Critical infrastructure operation process

Keywords

critical infrastructure, operation, prediction, climate-weather change

Abstract

The operation process of the critical infrastructure is considered and its operation states are introduced. The semi-Markov process is used to construct a general probabilistic model of the critical infrastructure operation process.

1. Introduction

The operation process of the critical infrastructure is considered and its operation states are introduced. The semi-Markov process is used to construct a general probabilistic model of the critical infrastructure operation process. To build this model, the vector of probabilities of the critical infrastructure operation process staying at the initial operation states, the matrix of probabilities of the critical infrastructure operation process transitions between the operation states, the matrix of conditional distribution functions and the matrix of conditional density functions of the critical infrastructure operation process conditional sojourn times at the operation states are defined. To describe the critical infrastructure operation process conditional sojourn times at the particular operation states, the uniform distribution, the triangular distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull distribution and the chimney distribution are suggested.

The operation process of a critical infrastructure is very complex and often it is difficult to analyze these critical infrastructure safety with respect to changing in time its operation process states and operating environment conditions that are essential in this analysis. The complexity of the critical infrastructure operation process and its influence on changing in time the critical infrastructure structure and its components’ safety parameters are essential in critical infrastructure safety analysis and protection. Usually, the critical infrastructure environment have either an explicit or an implicit strong influence on the critical infrastructure operation process. As a rule, some of the environmental events together with the infrastructure operation conditions define a set of different operation states of the critical infrastructure in which the critical infrastructure change its safety structure and its components’ safety parameters. In this report, we propose a convenient tool for analyzing this problem applying the semi-Markov model [Ferreira, Pacheco, 2007], [Glynn, Hass 2006], [Grabski, 2002], [Kolowrocki, 2014], [Linnios, Oprisan, 2005], [Mercier, 2008] of the critical infrastructure operation process, both without including critical infrastructure environment threats and with including them into this model.

2. Critical infrastructure operation process-modelling

2.1. Semi-Markov model of critical infrastructure operation process

We assume that the critical infrastructure during its operation process is taking \( v, v \in N \), different operation states \( z_1, z_2, \ldots, z_v \). Further, we define the critical infrastructure operation process \( Z(t), t \in 0, +\infty \), with discrete operation states from the set \( \{ z_1, z_2, \ldots, z_v \} \). Moreover, we assume that the critical infrastructure operation process \( Z(t) \) is a
semi-Markov process [Grabski, 2002], [Linnios, 2005], [Mercier, 2008], [Soszyńska, 2007], [Kołwowcoki, Soszyńska-Budny, 2011] with the conditional sojourn times $\theta_{zi}$ at the operation states $z_b$ when its next operation state is $z_i$, $b, l = 1, 2, ..., \nu, b \neq l$. Under these assumptions, the critical infrastructure operation process may be described by:

- the vector of the initial probabilities $p_{0}(0) = P(Z(0) = z_b), b = 1, 2, ..., \nu$, of the critical infrastructure operation process $Z(t)$ staying at particular operation states at the moment $t = 0$

$$\left[p_{0}(0)\right]_{\nu \times 1} = [p_1(0), p_2(0), ..., p_{\nu}(0)];$$

(1)

- the matrix of probabilities $p_{st}, b, l = 1, 2, ..., \nu, of the critical infrastructure operation process$ $Z(t)$ transitions between the operation states $z_b$ and $z_i$

$$\left[p_{st}\right]_{\nu \times \nu} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1\nu} \\ p_{21} & p_{22} & \cdots & p_{2\nu} \\ \vdots & \vdots & & \vdots \\ p_{\nu1} & p_{\nu2} & \cdots & p_{\nu\nu} \end{bmatrix},$$

(2)

where by formal agreement

$p_{sb} = 0$ for $b = 1, 2, ..., \nu$;

- the matrix of conditional distribution functions $H_{st}(t) = P(\theta_{zi} < t), b, l = 1, 2, ..., \nu, of the critical infrastructure operation process$ $Z(t)$ conditional sojourn times $\theta_{zi}$ at the operation states

$$\left[H_{st}(t)\right]_{\nu \times \nu} = \begin{bmatrix} H_{11}(t) & H_{12}(t) & \cdots & H_{1\nu}(t) \\ H_{21}(t) & H_{22}(t) & \cdots & H_{2\nu}(t) \\ \vdots & \vdots & & \vdots \\ H_{\nu1}(t) & H_{\nu2}(t) & \cdots & H_{\nu\nu}(t) \end{bmatrix},$$

(3)

where by formal agreement

$H_{sb}(t) = 0$ for $b = 1, 2, ..., \nu$.

