Blokus-Roszkowska Agnieszka

Kołowrocki Krzysztof
Gdynia Maritime University, Gdynia, Poland

Modelling Safety of Interconnected Critical Infrastructure Network Cascading

Keywords

Safety, modelling, interconnected, cascading, critical infrastructure

Abstract

In the paper the safety function of a multistate series network with dependent assets, with dependent subnetworks and with dependent assets of its subnetworks is determined. The multistate series-parallel network with dependent assets of its subnetworks and multistate series-"*m* out of *k*" network with dependent assets of its subnetworks is considered. Further the multistate parallel and "*m* out of n" networks with dependent assets are analyzed. The safety function of multistate parallel-series and "*m* out of *l*"-series networks with dependent assets of its subnetworks, and finally of multistate parallel-series and "*m* out of *l*"-series networks with dependent subnetworks and dependent assets of these subnetworks are determined. Proposed theoretical models of dependency are applied to the safety analysis of the exemplary electricity network. Finally, the obtained results are compared with results for the considered electricity network without assumption about dependencies between assets and subnetworks.

1. Introduction

The report is devoted to safety analysis of multistate critical infrastructure networks taking into account interaction and dependencies between their subnetworks and assets. The multistate approach to cascading effect modeling is proposed for networks with series, parallel, "*m* out of *n*", series-parallel, series-"*m* out of *k*", parallel-series and "*m* out of *l*"-series safety structure.

Critical infrastructures (CI) are usually interconnected and mutually dependent in various and complex ways, creating critical infrastructure network. They are interacting directly and indirectly at various levels of their complexity and operating activity [Kjølle et al., 2012], [Kotzanikolaou et al., 2013]. Identifying and modeling dependencies depend on the level of analysis. The selected level of analysis can vary from micro to macro level. Then, we can consider a holistic approach as in [Lauge et al., 2015] or a reductionistic approach in which elementary components are identified and their behaviour is described. For example, Svedsen and Wolthunsen [Svedsen, Wolthunsen, 2007] focus on the components of a critical infrastructure networks and they demonstrate several types of multi-dependency structures. This report also focus on the component level and analyze dependencies between assets of CI network and between subnetworks belonging to CI network.

Describing cascading effects in CI networks both the dependencies between subnetworks of this network and between their assets are considered. Then, after changing the safety state subset by some of assets in the subnetwork to the worse safety state subset, the lifetimes of remaining assets in this subnetwork in the safety state subsets decrease. Models of dependency and behavior of components can differ depending on the structural and material properties of the network, operational conditions and many other factors, as for example natural hazards. According to the equal load sharing rule, after changing the safety state subset by some of assets in the subnetwork to the worse safety state subset, the lifetimes of remaining assets in this subnetwork in the safety state subsets decrease equally depending, inter alia, on the number of these assets that have left the safety state subset. In the local load sharing model of dependency, after departure from the safety state subset by one of assets in the subnetwork the safety parameters of remaining assets are changing dependently of the coefficients of the network load growth. These coefficients are concerned with the distance from the asset that has got out of the safety state subset and can be interpreted in the metric sense as well as in the sense of relationships in the functioning of the network. Apart from the dependency of assets' departures from the safety states subsets, the dependencies between subnetworks are also taken into account.

In the report the safety function of a multistate series network with dependent assets, with dependent subnetworks and with dependent assets of its subnetworks is determined. The multistate series-parallel network with dependent assets of its subnetworks and multistate series-"*m* out of *k*" network with dependent assets of its subnetworks is considered. Further the multistate parallel and "*m* out of n" networks with dependent assets are analyzed. The safety function of multistate parallel-series and "*m* out of *l*"-series networks with dependent assets of its subnetworks, and finally of multistate parallel-series and "*m* out of *l*"-series networks with dependent subnetworks and dependent assets of these subnetworks are determined.

Proposed theoretical models of dependency are applied to the safety analysis of the exemplary electricity network. Finally, the obtained results are compared with results for the considered electricity network without assumption about dependencies between assets and subnetworks.

2. Multistate series CI network with dependent assets/subnetworks

2.1. Approach description

Describing cascading effects in a series network we can consider a network composed of dependent components. Then, we assume that after changing the safety state subset by one of components in a network to the worse safety state subset, the lifetimes of remaining components in the safety state subsets decrease. More exactly, we assume that these lifetimes decrease mostly for neighbour components in first line, then less for neighbour components in second line and so on and we call this rule of components dependency a local load sharing (LLS) rule. In other words, if the component E_j, $j = 1,\dots,n$, in the network gets out of the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$, the safety parameters of remaining components E_i, $i = 1,\dots,n$, $i \neq j$, in this network are changing dependently of the distance from the component E_j that has got out of this subset. The distance is defined by $d_{ij} = |i - j|$, $i,j = 1,2,\dots,n$ and the meaning of the distance index is illustrated in Figure 1.

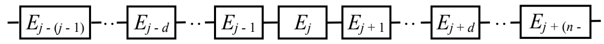

Figure 1. The meaning of the distance *d*.

We denote by $E[T_i(u)]$ and $E[T_{i/j}(u)]$, $i = 1,2,...,n$, $j = 1,2,...,n$, $u = 1,2,\dots,z$, the mean values of components' lifetimes $T_i(u)$ and $T_{i/j}(u)$, respectively, before and after departure of one fixed component E_j, $j = 1,\dots,n$, from the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$. With this notation, in considered local load sharing rule, the mean values of components lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\dots,z\}$, $\upsilon = u,u-1,\dots,1$, $u = 1,2,\dots,z$, are decreasing according to the following formula:

$$T_{i/j}(\upsilon) = q(\upsilon,d_{ij}) \cdot T_i(\upsilon),$$
$$E[T_{i/j}(\upsilon)] = q(\upsilon,d_{ij}) \cdot E[T_i(\upsilon)],\ i = 1,\dots,n,$$
$$j = 1,\dots,n,\ \upsilon = u,u-1,\dots,1, \tag{1}$$

where the coefficients of the network load growth $q(\upsilon,d_{ij})$, $0 < q(\upsilon,d_{ij}) \le 1$, $i = 1,\dots,n$, $j = 1,\dots,n$, and $q(\upsilon,0) = 1$ for $\upsilon = u,u-1,\dots,1$, $u = 1,2,\dots,z-1$, are non-increasing functions of components' distance $d_{ij} = |i - j|$ from the component that has got out of the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$. The distance between network assets can be interpreted in the metric sense as well as in the sense of relationships in the functioning of the network components.

Further, we define the safety function of a component E_i, $i = 1,\dots,n$, after departure of the component E_j, $j = 1,2,\dots,n$, from the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$,

$$S_{i/j}(t,\cdot) = [1, S_{i/j}(t,1),\dots,S_{i/j}(t,z)],\ t \ge 0,$$
$$i = 1,\dots,n,\ j = 1,\dots,n, \tag{2}$$

with the coordinates given by

$$S_{i/j}(t,\upsilon) = P(T_{i/j}(\upsilon) > t),\ t \ge 0,\ \upsilon = u,u-1,\dots,1,$$
$$u = 1,2,\dots,z-1,$$

$$S_{i/j}(t,\upsilon) = P(T_{i/j}(\upsilon) > t) = P(T_i(\upsilon) > t) = S_i(t,\upsilon),$$
$$\upsilon = u+1,\dots,z,\ u = 1,2,\dots,z-1. \tag{3}$$

2.2. Safety of a multistate series CI network with dependent assets

Then, we formulate the theorem concerned with safety of a series CI network composed of dependent asstes.

