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Integrated Impact Model on Critical Infrastructure Safety Related to Its Operation Process

Keywords

Critical infrastructure, impact, model, safety, operation process.

Abstract

The main objective of this paper is to present recently developed, the general safety analytical models of complex multistate technical systems related to their operation processes and to apply them practically to critical infrastructures. To realize this goal, the integrated model of critical infrastructure safety related to its operation process is proposed. The basic safety characteristics of this model are presented as the very practically significant. Furthermore, the unconditional safety functions of systems with different safety structures are determined under assumption that their safety functions are exponential. In case of the critical infrastructure safety analysis, its safety function and risk function which graph corresponds to the fragility curve, its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the component and critical infrastructure intensities of ageing/degradation and the coefficients of operation impact on component and critical infrastructure intensities of ageing are defined.

1. Introduction

The paper is devoted to safety modelling and prediction of critical infrastructure defined as a complex system in its operating environment that significant features are inside-system dependencies and outside-system dependencies, that in the case of its degradation have significant destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas. There are presented general safety analytical models of complex multistate technical systems related to their operation processes called complex technical systems. They are the integrated general models of complex technical systems, linking their multistate safety models and their operation processes models and considering variable at the different operation states their safety structures and their components' safety parameters. The conditional safety functions at the system particular operation states, the unconditional safety function and the risk function of the complex technical systems are defined. These joint models of the system safety and the variable in time system operation process are constructed for multistate series, parallel, "m out of n", consecutive "m out of n: F", series-parallel, parallel-series, series-"m out of k", "m_i out of l_i"-series, series-consecutive "m out of k: F" and consecutive "m_i out of l_i": F"-series systems. The joint models are applied to determining safety characteristics of these systems related to their varying in time safety structures and their components safety parameters. Under the assumption that the safety functions of the considered systems are exponential, the unconditional safety functions of these systems are determined.

Moreover, the set of practically useful indicators/indexes of critical infrastructure safety is proposed.

Most real technical systems are structurally very complex and they often have complicated operation processes. Large numbers of components and subsystems and their operating complexity cause that the evaluation and prediction of their safety is difficult. The time dependent interactions between the systems' operation processes operation states changing and the systems' structures and their components safety states changing processes are evident features of most real technical systems including critical infrastructures. The common safety and operation analysis of complex technical systems and critical infrastructures is of great value in the industrial practice. The convenient tools for analyzing this problem are the multistate system's safety modeling [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, 2014], [Xue, 1985], [Xue, Yang, 1995a-b] commonly used with the semi-Markov modeling [Ferreira, Pacheco, 2007], [Glynn, Hass, 2006], [Grabski, 2014], [Kołowrocki 20014], [Limnios, Oprisan, 2005], [Mercier, 2008] of the systems' operation processes, leading to the construction the joint general safety models of the complex technical systems related to their operation process [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011], [Kołowrocki, 2014] including critical infrastructures. The main objective of this report is to present recently developed, the general safety analytical models of complex multistate technical systems related to their operation processes [Kołowrocki, Soszyńska-Budny, 2011] and to apply them practically to real industrial systems and processes [Kołowrocki, Soszyńska, 2009b-c], [Kołowrocki, Soszyńska, 2010], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011] and critical infrastructures. In the case of critical infrastructure safety analysis, the determination of its safety function and its risk function which graph corresponds to the fragility curve, that are defined in the paper, are crutial indicators/indices for safety practitioners. Other practically significant discussed in the report critical infrastructure safety indices are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the component and critical infrastructure intensities of ageing/degradation and the coefficients of operation impact on component and critical infrastructure intensities of ageing.

2. System Operation at Variable Conditions

We assume that the system during its operation process is taking $v, v \in N$, different operation states $z_1, z_2, \ldots, z_v$. Further, we define the system operation process $Z(t)$, $t \in < 0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process [Grabski, 2014], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011] with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,\ldots, v$, $b \neq l.$ Under these assumptions, the system operation process may be described by:

- the vector $[p_b(0)]_{1\times v}$ of the initial probabilities $p_b(0) = P(Z(0) = z_b)$, $b = 1,2,\ldots, v$, of the system operation process $Z(t)$ staying at particular operation states at the moment $t = 0$;
- the matrix $[p_{bl}]_{v\times v}$ of probabilities p_{bl}, $b, l = 1,2,\ldots, v$, $b \neq l$, of the system operation process $Z(t)$ transitions between the operation states z_b and z_l ;
- the matrix $[H_{bl}(t)]_{v\times v}$ of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $t \geq 0$, $b, l = 1,2,\ldots, v$, $b \neq l$, of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states.