We introduce the matrix of the conditional density functions of the critical infrastructure operation process $Z(t)$ conditional sojourn times $\theta_{zi}$ at the operation states corresponding to the conditional distribution functions $H_{zi}(t)$

$$\left[h_{st}(t)\right]_{\nu \times \nu} = \begin{bmatrix} h_{11}(t) & h_{12}(t) & \cdots & h_{1\nu}(t) \\ h_{21}(t) & h_{22}(t) & \cdots & h_{2\nu}(t) \\ \vdots & \vdots & & \vdots \\ h_{\nu1}(t) & h_{\nu2}(t) & \cdots & h_{\nu\nu}(t) \end{bmatrix},$$

where

$h_{sb}(t) = \frac{d}{dt}[H_{sb}(t)]$ for $b, l = 1, 2, ..., \nu, b \neq l,$

and by formal agreement

$h_{bb}(t) = 0$ for $b = 1, 2, ..., \nu$.

We assume that the suitable and typical distributions suitable to describe the critical infrastructure operation process $Z(t)$ conditional sojourn times $\theta_{zi}, b, l = 1, 2, ..., \nu, b \neq l,$ in the particular operation states are defined in [Kołwowcoki, Soszyńska-Budny, 2011] and [EU-CIRCLE Report D2.1-GMU2, 2016].

3. Critical infrastructure operation process – prediction

3.1. Prediction of critical infrastructure operation process characteristics

Assuming that we have identified the unknown parameters of the critical infrastructure operation process semi-Markov model:

- the initial probabilities $p_{sb}(0), b = 1, 2, ..., \nu$, of the critical infrastructure operation process staying at the particular state $z_b$ at the moment $t = 0$;
- the probabilities $p_{bl}, b, l = 1, 2, ..., \nu, b \neq l$, of the critical infrastructure operation process transitions from the operation state $z_b$ into the operation state $z_l$;
- the distributions of the critical infrastructure operation process conditional sojourn times $\theta_{zi}, b, l = 1, 2, ..., \nu, b \neq l,$ at the particular operation states and their mean values $M_{bl} = E[\theta_{zi}], b, l = 1, 2, ..., \nu, b \neq l$;

we can predict this process basic characteristics.

As the mean values of the conditional sojourn times $\theta_{zi}$ are given by [Kołwowcoki, Soszyńska-Budny, 2011]

$$M_{bl} = E[\theta_{zi}] = \int_0^\infty dH_{zi}(t) = \int_0^\infty \theta_{zi}(t)dt,$$

(5)

$b, l = 1, 2, ..., \nu, b \neq l$,

then for the distinguished distributions (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], the mean values of the system operation process $Z(t)$...
conditional sojourn times \( \theta_{bl} \), \( b,l = 1,2,\ldots,v \), \( b \neq l \) at the particular operation states are respectively given by [Kolowrocki, Soszyńska-Budny, 2011]:

- for the uniform distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{2};
\]  
(6)

- for the triangular distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{3} + \frac{w_{bl}}{2}(y_{bl} - z_{bl})^2 - q_{bl}(x_{bl})^2;
\]

\[
+ \frac{w_{bl}}{6}[(x_{bl}z_{bl} - y_{bl}z_{bl}) + x_{bl}y_{bl}(x_{bl} + y_{bl})]
\]

\[
- \frac{(x_{bl})^3 + (y_{bl})^3}{3(y_{bl} - x_{bl})};
\]  
(7)

- for the double trapezium distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = \frac{x_{bl} + y_{bl} + z_{bl}}{2} + \frac{w_{bl}}{6}[(x_{bl})^2 - (z_{bl})^2] + \frac{w_{bl}}{2}[(y_{bl})^3 - (z_{bl})^3]
\]

\[
- \frac{w_{bl}}{6}[(2z_{bl})^2 - 5y_{bl}z_{bl} - (y_{bl})^2];
\]  
(8)