Proposition 7.1. If in a multistate series network assets are dependent according to the local load sharing rule and have safety functions (2.1)-(2.2), then its safety function is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1), \ldots, \boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \tag{4}$$

with the coordinates

$$\boldsymbol{S}_{LLS}(t,u) = \prod_{i=1}^{n} S_i(t,u+1)$$
$$+ \int_0^t \sum_{j=1}^{n} [\tilde{f}_j(a,u+1) \cdot \prod_{\substack{i=1 \\ i \neq j}}^{n} S_i(a,u+1) \cdot S_j(a,u)$$
$$\cdot \prod_{i=1}^{n} S_{i/j}(t-a,u)]da,\ u = 1,2,\ldots,z-1, \tag{5}$$

$$\boldsymbol{S}_{LLS}(t,z) = \prod_{i=1}^{n} S_i(t,z), \tag{6}$$

where:

$S_i(t,u+1)$ − the safety function coordinate of a component E_i, $i = 1,\ldots,n$,

$\tilde{f}_j(t,u+1)$ − the density function coordinate of a component E_j, $j = 1,\ldots,n$, corresponding to the distribution function $\tilde{F}_j(t,u+1)$, given by

$$\tilde{F}_j(t,u+1) = 1 - \frac{S_j(t,u+1)}{S_j(t,u)},$$
$$u = 1,2,\ldots,z-1,\ t \geq 0, \tag{7}$$

$S_j(t,u)$ − the safety function coordinate of a component E_j, $j = 1,\ldots,n$,

$S_{i/j}(t,u)$ − the safety function coordinate of a component E_i, $i = 1,\ldots,n$, after departure from the safety state subset $\{u+1,\ldots,z\}$, $u = 1,2,\ldots,z-1$, by the component E_j, $j = 1,\ldots,n$, such that

$$S_{i/j}(t-a,u) = \frac{S_{i/j}(t,u)}{S_i(a,u)},\ u = 1,2,\ldots,z-1,$$
$$0 < a < t,\ t \geq 0. \tag{8}$$

Further, we assume that assets E_i, $i = 1,\ldots,n$, of the network, have exponential safety functions

$$S_i(t,\cdot) = [1, S_i(t,1), \ldots, S_i(t,z)],\ t \geq 0, \tag{9}$$

with the coordinates

$$S_i(t,u) = \exp[-\lambda_i(u)t],\ u = 1,2,\ldots,z, \tag{10}$$

where $\lambda_i(u)$, $\lambda_i(u) \geq 0$, $i = 1,\ldots,n$, are components' intensities of departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. Then, according to the well known relationship between the lifetime mean value in this safety state subset and the intensity of departure from this safety state subset we get the formula for the intensities $\lambda_{i/j}(\upsilon)$, $i = 1,\ldots,n$, $j = 1,\ldots,n$, of components' departure from the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, after the departure of the jth component E_j, $j = 1,\ldots,n$, from that safety state subset. Namely, from (1), we obtain

$$\lambda_{i/j}(\upsilon) = \frac{\lambda_i(\upsilon)}{q(\upsilon,d_{ij})},\ \upsilon = u,u-1,\ldots,1. \tag{11}$$

Thus, considering (9)-(10) and (11), the components E_i, $i = 1,\ldots,n$, after the departure of the jth component E_j, $j = 1,\ldots,n$, from that safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, have the safety functions

$$S_{i/j}(t,\cdot) = [1, S_{i/j}(t,1), \ldots, S_{i/j}(t,u)],\ t \geq 0,\ i = 1,\ldots,n,$$
$$j = 1,\ldots,n, \tag{12}$$

with the coordinates

$$S_{i/j}(t,\upsilon) = \exp[-\frac{\lambda_i(\upsilon)}{q(\upsilon,d_{ij})}t],\ \upsilon = u,u-1,\ldots,1,$$
$$u = 1,2,\ldots,z-1, \tag{13}$$

$$S_{i/j}(t,\upsilon) = \exp[-\lambda_i(\upsilon)t],\ i = 1,\ldots,n, j = 1,\ldots,n,$$
$$\upsilon = u+1,\ldots,z,\ u = 1,2,\ldots,z-1. \tag{14}$$

Further for the exponential multistate series system with dependent components, the distribution function corresponding to the system component E_j , given by (7), takes form

$$\tilde{F}_j(t,u+1) = 1 - \frac{\exp[-\lambda_j(u+1)t]}{\exp[-\lambda_j(u)t]}$$
$$= 1 - \exp[-(\lambda_j(u+1) - \lambda_j(u))t], \tag{15}$$

and its corresponding density function is

$$\tilde{f}_j(t,u+1) = (\lambda_j(u+1) - \lambda_j(u))$$
$$\cdot \exp[-(\lambda_j(u+1) - \lambda_j(u))t],$$
$$u = 1,2,\ldots,z-1,\ t \geq 0. \tag{16}$$

Considering (12)-(15), in case the system components have exponential safety functions from *Proposition 7.1* we can obtain the following result.

Proposition 7.2. If in a multistate series network assets are dependent according to the local load sharing rule and have exponential safety functions (9)-(10), then its safety function is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1), \ldots, \boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \tag{17}$$

with the coordinates

$$\boldsymbol{S}_{LLS}(t,u) = \exp[-\sum_{i=1}^{n} \lambda_i(u+1)t]$$
$$+ \sum_{j=1}^{n} \frac{\lambda_j(u+1) - \lambda_j(u)}{\sum_{i=1}^{n} \lambda_i(u+1) - \sum_{i=1}^{n} \lambda_i(u)} \cdot [\exp[-\sum_{i=1}^{n} \frac{\lambda_i(u)}{q(u,d_{ij})} t]$$
$$- \exp[-(\sum_{i=1}^{n} \lambda_i(u+1) - \sum_{i=1}^{n} \lambda_i(u) + \sum_{i=1}^{n} \frac{\lambda_i(u)}{q(u,d_{ij})})t]],$$
$$u = 1,2,\ldots,z-1, \tag{18}$$

$$\boldsymbol{S}_{LLS}(t,z) = \exp[-\sum_{i=1}^{n} \lambda_i(z)t]. \tag{19}$$

Next, we consider a multistate series network composed of assets having identical exponential safety functions

$$S(t,\cdot) = [1, S(t,1), \ldots, S(t,z)],\ t \geq 0, \tag{20}$$

with the coordinates

$$S(t,u) = \exp[-\lambda(u)t], t \geq 0, \lambda(u) \geq 0,$$
$$u = 1,2,\ldots,z, \tag{21}$$

where $\lambda(u)$, $u = 1,2,\ldots,z$, are components' intensities of departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. Then, the intensities $\lambda_{i/j}(\upsilon)$, $i = 1,\ldots,n$, $j = 1,\ldots,n$, $\upsilon = u,u-1,\ldots,1$, of components' departure from this safety state subset after the departure of the jth component E_j, $j = 1,\ldots,n$, from (1), are given by

$$\lambda_{i/j}(\upsilon) = \frac{\lambda(\upsilon)}{q(\upsilon,d_{ij})},\ \upsilon = u,u-1,\ldots,1. \tag{22}$$

In this case *Proposition 7.2* takes the form presented below.