As the mean values $E[\theta_{bl}]$ of the conditional sojourn times θ_{bl} are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t), \ b, l = 1,2,\ldots, v, \ b \neq l, \quad (1)$$

then from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,\ldots, v$, of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,\ldots, v$, are given by [Soszyńska, 2010], [Kołowrocki, Soszyńska-Budny, 2011]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \ t \geq 0, \ b = 1,2,\ldots, v. \quad (2)$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,\ldots, v$, at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \ b = 1,2,\ldots, v, \quad (3)$$

where M_{bl} are defined by the formula (1).

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states $p_b(t) = P(Z(t) = z_b)$, $t \in < 0,+\infty)$, $b = 1,2,\ldots, v$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \ b = 1,2,\ldots, v, \quad (4)$$

where M_b, $b=1,2,...,v$, are given by (3), while the steady probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases}[\pi_b]=[\pi_b][p_{bl}]\\ \sum_{l=1}^{v}\pi_l=1.\end{cases} \quad (5)$$

In the case of a periodic system operation process, the limit transient probabilities p_b, $b=1,2,...,v$, at the operation states given by (4), are the long term proportions of the system operation process $Z(t)$ sojourn times at the particular operation states z_b, $b=1,2,...,v$.

Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b=1,2,...,v$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}_b=E[\hat{\theta}_b]=p_b\theta,\ b=1,2,...,v, \quad (6)$$

where p_b are given by (4).

3. Safety of Multistate Systems at Variable Operation Conditions

We assume that the changes of the operation states of the system operation process $Z(t)$ have an influence on the system multistate components E_i, $i=1,2,...,n$, safety and the system safety structure as well. Consequently, we denote the system multistate component E_i, $i=1,2,...,n$, conditional lifetime in the safety state subset $\{u,u+1,...,z\}$ while the system is at the operation state z_b, $b=1,2,...,v$, by $T_i^{(b)}(u)$ and its conditional safety function by the vector

$$[S_i(t,\cdot)]^{(b)}=[1,\ [S_i(t,1)]^{(b)},\ ...,\ [S_i(t,z)]^{(b)}\ ],$$
$$t\in<0,\infty),\quad b=1,2,...,v, \quad (7)$$

with the coordinates defined by

$$[S_i(t,u)]^{(b)}=P(T_i^{(b)}(u)>t|Z(t)=z_b) \quad (8)$$

for $t\in<0,\infty)$, $u=1,2,...,z$, $b=1,2,...,v$.

The safety function $[S_i(t,u)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the safety state subset $\{u,u+1,...,z\}$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b.

In the case, when the system components E_i, $i=1,2,...,n$, at the system operation process $Z(t)$ states z_b, $b=1,2,...,v$, have the exponential safety functions, the coordinates of the vector (7) are given by

$$[S_i(t,u)]^{(b)}=P(T_i^{(b)}(u)>t|Z(t)=z_b)$$
$$=\exp[-[\lambda_i(u)]^{(b)}t]$$
$$t\in<0,\infty),\ b=1,2,...,v,\ i=1,2,...,n. \quad (9)$$

Existing in (9) the intensities of ageing/degradation of the system components E_i, $i=1,2,...,n$, (the intensities of the system components E_i, $i=1,2,...,n$, departure from the safety state subset $\{u,u+1,...,z\}$) at the system operation states z_b, $b=1,2,...,v$, i.e. the coordinates of the vector

$$[\lambda_i(\cdot)]^{(b)}=[0,\ [\lambda_i(1)]^{(b)},\ \ldots,[\lambda_i(z)]^{(b)}\ ],$$
$$t\in<0,+\infty),\ b=1,2,...,v,\ i=1,2,...,n, \quad (10)$$

are given by

$$[\lambda_i(u)]^{(b)}=\rho_i^{(b)}(u)\cdot\lambda_i(u),$$
$$u=1,2,...,z,\ b=1,2,...,v,\ i=1,2,...,n, \quad (11)$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i=1,2,...,n$, (the intensities of the system components E_i, $i=1,2,...,n$, departure from the safety state subset $\{u,u+1,...,z\}$) without operation process impact, i.e. the coordinate of the vector

$$\lambda_i(\cdot)=[0,\ \lambda_i(1),\ \ldots,\lambda_i(z)\ ],\ i=1,2,...,n, \quad (12)$$

and

$$[\rho_i(u)]^{(b)},\ u=1,2,...,z,\ b=1,2,...,v,$$
$$i=1,2,...,n, \quad (13)$$

are the coefficients of operation impact on the system components E_i, $i=1,2,...,n$, intensities of ageing (the coefficients of operation impact on critical infrastructure component E, i = 1,2,...,n, intensities of departure from the safety state subset $\{u, u+1, ..., z\}$) at the system operation states z_b, $b = 1,2,...,v$, i.e. the coordinate of the vector

$$[\rho_i\,(\cdot)]^{(b)} = [0,\ [\rho_i\,(1)]^{(b)},\ \ldots, [\rho_i\,(z)]^{(b)}\ ],$$
$$b = 1,2,...,\nu,\ \ i = 1,2,...,n. \quad (14)$$

