safety parameters identification, makes its safety prediction much more precise.

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