Proposition 7.3. If in a multistate series network assets are dependent according to the local load sharing rule and have identical exponential safety functions (20)-(21), then its safety function is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1), \ldots, \boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \tag{23}$$

with the coordinates

$$\boldsymbol{S}_{LLS}(t,u) = \exp[-n\lambda(u+1)t]$$
$$+ \frac{1}{n} \sum_{j=1}^{n} [\exp[-\lambda(u) \sum_{i=1}^{n} \frac{1}{q(u,d_{ij})} t]$$
$$- \exp[-(n\lambda(u+1) - n\lambda(u) + \lambda(u) \sum_{i=1}^{n} \frac{1}{q(u,d_{ij})})t]],$$
$$u = 1,2,\ldots,z-1, \tag{24}$$

$$\boldsymbol{S}_{LLS}(t,z) = \exp[-n\lambda(z)t]. \tag{25}$$

2.3. Safety of a multistate series CI network with dependent subnetworks

Further, we consider a series network composed of k dependent subnetworks, presented in Figure 2. We assume the local load sharing model of dependency between subnetworks. Then, after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by the subnetwork N_g, $g = 1,2,\ldots,k$, the safety parameters of assets of remaining subnetworks are changing dependently of the coefficients of the network load growth concerned with the distance from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. Within a single subnetwork the assets are independent and linked in series.

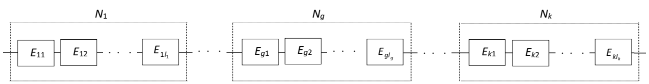

Figure 2. The scheme of a series network of k dependent subnetworks.

We assume that in the i-th subnetwork N_i, $i = 1,2,...,k$, there are l_i components, denoted by E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$ with exponential safety functions of the form

$$S_{ij}(t,\cdot) = [1, S_{ij}(t,1), \ldots, S_{ij}(t,z)],\ t \in (-\infty,\infty), \tag{26}$$

where

$$S_{ij}(t,u) = 1 \text{ for } t < 0,\ S_{ij}(t,u) = \exp[-\lambda_{ij}(u)t]$$
$$\text{for } t \geq 0,\ \lambda_{ij}(u) > 0,\ i = 1,2,...,n, j = 1,2,...,l_i,$$
$$u = 1,2,\ldots,z. \quad (27)$$

We denote by $E[T_{i,j}(u)]$ and $E[T_{i/g,j}(u)]$, $i = 1,2,...,k$, $g = 1,2,...,k$, $j = 1,2,...,l_i$, $u = 1,2,\ldots,z$, the mean values of the lifetimes of ith subnetwork assets $T_{i,j}(u)$ and $T_{i/g,j}(u)$, respectively, before and after departure of one fixed subnetwork S_g, $g = 1,\ldots,k$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. With this notation, in LLS model used between subnetworks, the mean values of their components lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, are decreasing according to the following formula:

$$E[T_{i/g,j}(\upsilon)] = q(\upsilon,d_{ig}) \cdot E[T_{i,j}(\upsilon)],\ i = 1,2,...,k,$$
$$g = 1,2,...,k, j = 1,2,...,l_i,\ \upsilon = u,u-1,\ldots,1, \quad (28)$$

where the coefficients of the network load growth $q(\upsilon,d_{ig})$, $0 < q(\upsilon,d_{ig}) \leq 1$ for $i = 1,2,...,k$, $g = 1,2,...,k$, and $q(\upsilon,0) = 1$ for $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z-1$, are functions of distance $d_{ig} = |i - g|$ from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. The distance between subnetworks can be interpreted in the metric sense as well as in the sense of relationships in the functioning of the network.

According to the well-known relationship between the lifetime mean value in this safety state subset and the intensity of departure from this safety state subset we can determine the intensities $\lambda_{i/g,j}(\upsilon)$, $i = 1,2,...,k$, $g = 1,2,...,k$, $j = 1,2,...,l_i$, of departure from the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, of ith subnetwork assets after the departure of the subnetwork S_g, $g = 1,\ldots,k$. Namely, from (28), we obtain

$$\lambda_{i/g,j}(\upsilon) = \frac{\lambda_{ij}(\upsilon)}{q(\upsilon,d_{ig})},\ i = 1,2,...,k, g = 1,2,...,k,$$
$$j = 1,2,...,l_i,\ \upsilon = u,u-1,\ldots,1. \quad (29)$$

Then, the safety function of such series network composed of k dependent multistate subnetworks N_i, $i = 1,2,\ldots,k$, is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1),\ldots,\boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \quad (30)$$

with the coordinates

$$\boldsymbol{S}_{LLS}(t,u) = \exp[-\sum_{i=1}^{k}\sum_{j=1}^{l_k}\lambda_{ij}(u+1)t]$$
$$+ \sum_{g=1}^{k}\frac{\sum_{j=1}^{l_k}(\lambda_{gj}(u+1)-\lambda_{gj}(u))}{\sum_{i=1}^{k}\sum_{j=1}^{l_k}(\lambda_{ij}(u+1)-\lambda_{ij}(u))}$$
$$\cdot [\exp[-\sum_{i=1}^{k}\sum_{j=1}^{l_k}\frac{\lambda_{ij}(u)}{q(u,d_{ig})}t]$$
$$-\exp[-\sum_{i=1}^{k}\sum_{j=1}^{l_k}(\lambda_{ij}(u+1)-\lambda_{ij}(u)+\frac{\lambda_{ij}(u)}{q(u,d_{ig})})t]],$$
$$u = 1,2,\ldots,z-1, \quad (31)$$

$$\boldsymbol{S}_{LLS}(t,z) = \exp[-\sum_{i=1}^{k}\sum_{j=1}^{l_i}\lambda_{ij}(z)t]. \quad (32)$$

2.4. Safety of a multistate series CI network with dependent assets of its subnetworks

Further, we consider a series network composed of k independent subnetworks, presented in Figure 3. We assume that in the i-th series subnetwork N_i, $i = 1,2,...,k$, there are l_i components dependent according to LLS rule, denoted by E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$. Then, $T_{ij}(u)$, $i = 1,2,...,k$, $j = 1,2,...,l$, k, $l \in N$, are random variables representing lifetimes of components E_{ij} in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,\ldots,z$. More exactly, we assume that after changing the safety state subset by one of assets E_{ig_i}, $g_i = 1,2,...,l_i$, in the i-th series subnetwork N_i, $i = 1,2,...,k$, to the worse safety state subset, the lifetimes of remaining assets in this subnetwork in the safety state subsets decrease. The local load sharing model of components dependency is described in Section 2.1.

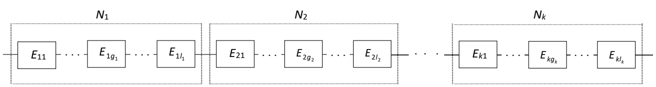

Figure 3. The scheme of a series network of k subnetworks with dependent assets.

We denote by $E[T_{i,j}(u)]$ and $E[T_{i,j/g_i}(u)]$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, $g_i = 1,2,...,l_i$, $u = 1,2,\ldots,z$, the mean values of components' lifetimes respectively, before and after departure of one fixed component E_{ig_i}, $g_i = 1,2,...,l_i$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in the i-th subnetwork N_i, $i = 1,2,...,k$. The safety parameters of remaining components E_{ij}, $j = 1,2,...,l_i$, $j \neq g_i$, in this subnetwork are changing dependently of the distance from the component

E_{ig_i}. Then, the mean values of these components lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, are decreasing according to the following formula:

$$E[T_{i,j/g_i}(\upsilon)] = q(\upsilon, d_{jg_i}) \cdot E[T_{i,j}(\upsilon)],\ i = 1,2,...,k,$$
$$j = 1,2,...,l_i,\ g_i = 1,2,...,l_i,\ \upsilon = u, u-1,\ldots,1, \qquad (33)$$

where the coefficients of the network load growth $q(\upsilon, d_{jg_i})$ are functions of components' distance $d_{jg_i} = |j - g_i|$, $i = 1,2,...,k$, from the component that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$.