The system component safety function (7), the system components intensities' of ageing (4) and the coefficients of the operation impact on the system components intensities of ageing (14) are main system component safety indices.

Similarly, we denote the system conditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ while the system is at the operation state z_b, $b = 1,2,...,v$, by $T^{(b)}(u)$ and the conditional safety function of the system by the vector

$$[\boldsymbol{S}(t,\cdot)]^{(b)} = [1,\ [\boldsymbol{S}(t,1)]^{(b)}, ...,\ [\boldsymbol{S}(t,z)]^{(b)}], \quad (15)$$

with the coordinates defined by

$$[\boldsymbol{S}(t,u)]^{(b)} = P(T^{(b)}(u) > t \,|\, Z(t) = z_b) \quad (16)$$

for $t \in <0,\infty)$, $u = 1,2,..., z$, $b = 1,2,...,\nu$.

The safety function $[\boldsymbol{S}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is greater than t, while the system operation process $Z(t)$ is at the operation state z_b.

Further, we denote the system unconditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ by $T(u)$ and the unconditional safety function of the system by the vector

$$\boldsymbol{S}(t,\cdot) = [1,\ \boldsymbol{S}(t,1), ...,\ \boldsymbol{S}(t,z)], \quad (17)$$

with the coordinates defined by

$$\boldsymbol{S}(t,u) = P(T(u) > t) \quad (18)$$

for $t \in <0,\infty)$, $u = 1,2,..., z$.

In the case when the system operation time θ is large enough, the coordinates (18) of the unconditional safety function of the system defined by (17) are given by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)}$$

for $t \ge 0$, $u = 1,2,..., z$, (19)

where $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,..., z$, $b = 1,2,...,\nu$, are the coordinates of the system conditional safety functions defined by (15)-(16) and p_b, $b = 1,2,...,\nu$, are the system operation process limit transient probabilities given by (4).

The exemplary graph of a five-state (z = 4) critical infrastructure safety function

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)],\ t \in <0,\infty),$$

is shown in Figure 1.

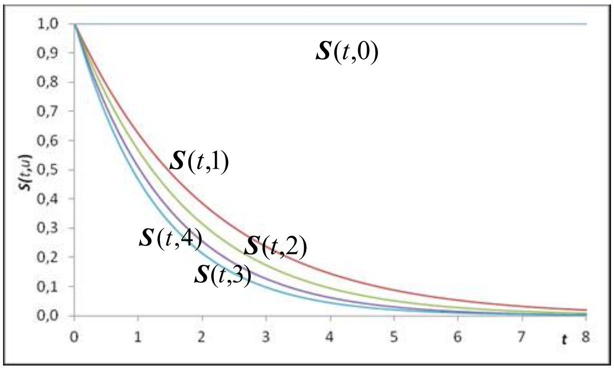

Figure 1 The graphs of a five-state critical infrastructure safety function $\boldsymbol{S}(t,\cdot)$ coordinates

The mean value of the system unconditional lifetime $T(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is given by [Kołowrocki, Soszyńska-Budny, 2011]

$$\mu(u) \cong \sum_{b=1}^{\nu} p_b \mu_b(u),\ \ u = 1,2,..., z, \quad (20)$$

where $\mu_b(u)$ are the mean values of the system conditional lifetimes $T^{(b)}(u)$ in the safety state subset $\{u, u+1, ..., z\}$ at the operation state z_b, $b = 1,2,...,\nu$, given by

$$\mu_b(u) = \int_0^\infty [\boldsymbol{S}(t,u)]^{(b)} dt,\ \ u = 1,2,..., z, \quad (21)$$

$[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,..., z$, $b = 1,2,...,\nu$, are defined by (15)-(16) and p_b are given by (4). Whereas, the variance of the system unconditional lifetime $T(u)$ is given by

$$\sigma^2(u) = 2\int_0^\infty t\ \boldsymbol{S}(t,u)dt - [\mu(u)]^2,\ \ u = 1,2,..., z, \quad (22)$$

where $\boldsymbol{S}(t,u)$, $u = 1,2,...,z$, are given by (17)-(18) and $\mu(u)$, $u = 0,1,...,z$, are given by (20).