- for the quasi-trapezium distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = \frac{q_{bl}}{2}\left[(z_{bl})^2 - (x_{bl})^2\right]
\]

\[
- \frac{A_{bl} - q_{bl}}{6}[2(z_{bl})^2 - 5x_{bl}z_{bl} - (x_{bl})^2]
\]

\[
+ \frac{A_{bl}}{2}[(z_{bl})^2 - (x_{bl})^2] + \frac{w_{bl}}{2}[(y_{bl})^3 - (z_{bl})^3]
\]

\[
- \frac{w_{bl}}{6}[(2z_{bl})^2 - 5y_{bl}z_{bl} - (y_{bl})^2];
\]  
(9)

- for the exponential distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = x_{bl} + \frac{1}{\alpha_{bl}};
\]  
(10)

- for the Weibull distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = x_{bl} + \frac{1}{\alpha_{bl}^\beta_{bl}} \Gamma(1 + \frac{1}{\beta_{bl}}),
\]  
(11)

where

\[
\Gamma(u) = \int_0^\infty t^{u-1}e^{-t} dt, \quad u > 0,
\]
is the gamma function;

- for the chimney distribution

\[
M_{\theta}\theta = E[\theta_{bl}] = \frac{1}{2}[A_{bl}(x_{bl} + z_{bl}^2) + C_{bl}(z_{bl}^2 + z_{bl}^3)]
\]

\[
+ D_{bl}(z_{bl}^2 + y_{bl})].
\]  
(12)

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times \( \theta_{bl}, b, l = 1,2,\ldots,v \), of the system operation process \( Z(t) \) at the operation states \( z_{bl}, b, l = 1,2,\ldots,v \), are given by [Kołowrocki, Soszyńska-Budny, 2011]

\[
H_{bl}(t) = \sum_{i=1}^{v} p_{bl} H_{bl}(t), \quad b, l = 1,2,\ldots,v.
\]  
(13)

Hence, the mean values \( E[\theta_{bl}] \) of the system operation process \( Z(t) \) unconditional sojourn times \( \theta_{bl}, b, l = 1,2,\ldots,v \), at the operation states are given by

\[
M_{\theta}\theta = E[\theta_{bl}] = \sum_{i=1}^{v} p_{bl} M_{\theta}\theta, \quad b, l = 1,2,\ldots,v,
\]  
(14)

where \( M_{\theta}\theta \) are defined by the formula (5) in a case of any distribution of sojourn times \( \theta_{bl} \) and by the formulae (2.6)-(2.12) in the cases of particular defined respectively by (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.

The limit values of the system operation process \( Z(t) \) transient probabilities at the particular operation states

\[
p_{bl}(t) = P(Z(t) = z_{bl}), \quad t \in 0,+,\ldots, b, l = 1,2,\ldots,v,
\]
are given by [Kołowrocki, Soszyńska-Budny, 2011]

\[
p_{bl} = \lim_{t \to +\infty} p_{bl}(t) = \frac{\pi_{bl} M_{\theta\theta}}{\sum_{i=1}^{v} \pi_{bl} M_{\theta\theta}}, \quad b, l = 1,2,\ldots,v,
\]  
(15)

where \( M_{\theta\theta}, b, l = 1,2,\ldots,v \), are given by (2.14), while the steady probabilities \( \pi_{bl} \) of the vector \( \pi_{bl} \) satisfy the system of equations

\[
[\pi_{bl}] = [\pi_{bl}][p_{bl}]
\]

\[
\sum_{i=1}^{v} \pi_{bl} = 1.
\]  
(16)
In the case of a periodic system operation process, the limit transient probabilities \( p_b, b = 1,2,\ldots,v \), at the operation states defined by (15), are the long term proportions of the system operation process \( Z(t) \) sojourn times at the particular operation states \( z_b, b = 1,2,\ldots,v \).

Other interesting characteristics of the system operation process \( Z(t) \) possible to obtain are its total sojourn times \( \hat{\theta}_b \) at the particular operation states \( z_b, b = 1,2,\ldots,v \), during the fixed system operation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times \( \hat{\theta}_b \) at the particular operation states \( z_b, \) for sufficiently large operation time \( \theta \), have approximately normal distributions with the expected value given by

\[
\hat{M}_b = E[\hat{\theta}_b] = p_b \theta, \quad b = 1,2,\ldots,v, \tag{17}
\]

where \( p_b \) are given by (15).

4. Conclusions

The probabilistic model of the critical infrastructure operation process presented in this paper is the basis for further considerations in particular tasks of the EU-CIRCLE project. This model will be developed in order to construct the integrated model of critical infrastructure Safety (IMCIS) Related to Its Operation Process – IMCIS Model 1.

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