If components have exponential safety functions given by (26)-(27), then after departure of the asset E_{ig_i}, $g_i = 1,2,...,l_i$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in the i-th subnetwork N_i, $i = 1,2,...,k$, the intensities $\lambda_{i,j/g_i}(\upsilon)$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, $g_i = 1,2,...,l_i$, of departure from the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, of remaining assets E_{ij}, $j = 1,2,...,l_i$, $j \neq g_i$, in this subnetwork are given by

$$\lambda_{i,j/g_i}(\upsilon) = \frac{\lambda_{ij}(\upsilon)}{q(\upsilon, d_{jg_i})},\ i = 1,2,...,k,\ j = 1,2,...,l_i,$$
$$g_i = 1,2,...,l_i,\ \upsilon = u, u-1,\ldots,1. \qquad (34)$$

Taking into account (34), the safety function of such series network composed of k multistate subnetworks N_i, $i = 1,2,\ldots,k$, with dependent assets is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1),\ldots,\boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \qquad (35)$$

with the coordinates

$$\boldsymbol{S}_{LLS}(t,u) = \prod_{i=1}^{k}[\exp[-\sum_{j=1}^{l_i}\lambda_{ij}(u+1)t]$$
$$+\sum_{g_i=1}^{l_i}\frac{\lambda_{ig_i}(u+1)-\lambda_{ig_i}(u)}{\sum_{j=1}^{l_i}(\lambda_{ij}(u+1)-\lambda_{ij}(u))}\cdot[\exp[-\sum_{j=1}^{l_i}\frac{\lambda_{ij}(u)}{q(u,d_{jg_i})}t]$$
$$-\exp[-\sum_{j=1}^{l_i}(\lambda_{ij}(u+1)-\lambda_{ij}(u)+\frac{\lambda_{ij}(u)}{q(u,d_{jg_i})})t]]],$$
$$u = 1,2,\ldots,z-1, \qquad (36)$$

$$\boldsymbol{S}_{LLS}(t,z) = \exp[-\sum_{i=1}^{k}\sum_{j=1}^{l_i}\lambda_{ij}(z)t]. \qquad (37)$$

3. Multistate series-parallel CI network with dependent assets of its subnetworks

3.1. Approach description

In this section we consider a network composed of k multistate series subnetworks working independently with dependent assets. Subnetworks are linked in parallel and form a series-parallel system, with a scheme given in Figure 4. We assume that in the i-th series subnetwork N_i, $i = 1,2,...,k$, there are l_i components dependent according to LLS rule, denoted by E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$. Similarly as in Section 7.2.4, we assume that after changing the safety state subset by one of assets E_{ig_i}, $g_i = 1,2,...,l_i$, in the i-th series subnetwork N_i, $i = 1,2,...,k$, to the worse safety state subset, the lifetimes of remaining assets in this subnetwork in the safety state subsets decrease.

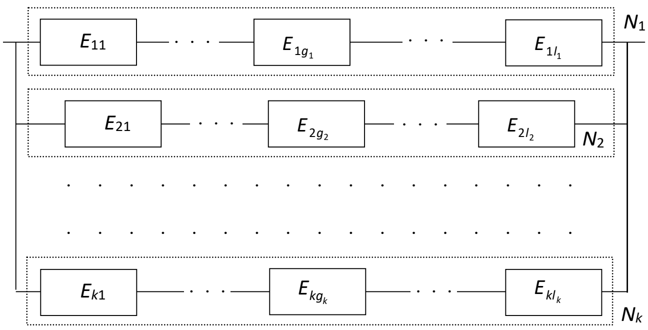

Figure 4. The scheme of a series-parallel network of k subnetworks with dependent assets.

3.2. Safety of a series-parallel CI network with dependent assets of its subnetworks

We assume that in the i-th subnetwork N_i, $i = 1,2,...,k$, there are l_i dependent assets with exponential safety functions (26)-(27). Then, after departure of the asset E_{ig_i}, $g_i = 1,2,...,l_i$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in the i-th subnetwork N_i, $i = 1,2,...,k$, the intensities $\lambda_{i,j/g_i}(\upsilon)$, $i = 1,2,...,k$, $j = 1,2,...,l_i$, $g_i = 1,2,...,l_i$, of departure from the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, of remaining assets E_{ij}, $j = 1,2,...,l_i$, $j \neq g_i$, in this subnetwork are given by (34).

Linking the results for a multistate series network having assets dependent according to LLS rule with the safety function of a parallel network with independent components, given in [Kołowrocki, 2014], [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006], we obtain following result.

Proposition 7.4. If in a multistate series-parallel network, there are k series subnetworks N_i, $i = 1,2,\ldots,k,$ with assets dependent according to the local load sharing rule and having exponential safety functions (26)-(27), then its safety function is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1), \ldots, \boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \tag{38}$$

with the coordinates

$$\begin{aligned}\boldsymbol{S}_{LLS}(t,u) &= 1 - \prod_{i=1}^{k}[1 - \exp[-\sum_{j=1}^{l_i}\lambda_{ij}(u+1)t] \\ &- \sum_{g_i=1}^{l_i}\frac{\lambda_{ig_i}(u+1) - \lambda_{ig_i}(u)}{\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u))} \cdot [\exp[-\sum_{j=1}^{l_i}\frac{\lambda_{ij}(u)}{q(u,d_{jg_i})}t] \\ &- \exp[-\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u) + \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})})t]]], \\ u &= 1,2,\ldots,z-1,\end{aligned} \tag{39}$$

$$\boldsymbol{S}_{LLS}(t,z) = 1 - \prod_{i=1}^{k}[1 - \exp[-\sum_{j=1}^{l_i}\lambda_{ij}(z)t]]. \tag{40}$$

4. Multistate series-"*m* out of *k*" CI network with dependent assets of its subnetworks

4.1. Approach description

We consider a multistate series-"*m* out of *k*" network, with a scheme given in Figure 5. The network is composed of *k* series subneworks working independently linked in a "*m* out of *k*" safety structure. We assume that in the *i*-th series subnetwork N_i, i = 1,2,...,k, there are l_i components dependent according to LLS rule, denoted by E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i. The local load sharing model of components dependency is described in Section 2.1.

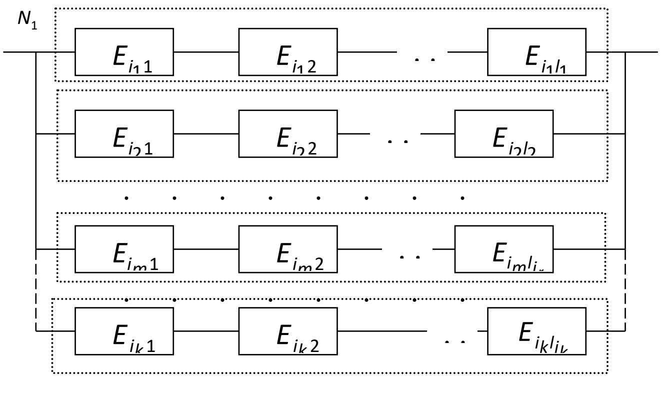

Figure 5. The scheme of a series-"*m* out of *k*" network safety structure

4.2. Safety of a series-"m out of k" network with dependent assets of its subnetworks

Then linking the results for a multistate series network assuming its components' dependency with the safety function of a "*m* out of *k*" system with independent components [Kołowrocki, 2014], [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006], we obtain following proposition.