According to (1.19) in [Kołowrocki, Soszyńska-Budny, 2011], we get the following formulae for the mean values of the unconditional lifetimes of the system in particular safety states

$$\bar{\mu}(u) = \mu(u) - \mu(u+1),$$
$$u = 0,1,...,z-1, \ \bar{\mu}(z) = \mu(z), \quad (23)$$

where $\mu(u)$, $u = 0,1,...,z$, are given by (20).
Moreover, according (1.20)-(1.21) in [Kołowrocki, Soszyńska-Budny, 2011], if r is the system critical safety state, then the system risk function

$$\boldsymbol{r}(t) = P(S(t) < r \mid S(0) = z) = P(T(r) \le t),$$
$$t \in <0,\infty), \quad (24)$$

defined as a probability that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment $t = 0$ [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011] is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r), \ t \in <0,\infty), \quad (25)$$

where $\boldsymbol{S}(t,r)$ is the coordinate of the system unconditional safety function given by (19) for $u = r$.
The graph of the system risk function presented in Figure 2 is called the fragility curve of the system.

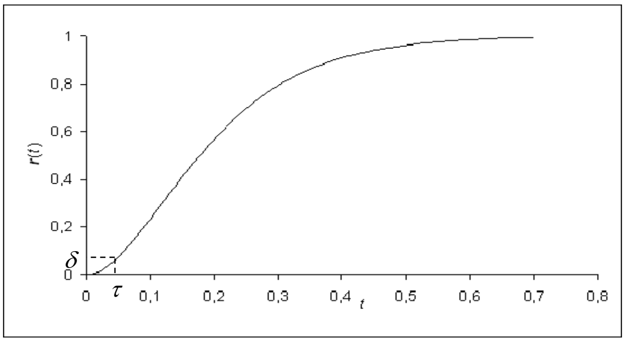

Figure 2 The graph (the fragility curve) of a system risk function $\boldsymbol{r}(t)$

The system safety function, the system risk function and the system fragility curve are main system safety factors. Other practically useful system safety factors are:

- the mean value of the unconditional system lifetime $T(r)$ up to the exceeding the critical safety state r given by

$$\mu(r) \cong \sum_{b=1}^{\nu} p_b \mu_b(r), \quad (26)$$

where $\mu_b(r)$ are the mean values of the system conditional lifetimes $T^{(b)}(r)$ in the safety state subset $\{r, r+1,...,z\}$ at the operation state z_b, $b = 1,2,...,\nu$, given by

$$\mu_b(r) = \int_0^\infty [\boldsymbol{S}(t,r)]^{(b)} dt, \ b = 1,2,...,\nu, \quad (27)$$

$[\boldsymbol{S}(t,r)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are defined by (15)-(16) and p_b are given by (4);

- the standard deviation of the system lifetime $T(r)$ up to the exceeding the critical safety state r given by

$$\sigma(r) = \sqrt{n(r) - [\mu(r)]^2}, \quad (28)$$

where

$$n(r) = 2\int_0^\infty t\, \boldsymbol{S}(t,r) dt, \quad (29)$$

where $\boldsymbol{S}(t,r)$ is given by (19) and $\mu(r)$ is given by (20) for $u = r$.

- the moment τ the system risk function exceeds a permitted level δ given by

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (30)$$

and illustrated in Figure 2, where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (25).

Other critical infrastructure safety indices are:

- the intensities of ageing/degradation of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$) related to the operation process impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}(t,\cdot) = [0, \boldsymbol{\lambda}(t,1), ..., \boldsymbol{\lambda}(t,z)], \ t \in <0,+\infty), \quad (31)$$

where

$$\boldsymbol{\lambda}(t,u) = \frac{-\dfrac{d\boldsymbol{S}(t,u)}{dt}}{\boldsymbol{S}(t,u)}, \ t \in <0,+\infty), \ u = 1,2,...,z; \quad (32)$$