Proposition 7.5. If in a multistate series-"*m* out of *k*" network, its subnetworks are working independently and assets of these series subnetworks are dependent according to the local load sharing, then its safety function is given by the vector

$$\overline{\boldsymbol{S}}_{LLS}^{(m)}(t,\cdot) = [1, \overline{\boldsymbol{S}}_{LLS}^{(m)}(t,1), \ldots, \overline{\boldsymbol{S}}_{LLS}^{(m)}(t,z)],\ t \geq 0, \tag{41}$$

with the coordinates

$$\begin{aligned}&\overline{\boldsymbol{S}}_{LLS}^{(m)}(t,u) \\ &= 1 - \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+\ldots+r_k \leq m-1}}^{1}\prod_{i=1}^{k}[\boldsymbol{S}_{LLS}^{(i)}(t,u)]^{r_i}[1 - \boldsymbol{S}_{LLS}^{(i)}(t,u)]^{1-r_i}, \\ &u = 1,2,\ldots,z,\end{aligned} \tag{42}$$

or by the vector

$$\overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,\cdot) = [1, \overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,1), \ldots, \overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,z)],\ t \geq 0, \tag{43}$$

with the coordinates

$$\begin{aligned}&\overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,u) = \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+\ldots+r_k \leq \overline{m}}}^{1}\prod_{i=1}^{k}[1 - \boldsymbol{S}_{LLS}^{(i)}(t,u)]^{r_i}[\boldsymbol{S}_{LLS}^{(i)}(t,u)]^{1-r_i}, \\ &\overline{m} = k - m,\ u = 1,2,\ldots,z,\end{aligned} \tag{44}$$

where $\boldsymbol{S}_{LLS}^{(i)}(t,u)$, $u = 1,2,\ldots,z,$ is the safety function coordinate, given by (5)-(6), of the *i*-th series subnetwork i = 1,2,...,k with local load sharing model of dependency.

We assume that in the *i*-th subnetwork N_i, i = 1,2,...,k, there are l_i components, denoted by E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i with exponential safety functions (26)-(27). Then, applying (41)-(42) or (43)-(44) respectively, from *Proposition 7.5* we can obtain immediately the following result.

Proposition 7.6. If in a multistate series-"*m* out of *k*" network, there are k series subnetworks N_i, $i = 1,2,\ldots,k,$ with assets dependent according to the local load sharing rule and having exponential safety functions (26)-(27), then its safety function is given by the vector

$$\overline{\boldsymbol{S}}_{LLS}^{(m)}(t,\cdot) = [1, \overline{\boldsymbol{S}}_{LLS}^{(m)}(t,1), \ldots, \overline{\boldsymbol{S}}_{LLS}^{(m)}(t,z)],\ t \geq 0, \quad (45)$$

with the coordinates

$$\overline{\boldsymbol{S}}_{LLS}^{(m)}(t,u) = 1 - \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+\ldots+r_k \leq m-1}}^{1} \prod_{i=1}^{k} [\exp[-\sum_{j=1}^{l_i} \lambda_{ij}(u+1)t]$$
$$+ \sum_{g_i=1}^{l_i} \frac{\lambda_{ig_i}(u+1) - \lambda_{ig_i}(u)}{\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u))} \cdot [\exp[-\sum_{j=1}^{l_i} \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})} t]$$
$$- \exp[-\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u) + \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})})t]]]^{r_i}$$
$$\cdot [1 - \exp[-\sum_{j=1}^{l_i} \lambda_{ij}(u+1)t]$$
$$- \sum_{g_i=1}^{l_i} \frac{\lambda_{ig_i}(u+1) - \lambda_{ig_i}(u)}{\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u))}$$
$$\cdot [\exp[-\sum_{j=1}^{l_i} \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})} t]$$
$$- \exp[-\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u) + \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})})t]]]^{1-r_i},$$
$$u = 1,2,\ldots,z-1, \quad (46)$$

$$\overline{\boldsymbol{S}}_{LLS}^{(m)}(t,z) = 1$$
$$- \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+\ldots+r_k \leq m-1}}^{1} \prod_{i=1}^{k} [\exp[-\sum_{j=1}^{l_i} \lambda_{ij}(z)t]]^{r_i} \quad (47)$$
$$[1 - \exp[-\sum_{j=1}^{l_i} \lambda_{ij}(z)t]]^{1-r_i}$$

or by the vector

$$\overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,\cdot) = [1, \overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,1), \ldots, \overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,z)],\ t \geq 0, \quad (48)$$

with the coordinates

$$\overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,u) = \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+\ldots+r_k \leq \overline{m}}}^{1} \prod_{i=1}^{k} [1 - \exp[-\sum_{j=1}^{l_i} \lambda_{ij}(u+1)t]$$
$$- \sum_{g_i=1}^{l_i} \frac{\lambda_{ig_i}(u+1) - \lambda_{ig_i}(u)}{\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u))} \cdot [\exp[-\sum_{j=1}^{l_i} \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})} t]$$
$$- \exp[-\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u) + \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})})t]]]^{r_i}$$
$$[\exp[-\sum_{j=1}^{l_i} \lambda_{ij}(u+1)t] + \sum_{g_i=1}^{l_i} \frac{\lambda_{ig_i}(u+1) - \lambda_{ig_i}(u)}{\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u))}$$
$$\cdot [\exp[-\sum_{j=1}^{l_i} \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})} t]$$
$$- \exp[-\sum_{j=1}^{l_i}(\lambda_{ij}(u+1) - \lambda_{ij}(u) + \frac{\lambda_{ij}(u)}{q(u,d_{jg_i})})t]]]^{1-r_i}$$
$$\overline{m} = k - m,\ u = 1,2,\ldots,z-1, \quad (49)$$

$$\overline{\boldsymbol{S}}_{LLS}^{(\overline{m})}(t,z) = \sum_{\substack{r_1,r_2,\ldots,r_k=0 \\ r_1+\ldots+r_k \leq \overline{m}}}^{1} \prod_{i=1}^{k} [1 - \exp[-\sum_{j=1}^{l_i} \lambda_{ij}(z)t]]^{r_i}$$
$$[\exp[-\sum_{j=1}^{l_i} \lambda_{ij}(z)t]]^{1-r_i},$$
$$\overline{m} = k - m. \quad (50)$$

5. Multistate parallel CI networks with dependent assets

5.1. Approach description

For a parallel network composed of subnetworks we assume that after decreasing the safety state by one of the subnetworks the increased load can be shared equally among the remaining subnetworks. More generally, we assume that after leaving the safety state subset by some of subnetworks, the lifetimes of remaining subnetworks decrease equally depending on the number of these subnetworks that have left the safety state subset. Additionally these changes are influenced by the component stress proportionality correction coefficient, concerned with features of particular network and its components. More exactly, if $\omega, \omega = 0,1,2,\ldots,n-1$, subnetworks are out of the safety state subset $\{u,u+1,...,z\}$, the mean values of the lifetimes $T_i'(u)$ in the safety state subset $\{u,u+1,...,z\}$ of the remaining subnetworks become less according to the formula

$$E[T_i'(u)] = c(u)\frac{n-\omega}{n}E[T_i(u)],\ i = 1,2,\ldots,n,$$
$$u = 1,2,\ldots,z, \quad (51)$$

where $c(u)$ is the component stress proportionality correction coefficient for each u, $u = 1,2,\ldots,z$, [Kołowrocki, 2013]. This model of equal load sharing (ELS) is often applied to parallel or "m out of n" systems and has been analyzed in [Blokus-Roszkowska, Kołowrocki, 2014a,b].

Hence, for case of network with dependent subnetworks having identical exponential safety functions of the form

$$S_i(t,\cdot) = [1, S_i(t,1), \ldots, S_i(t,z)], \ t \geq 0,$$
$$i = 1,2,\ldots,n, \tag{52}$$

with the coordinates

$$S_i(t,u) = \exp[-\lambda(u)t], \ t \geq 0, \ i = 1,2,\ldots,n,$$
$$u = 1,2,\ldots,z, \tag{53}$$

we get following formula for intensities of departure from the safety state subset $\{u,u+1,...,z\}$, of remaining subnetworks

$$\lambda^{(\omega)}(u) = \frac{n}{n-\omega}\frac{\lambda(u)}{c(u)},$$
$$\omega = 0,1,\ldots,n-1, \ u = 1,2,\ldots,z. \tag{54}$$

5.2. Safety of multistate parallel networks with dependent assets

Proposition 7.7. If in a multistate parallel network subnetworks are dependent according to the equal load sharing rule and have identical exponential safety functions (52)-(53), then its safety function is given by the vector

$$\boldsymbol{S}_{ELS}(t,\cdot) = [1, \boldsymbol{S}_{ELS}(t,1), \ldots, \boldsymbol{S}_{ELS}(t,z)], \ t \geq 0, \tag{55}$$

with the coordinates

$$\boldsymbol{S}_{ELS}(t,u) = \sum_{\omega=0}^{n-1} \frac{\left[\frac{n\lambda(u)}{c(u)}t\right]^{\omega}}{\omega!} \exp[-\frac{n\lambda(u)}{c(u)}t],$$
$$u = 1,2,\ldots,z. \tag{56}$$

6. Multistate "*m* out of *n*" CI networks with dependent assets

6.1. Approach description

For a "*m* out of *n*" network composed of subnetworks, similarly as for a parallel network, we assume equal load sharing model of dependency. Then, after decreasing the safety state by some of the subnetworks the increased load can be shared equally among the remaining subnetworks. More exacly, if $\omega, \omega = 0,1,2,\ldots,n-m,$ subnetworks of a network are out of the safety state subset $\{u,u+1,...,z\}$, u = 1, 2,…,z, the mean values of the lifetimes in this safety state subset of the remaining subnetworks are given by (7.51). Then, in case subnetworks have identical exponential safety functions given by (52)-(53), the intensities of departure from the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,\ldots,z$, of remaining subnetworks are given by

$$\lambda^{(\omega)}(u) = \frac{n}{n-\omega}\frac{\lambda(u)}{c(u)}, \ \omega = 0,1,\ldots,n-m,$$
$$u = 1,2,\ldots,z. \tag{57}$$

6.2. Safety of multistate "m out of n" networks with dependent assets

Proposition 7.8. If in a multistate "*m* out of *n*" network subnetworks are dependent according to the equal load sharing rule and have identical exponential safety functions (52)-(53), then its safety function is given by the vector

$$\boldsymbol{S}_{ELS}(t,\cdot) = [1, \boldsymbol{S}_{ELS}(t,1), \ldots, \boldsymbol{S}_{ELS}(t,z)], \tag{58}$$

with the coordinates

$$\boldsymbol{S}_{ELS}(t,u) = \sum_{\omega=0}^{n-m} \frac{\left[\frac{n\lambda(u)}{c(u)}t\right]^{\omega}}{\omega!} \exp[-\frac{n\lambda(u)}{c(u)}t], \ t \geq 0,$$
$$u = 1,2,\ldots,z. \tag{59}$$

7. Multistate parallel-series network with dependent assets of its subnetworks

7.1. Approach description

In this section we consider a multistate parallel-series network with a scheme presented in Figure 6. We assume that k is a number of parallel subnetworks working independently linked in series. In ith subnetowork we assume there are l_i, $i = 1,2,...,k$, assets dependent according to the equal load sharing rule, described in Section 7.5.1. Then, we assume that after leaving the safety state subset by some of assets in a subnetwork, the lifetimes of remaining assets in this subnetwork decrease equally depending on the number of these assets that have left the safety state subset. Additionally these changes are influenced by the component stress proportionality correction coefficient $c_i(u)$, $i = 1,2,...,k$, $u = 1,2,\ldots,z$, concerned with features of ith subnetowork and its assets. We denote by E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, components of a network and assume that all assets E_{ij} have the same safety state set as before $\{0,1,...,z\}$. Then, $T_{ij}(u)$, $i = 1,2,...,k$, $j = 1,2,..., l_i$, are random variables representing lifetimes of assets E_{ij} in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,\ldots,z$.

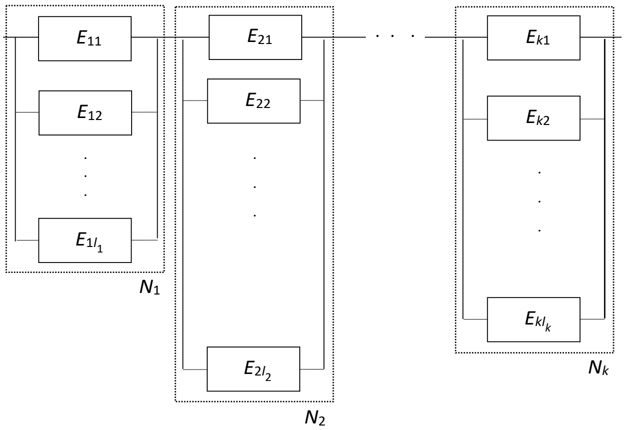

Figure 6. The scheme of a parallel-series network safety structure

We assume similarly as in formula (54) for a multistate parallel network that if $\omega_i, \omega_i = 0,1,2,\ldots,l_i-1$, assets in *i*th parallel subnetwork *i* = 1,2,...,*k*, are out of the safety state subset {*u*,*u*+1,...,*z*}, *u* = 1,2,…,*z*, the mean values of lifetimes $T_{ij}'(u)$ in the safety state subset {*u*,*u*+1,...,*z*} of this subnetwork remaining assets are given by [Kołowrocki, 2013]

$$E[T_{ij}'(u)] = c_i(u)\frac{l_i-\omega_i}{l_i}E[T_{ij}(u)],$$
$$i=1,2,\ldots,k,\ j=1,2,\ldots,l_i,\ \omega_i=0,1,2,\ldots,l_i-1,$$
$$u=1,2,\ldots,z. \quad (60)$$

We assume that in *i*-th parallel subnetwork N_i, *i* = 1,2,...,*k*, assets are dependent according to ELS rule and have identical exponential safety functions of the form

$$S_{ij}(t,\cdot) = [1, S_{ij}(t,1),\ldots,S_{ij}(t,z)],\ t \geq 0,\ i=1,2,\ldots,k,$$
$$j=1,2,\ldots,l_i, \quad (61)$$

where

$$S_{ij}(t,u) = \exp[-\lambda_i(u)t],\ t\geq 0,\ i=1,2,\ldots,k,$$
$$j=1,2,\ldots,l_i,\ u=1,2,\ldots,z, \quad (62)$$

with the intensity of departure $\lambda_i(u)$ from the safety state subset {*u*,*u*+1,...,*z*}, *u* = 1,2,…,*z*. Then, after the departure of $\omega_i, \omega_i = 0,1,2,\ldots,l_i-1$, assets from this safety state subset in the *i*th subnetwork *i* = 1,2,...,*k*, we get following formula for the intensities of departure from this subset of remaining assets in the *i*th subnetowork

$$\lambda_i^{(\omega)}(u) = \frac{l_i}{l_i-\omega_i}\frac{\lambda_i(u)}{c_i(u)},\ i=1,2,\ldots,k,$$
$$\omega_i = 0,1,2,\ldots,l_i-1,\ u=1,2,\ldots,z. \quad (63)$$

7.2. Safety of a multistate parallel-series network with dependent assets of its subnetworks

Considering results, for a parallel network with assets dependent according to the equal load sharing rule, given in *Proposition 7.7* and linking these results with the safety function of a series network with independent subnetworks, we can obtain the formula for the safety function of a parallel-series network in the form of following proposition.