- the coefficients of operation process impact on the critical infrastructure intensities of ageing (the coefficients of operation process impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, ..., z\}$), i.e. the coordinates of the vector

$$\boldsymbol{\rho}(t,\cdot) = [0,\ \boldsymbol{\rho}(t,1), \ldots, \boldsymbol{\rho}(t,z)\ ],\ t \in <0,+\infty), \quad (33)$$

where

$$\boldsymbol{\lambda}(t,u) = \boldsymbol{\rho}(t,u) \cdot \lambda(t,u),\ t \in <0,+\infty),$$
$$u = 1,2,...,z, \quad (34)$$

and $\lambda(t,u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u, u+1, ..., z\}$) without of climate-weather impact, i.e. the coordinate of the vector

$$\lambda(t,\cdot) = [0,\ \lambda(t,1), \ldots, \lambda(t,z)\ ],\ t \in <0,+\infty). \quad (35)$$

In the case, when the critical infrastructure has the exponential safety functions, i.e.

$$\boldsymbol{S}(t,\cdot) = [0,\ \boldsymbol{S}(t,1), \ldots, \boldsymbol{S}(t,z)\ ],\ t \in <0,+\infty), \quad (36)$$

where

$$\boldsymbol{S}(t,u) = \exp[-\boldsymbol{\lambda}(u)t],\ t \in <0,+\infty),$$
$$\boldsymbol{\lambda}(u) \geq 0,\ u = 1,2,\ldots,z, \quad (37)$$

the critical infrastructure safety indices defined by (31)-(35) take forms:
the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1, ..., z\}$) related to the operation impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}(\cdot) = [0,\ \boldsymbol{\lambda}(1), \ldots, \boldsymbol{\lambda}(z)\ ], \quad (38)$$

- the coefficients of the operation impact on the scritical infrastructure intensities of ageing (the coefficients of the climate-weatcher impact on critical infrastructure intensities of departure from the safety state subset $\{u, u+1, ..., z\}$), i.e. the coordinate of the vector

$$\boldsymbol{\rho}(\cdot) = [0,\ \boldsymbol{\rho}(1), \ldots, \boldsymbol{\rho}(z)\ ], \quad (39)$$

where

$$\boldsymbol{\lambda}(u) = \boldsymbol{\rho}(u) \cdot \lambda(u),\ u = 1,2,...,z. \quad (40)$$

and $\boldsymbol{\lambda}(u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u, u+1, ..., z\}$) without of operation impact, i.e. the coordinate of the vector

$$\lambda(\cdot) = [0,\ \lambda(1), \ldots, \lambda(z)\ ]. \quad (41)$$

4. Safety of Multistate Exponential Systems at Variable Operation Conditions

We assume that the system components at the system operation states have the exponential safety functions. This assumption and the results given in Chapter 1 [Kołowrocki, Soszyńska-Budny, 2011] yield the following results formulated in the form of the following proposition.

Proposition 1
If components of the multi-state system at the operation states z_b, $b = 1,2,...,\nu$, have the exponential safety functions given by

$$[S_i(t,\cdot)]^{(b)} = [1, [S_i(t,1)]^{(b)}, \ldots, [S_i(t,z)]^{(b)}],$$
$$t \in <0,\infty),\ b = 1,2,...,\nu,\ i = 1,2,...,n, \quad (42)$$

with the coordinates

$$[S_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t \mid Z(t) = z_b)$$
$$= \exp[-[\lambda_i(u)]^{(b)} t],$$
$$t \in <0,\infty),\ b = 1,2,...,\nu,\ i = 1,2,...,n, \quad (43)$$

and the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1, ..., z\}$) related to operation impact, existing in (2), are given by

$$[\lambda_i(u)]^{(b)} = \rho_i^{(b)}(u) \cdot \lambda_i(u),\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu,\ i = 1,2,...,n, \quad (44)$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1, ..., z\}$) without operation impact and

$$[\rho_i(u)]^{(b)},\ u = 1,2,...,z,\ b = 1,2,...,\nu,$$

$$i = 1,2,...,n, \tag{45}$$

are the coefficients of operation impact on the system components E_i, i = 1,2,...,n, intensities E_i, $i = 1,2,..., n$, of ageing (the coefficients of operation impact on critical infrastructure component E, i = 1,2,...,n, intensities of departure from the safety state subset $\{u, u+1,..., z\}$) without operation impact, in the case of series, parallel, "m out of n", consecutive "m out of n: F" systems
and respectively by

$$[S_{ij}(t,\cdot)]^{(b)} = [1, [S_{ij}(t,1)]^{(b)}, \ldots, [S_{ij}(t,z)]^{(b)}],$$
$$t \in < 0, \infty),\ b = 1,2,..., v,$$
$$i = 1,2,...,k, j = 1,2,...,l_i, \tag{46}$$

with the coordinates

$$[S_{ij}(t,u)]^{(b)} = P(T_{ij}^{(b)}(u) > t | Z(t) = z_b)$$
$$= \exp[-[\lambda_{ij}(u)]^{(b)} t],\ t \in < 0, \infty),\ b = 1,2,..., v,$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i, \tag{47}$$