Proposition 7.9. If in a multistate parallel-series network, there are *k* parallel subnetworks N_i, $i=1,2,\ldots,k$, with assets dependent according to the equal load sharing rule and having exponential safety functions (61)-(62), then its safety function is given by the vector

$$\boldsymbol{S}_{ELS}(t,\cdot) = [1, \boldsymbol{S}_{ELS}(t,1),\ldots,\boldsymbol{S}_{ELS}(t,z)], \quad (64)$$

with the coordinates

$$\boldsymbol{S}_{ELS}(t,u) = \prod_{i=1}^{k}\Big[\sum_{\omega_i=0}^{l_i-1}\frac{\big[\frac{l_i\lambda_i(u)}{c_i(u)}t\big]^{\omega_i}}{\omega_i!}\exp\Big[-\frac{l_i\lambda_i(u)}{c_i(u)}t\Big]\Big],$$
$$t\geq 0,\ u=1,2,\ldots,z. \quad (65)$$

8. Multistate "*m* out of *l*"-series networks with dependent assets of their subnetworks

8.1. Approach description

Here, we consider a multistate "m_i out of l_i"-series network composed of *k* linked in series "m_i out of l_i", *i* = 1,2,...,*k*, subnetworks. The scheme of such network is presented in Figure 7. We assume that assets in each "m_i out of l_i", *i* = 1,2,...,*k*, subnetwork are dependent according to the equal load sharing rule and subnetworks are working independently.

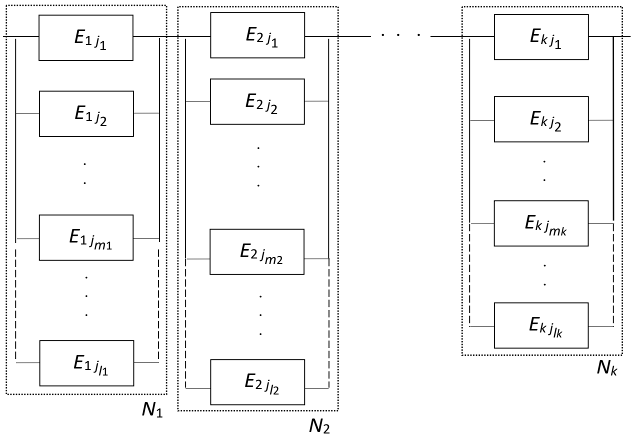

Figure 7. The scheme of a "m_i out of l_i"-series network.

In each "m_i out of l_i" subnetwork there are l_i, i = 1,2,...,k, assets that are dependent according to the equal load sharing rule, described in Section 7.6.1. We assume that if $\omega_i, \omega_i = 0,1,2,\ldots,l_i - m_i$, assets in ith "m_i out of l_i", i = 1,2,...,k, subnetwork are out of the safety state subset {u,u+1,...,z}, u = 1,2,…,z, the mean values of lifetimes $T_{ij}'(u)$ in the safety state subset {u,u+1,...,z} of this subnetwork remaining assets are given by [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006]

$$E[T_{ij}'(u)] = c_i(u)\frac{l_i - \omega_i}{l_i}E[T_{ij}(u)],$$
$$i = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i,\ \omega_i = 0,1,2,\ldots,l_i - m_i,$$
$$u = 1,2,\ldots,z. \tag{66}$$

We assume that in i-th subnetwork N_i, i = 1,2,...,k, assets are dependent according to ELS rule and have identical exponential safety functions (61)-(62). Then, the intensities of departure from the safety state subset {u,u+1,...,z}, u = 1,2,…,z, of remaining assets in the ith, i = 1,2,...,k, subnetwork are given by

$$\lambda_i^{(\omega)}(u) = \frac{l_i}{l_i - \omega_i}\frac{\lambda_i(u)}{c_i(u)},\ i = 1,2,\ldots,k,$$
$$\omega_i = 0,1,2,\ldots,l_i - m_i,\ u = 1,2,\ldots,z. \tag{67}$$

8.2. Safety of a multistate "m out of l"-series system with dependent assets of its subnetworks

Proposition 7.9 slight extension yields the following result.

Proposition 7.10. If in a multistate "m_i out of l_i"-series network, there are k "m_i out of l_i" subnetworks N_i, $i = 1,2,\ldots,k$, with assets dependent according to the equal load sharing rule and having exponential safety functions (61)-(62), then its safety function is given by the vector

$$\boldsymbol{S}_{ELS}(t,\cdot) = [1, \boldsymbol{S}_{ELS}(t,1),\ldots,\boldsymbol{S}_{ELS}(t,z)], \tag{68}$$

with the coordinates

$$\boldsymbol{S}_{ELS}(t,u) = \prod_{i=1}^{k}[\sum_{\omega_i=0}^{l_i-m_i}\frac{[\frac{l_i\lambda_i(u)}{c_i(u)}t]^{\omega_i}}{\omega_i!}\exp[-\frac{l_i\lambda_i(u)}{c_i(u)}t]],$$
$$t \geq 0,\ u = 1,2,\ldots,z. \tag{69}$$

9. Multistate parallel-series networks with dependent subnetworks and dependent assets of these subnetworks

9.1. Approach description

Considering cascading effects in networks with more complex structures we can link the results of safety analysis for previously described dependency models. Then, apart from the dependency of subnetworks' departures from the safety states subsets we can take into account the dependencies between assets in subnetworks. This way we can proceed with parallel-series and "m out of l"-series networks assuming the dependence between their parallel, respectively "m out of l", subnetworks according to the local load sharing rule and the dependence between their assets in subnetworks according to the equal load sharing rule. Further, such model of dependency we will call a mixed load sharing (MLS) model.

9.2. Safety of a multistate parallel-series network with dependent subnetworks and dependent assets of these subnetworks

In this section, we propose a mixed load sharing model of dependency between subnetworks and between assets in these subnetworks. We consider a multistate parallel-series network composed of k parallel subnetworks N_i, i = 1,2,…,k, connected in series, illustrated in Figure 8. Further, by E_{ij}, i = 1,2,…,k, j = 1,2,…,l_i, we denote the jth asset being in the ith subsystem N_i, and we assume that assets in the ith subnetwork have identical exponential safety functions, given by (61)-(62).