and the intensities of ageing of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, (the intensities of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, departure from the safety state subset $\{u, u+1,..., z\}$) related to operation impact, existing in (47), are given by

$$[\lambda_{ij}(u)]^{(b)} = \rho_{ij}^{(b)}(u) \cdot \lambda_{ij}(u),$$
$$u = 1,2,..., z,\ b = 1,2,..., v,$$
$$i = 1,2,...,k, j = 1,2,...,l_i, \tag{48}$$

where $\lambda_{ij}(u)$ are the intensities of ageing of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, (the intensities of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, departure from the safety state subset $\{u, u+1,..., z\}$) without operation impact and

$$[\rho_{ij}(u)]^{(b)},\ u = 1,2,..., z,\ b = 1,2,..., v,$$
$$i = 1,2,...,k, j = 1,2,...,l_i, \tag{49}$$

are the coefficients of operation impact on the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, intensities of ageing (the coefficients of operation impact on critical infrastructure component E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, intensities of departure from the safety state subset $\{u, u+1,..., z\}$) without climate-weather change impact, in the case of series-parallel, parallel-series, series-"m out of k", "m_i out of l_i"-series, series-consecutive "m out of k: F" and consecutive "m_i out of l_i: F"-series systems and the system operation time θ is large enough, then its multistate unconditional safety function is given by the vector:

i) for a series system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{50}$$

where

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b \exp[-\sum_{i=1}^{n} [\lambda_i(u)]^{(b)} t] \tag{51}$$

for $t \geq 0$, $u = 1,2,..., z$;

ii) for a parallel system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)] \quad \text{for } t \geq 0, \tag{52}$$

where

$$\boldsymbol{S}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \prod_{i=1}^{n} [1 - \exp[-[\lambda_i(u)]^{(b)} t]] \tag{53}$$

for $t \geq 0$, $u = 1,2,..., z$;

iii) for a "m out of n" system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{54}$$

where

$$\boldsymbol{S}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \sum_{\substack{r_1, r_2, ..., r_n = 0 \\ r_1 + r_2 + ... + r_n \leq m-1}}^{1} \prod_{i=1}^{n} \exp[-r_i [\lambda_i(u)]^{(b)} t]$$
$$\cdot [1 - \exp[-[\lambda_i(u)]^{(b)} t]]^{1-n} \tag{55}$$

for $t \geq 0$, $u = 1,2,...,z$,
or

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{56}$$

where

$$\boldsymbol{S}(t,u)$$
$$\cong \sum_{b=1}^{v'} p'_b \sum_{\substack{r_1, r_2, ..., r_n = 0 \\ r_1 + r_2 + ... + r_n \leq \bar{m}}}^{1} \prod_{i=1}^{n} [1 - \exp[-[\lambda_i(u)]^{(b)} t]]^{r_i}$$
$$\cdot \exp[-(1 - r_i)[\lambda_i(u)]^{(b)} t] \tag{57}$$

for $t \geq 0$, $u = 1,2,...,z$, and $\overline{m} = n - m$;

iv) for a consecutive "m out of n: F" system

$$\boldsymbol{CS}(t,\cdot) = [1, \boldsymbol{CS}(t,1),..., \boldsymbol{CS}(t,z)] \text{ for } t \geq 0, \tag{58}$$

where

$$\boldsymbol{CS}(t,u) \cong \sum_{b=1}^{\nu'} p_b [\boldsymbol{CS}(t,u)]^{(b)} \tag{59}$$

for $t \geq 0$, $u = 1,2,...,z$, and $[\boldsymbol{CS}(t,u)]^{(b)}$, $t \geq 0$, $b = 1,2,...,\nu'$, are given by

$$[\boldsymbol{CS}(t,u)]^{(b)} \cong \begin{cases} 1 & \text{for } n < m, \\ 1 - \sum_{b=1}^{\nu'} q_b \prod_{i=1}^{n} [1 - \exp[-[\lambda_i(u)]^{(b)} t]] & \text{for } n = m, \\ \sum_{b=1}^{\nu'} q_b [\exp[-[\lambda_n(u)]^{(b)} t][\boldsymbol{CS}_{n-1}(t,u)]^{(b)} & \\ + \sum_{i=1}^{m-1} \exp[-[\lambda_{n-i}(u)]^{(b)} t][\boldsymbol{CS}_{n-i-1}(t,u)]^{(b)} & \\ \prod_{j=n-i+1}^{n} [1 - \exp[-[\lambda_j(u)]^{(b)} t]] & \text{for } n > m, \end{cases} \tag{60}$$

for $t \geq 0$, $u = 1,2,...,z$;

v) for a series-parallel system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{61}$$

where

$$\boldsymbol{S}(t,u) \cong 1 - \sum_{b=1}^{\nu'} p_b \prod_{i=1}^{k} [1 - \exp[-\sum_{j=1}^{l_i} [\lambda_{ij}(u)]^{(b)} t]] \tag{62}$$

for $t \geq 0$, $u = 1,2,...,z$;

vi) for a parallel-series system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{63}$$

where

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu'} p_b \prod_{i=1}^{k} [1 - \prod_{j=1}^{l_i} [1 - \exp[-[\lambda_{ij}(u)]^{(b)} t]]] \tag{64}$$

for $t \geq 0$, $u = 1,2,...,z$;

vii) for a series-"m out of k" system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{65}$$

where

$$\boldsymbol{S}(t,u) \cong 1 - \sum_{b=1}^{\nu'} p_b \sum_{\substack{r_1,r_2,...,r_k=0 \\ r_1+r_2+...+r_k \leq m-1}}^{1} \prod_{i=1}^{k} [\prod_{j=1}^{l_i} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_i} \cdot [1 - \prod_{j=1}^{l_i} \exp[-[\lambda'_{ij}(u)]^{(b)} t]]^{1-r_i} \tag{66}$$

for $t \geq 0$, $u = 1,2,...,z$,

or

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{67}$$

where

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu'} p_b \sum_{\substack{r_1,r_2,...,r_k=0 \\ r_1,r_2,...+r_k \leq \overline{m}}}^{1} \prod_{i=1}^{k} [1 - [\prod_{i=1}^{l_i} \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_i} \cdot [\prod_{j=1}^{l_i} \exp[-[\lambda'_{ij}(u)]^{(b)} t]]^{1-r_i} \tag{68}$$

for $t \geq 0$, $\overline{m} = k - m$, $u = 1,2,...,z$;

viii) for a "m_i out of l_i"-series system

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{69}$$

where

$$S(t,u) \cong \sum_{b=1}^{\nu'} p_b \prod_{i=1}^{k} [1 - \sum_{\substack{r_1,r_2,...,r_{l_i}=0 \\ r_1+r_2+...+r_{l_i} \leq m_i-1}}^{1} \prod_{j=1}^{l_i} \exp[-r_j [\lambda_{ij}(u)]^{(b)} t]] \cdot [1 - \exp[-[\lambda_{ij}(u)]^{(b)} t]]^{1-r_j}] \tag{70}$$

for $t \geq 0$, $u = 1,2,...,z$,

or

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),..., \boldsymbol{S}(t,z)] \text{ for } t \geq 0, \tag{71}$$

where

$$\boldsymbol{S}(t,u)$$
$$\cong \sum_{b=1}^{v'} p_b \prod_{i=1}^{k} [\sum_{\substack{r_1,r_2,\ldots,r_{l_i}=0 \\ r_1+r_2+\ldots+r_{l_i} \le \overline{m}_i}}^{1} \prod_{j=1}^{l_i} [1-\exp[-[\lambda_{ij}(u)]^{(b)} t]]^{r_j}$$
$$\cdot \exp[-(1-r_j)[\lambda_{ij}(u)]^{(b)} t]] \quad (72)$$

for $t \ge 0$, $\overline{m}_i = l_i - m_i$, $i = 1,2,\ldots,k$, $u = 1,2,\ldots,z$;

ix) for a series-consecutive "*m* out of *k*: F" system

$$\boldsymbol{CS}(t,\cdot) = [1, \boldsymbol{CS}(t,1), \ldots, \boldsymbol{CS}(t,z)] \text{ for } t \ge 0, \quad (73)$$

where

$$\boldsymbol{CS}(t,u) \cong \sum_{b=1}^{v'} p_b [\, \boldsymbol{CS}(t,u)]^{(b)} \quad (74)$$

for $t \ge 0$, $u = 1,2,\ldots,z$, and $[\boldsymbol{CS}(t,u)]^{(b)}$, $b = 1,2,\ldots,v'$, are given by

$$[\boldsymbol{CS}(t,u)]^{(b)}$$
$$= \begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k} [1-\exp[-\sum_{j=1}^{l_i} \lambda_{ij}(u) t]] & \text{for } k = m, \\ \exp[-\sum_{j=1}^{l_k} [\lambda_{kj}(u)]^{(b)} t][\boldsymbol{CS}_{k-1;l_1,l_2,\ldots,l_k}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m-1} [\exp[-\sum_{v=1}^{l_{k-j}} [\lambda_{k-jv}(u)]^{(b)} t]] & \\ \cdot [\boldsymbol{CS}_{k-j-1;l_1,l_2,\ldots,l_k}(t,u)]^{(b)} & \\ \cdot \prod_{i=k-j+1}^{k} [1-\exp[-\sum_{v=1}^{l_i} [\lambda_{iv}(u)]^{(b)} t]] & \text{for } k > m, \end{cases}$$

for $t \ge 0$, $u = 1,2,\ldots,z$; (75)

x) for a consecutive "m_i out of l_i: F"-series system

$$\boldsymbol{CS}(t,\cdot) = [1, \boldsymbol{CS}(t,1), \ldots, \boldsymbol{CS}(t,z)] \text{ for } t \ge 0, \quad (76)$$

where

$$\boldsymbol{CS}(t,u) \cong \sum_{b=1}^{v'} p'_b \prod_{i=1}^{k} [\boldsymbol{CS}_{i,l_i}(t,u)]^{(b)} \quad (77)$$

for $t \ge 0$, $u = 1,2,\ldots,z$, and $[\boldsymbol{CS}_{i,l_i}(t,u)]^{(b)}$, i=1, 2,… , k, $b = 1,2,\ldots,v'$, are given by

$$[\boldsymbol{CS}(t,u)]^{(b)}$$
$$= \begin{cases} 1 & \text{for } l_i < m_i, \\ 1 - \prod_{j=1}^{l_i} [1-\exp[-[\lambda_{ij}(u)]^{(b)} t]] & \text{for } l_i = m_i, \\ \exp[-[\lambda_{il_i}(u)]^{(b)} t][\boldsymbol{CS}_{i,l_i-1}(t,u)]^{(b)} & \\ + \sum_{j=1}^{m_i-1} \exp[-[\lambda_{il_i-j}(u)]^{(b)} t][\boldsymbol{CS}_{i,l_i\text{-}j\text{-}1}(t,u)]^{(b)} & \\ \cdot \prod_{v=l_i-j+1}^{l_i} [1-\exp[-[\lambda_{iv}(u)]^{(b)} t]] & \text{for } l_i > m_i, \end{cases} \quad (78)$$

for $t \ge 0$, $u = 1,2,\ldots,z$.

Remark 1
The formulae for the safety functions stated in *Proposition 1* are valid for the considered systems under the assumption that they do not change their structure shapes at different operation states z'_b, $b = 1,2,\ldots,v'$. This limitation can be simply omitted by the replacement in these formulae the system's structure shape constant parameters n, m, k, m_i, l_i, respectively by their changing at different operation states z'_b, $b = 1,2,\ldots,v'$, equivalent structure shape parameters $n^{(b)}$, $m^{(b)}$, $k^{(b)}$, $m_i^{(b)}$, $l_i^{(b)}$, $b = 1,2,\ldots,v$.
For the exponential complex technical systems, considered in *Proposition 1*, we determine the mean values $\mu'(u)$ and the standard deviations $\sigma'(u)$ of the unconditional lifetimes of the system in the safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, the mean values $\bar{\mu}'(u)$ of the unconditional lifetimes of the system in the particular safety states u, $u = 1,2,\ldots,z$, the system risk function $\boldsymbol{r}'(t)$ and the moment τ' when the system risk function exceeds a permitted level δ respectively defined by (20)-(25), after substituting for $\boldsymbol{S}'(t,u)$, $u = 1,2,\ldots,z$, the coordinates of the unconditional safety functions given respectively by (50)-(78)

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5. Conclusions

The integrated general model of complex systems' safety, linking their safety models and their operation processes models and considering variable at different operation states their safety structures and their components safety parameters was constructed. The material given in this report delivers the basis for procedures and algorithms that allow to find the main an practically important safety characteristics of the critical infrastructures defined as complex technical systems at the variable operation conditions. Next the results are applied to the safety evaluation of the port oil piping transportation system and the maritime ferry technical system. The predicted safety characteristics of these exemplary critical infrastructures operating at the variable conditions are different from those determined for this system operating at constant conditions [Kołowrocki, Soszyńska-Budny, 2011]. This fact justifies the sensibility of considering real systems at the variable operation conditions that is appearing out in a natural way from practice. This approach, upon the sufficient accuracy of the critical infrastructures' operation processes and the critical infrastructures' components safety parameters identification, makes their safety prediction much more precise than in the case of omitting their operation processes impacts.

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