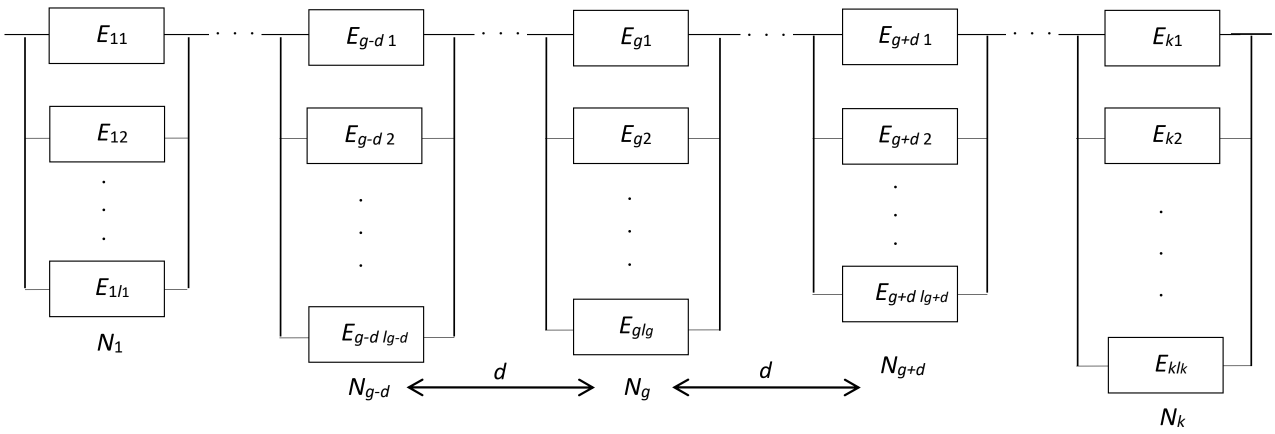

Figure 8. The scheme of a parallel-series network.

In the ith parallel subnetwork N_i, $i = 1,2,\ldots,k$, we consider dependency of its l_i assets according to the equal load sharing model, presented in Section 5.1. Then, after departure from the safety state subset

$\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by $\omega_i, \omega_i = 0,1,2,\ldots,l_i - 1$, assets of the subnetwork, the intensities of departure from this safety state subset of the remaining assets in the subnetwork are given by (63).

Further, between these subnetworks, linked in series, we assume the local load sharing model of dependency, presented in Section 7.2.1. Then, we assume that after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by the subnetwork N_g, $g = 1,2,\ldots,k$, the safety parameters of assets of remaining subnetworks are changing dependently of the distance from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, expressed by index d. However, within a single subnetwork the changes of the safety parameters for all of its assets are on the same level according to the equal load sharing rule. The meaning of the distance d in mixed load sharing model is illustrated in Figure 8.

We denote by $E[T_{i,j}(u)]$ and $E[T_{i/g,j}(u)]$, $i = 1,2,...,k$, $g = 1,2,...,k$, $j = 1,2,...,l_i$, $u = 1,2,\ldots,z$, the mean values of the lifetimes of ith subnetwork assets $T_{i,j}(u)$ and $T_{i/g,j}(u)$, respectively, before and after departure of one fixed subnetwork N_g, $g = 1,\ldots,k$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. With this notation, in the local load sharing model used between subsnetworks, the mean values of their components lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u\text{-}1,\ldots,1$, $u = 1,2,\ldots,z$, are decreasing, using (1), according to the following formula:

$$E[T_{i/g,j}(\upsilon)] = q(\upsilon, d_{ig}) \cdot E[T_{i,j}(\upsilon)],\ i = 1,2,...,k,$$
$$g = 1,2,...,k, j = 1,2,...,l_i,\ \upsilon = u, u-1,\ldots,1, \qquad (70)$$

where the coefficients of the network load growth $q(\upsilon,d_{ig})$, $0 < q(\upsilon,d_{ig}) \le 1$ for $i = 1,2,...,k$, $g = 1,2,...,k$, and $q(\upsilon,0) = 1$ for $\upsilon = u,u\text{-}1,\ldots,1$, $u = 1,2,\ldots,z\text{-}1$, are non-increasing functions of subnetworks' distance $d_{ig} = |i - g|$ from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. The distance between subnetworks can be interpreted in the metric sense as well as in the sense of relationships in the network functioning.

Considering results, given in [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006], concerned with Erlang distribution of network lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in case assets of parallel network are dependent according to the equal load sharing rule, and linking this result with the safety function of a series network with assets dependent according to the local load sharing rule and having Erlang safety functions, presented in [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006], we can obtain the safety function of a multistate parallel-series network with mixed model of dependency. Then, applying Proposition 7.1 for series network composed of k subnetworks N_i, $i = 1,2,\ldots,k$, and using fact that the ith subnetwork has Erlang safety functions with the shape parameter l_i and with the intensity parameter $l_i\lambda_i(u)/c_i(u)$, $u = 1,2,\ldots,z$, we immediately get the following result.

Proposition 7.11. If in a multistate parallel-series netowork, there are k parallel subnetworks N_i, $i = 1,2,\ldots,k$, dependent according to the local load sharing rule and assets of these parallel subnetworks are dependent according to the equal load sharing rule and have exponential safety functions (61)-(62), then its safety function is given by the vector

$$\boldsymbol{S}_{MLS}(t,\cdot) = [1, \boldsymbol{S}_{MLS}(t,1), \ldots, \boldsymbol{S}_{MLS}(t,z)],\ t \geq 0, \quad (71)$$

with the coordinates

$$\boldsymbol{S}_{MLS}(t,u) = \prod_{i=1}^{k} [\sum_{\omega=0}^{l_i-1} \frac{[\frac{l_i\lambda_i(u+1)}{c_i(u+1)} t]^{\omega}}{\omega!}]$$
$$\cdot \exp[-\frac{l_i\lambda_i(u+1)}{c_i(u+1)} t]$$
$$+ \sum_{g=1}^{k} \int_0^t \tilde{f}_g(a,u+1) \cdot \prod_{\substack{i=1 \\ i\neq g}}^{k} [\sum_{\omega=0}^{l_i-1} \frac{[\frac{l_i\lambda_i(u+1)}{c_i(u+1)} a]^{\omega}}{\omega!}]$$
$$\cdot \sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u)}{c_g(u)} a]^{\omega}}{\omega!}$$
$$\cdot \exp[-(\sum_{\substack{i=1 \\ i\neq g}}^{k} \frac{l_i\lambda_i(u+1)}{c_i(u+1)} + \frac{l_g\lambda_g(u)}{c_g(u)}) a]$$
$$\cdot [\prod_{i=1}^{k} \frac{\sum_{\omega=0}^{l_i-1} \frac{[\frac{l_i\lambda_i(u)}{c_i(u)q(u,d_{ig})} t]^{\omega}}{\omega!}}{\sum_{\omega=0}^{l_i-1} \frac{[\frac{l_i\lambda_i(u)}{c_i(u)} a]^{\omega}}{\omega!}}$$
$$\cdot \exp[-\frac{l_i\lambda_i(u)}{c_i(u)q(u,d_{ig})} t + \frac{l_i\lambda_i(u)}{c_i(u)} a]] da,$$
$$u = 1,2,\ldots,z-1, \quad (72)$$

where $\tilde{f}_g(a,u+1)$ is given by

$$\tilde{f}_g(a,u+1)$$
$$= [\frac{\sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u+1)}{c_g(u+1)} a]^{\omega}}{\omega!} \cdot \sum_{\omega=0}^{l_g-2} \frac{[\frac{l_g\lambda_g(u)}{c_g(u)}]^{\omega+1} a^{\omega}}{\omega!}}{[\sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u)}{c_g(u)} a]^{\omega}}{\omega!}]^2}$$
$$- \frac{\sum_{\omega=0}^{l_g-2} \frac{[\frac{l_g\lambda_g(u+1)}{c_g(u+1)}]^{\omega+1} a^{\omega}}{\omega!} \cdot \sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u)}{c_g(u)} a]^{\omega}}{\omega!}}{[\sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u)}{c_g(u)} a]^{\omega}}{\omega!}]^2}$$
$$+ \frac{\sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u+1)}{c_g(u+1)} a]^{\omega}}{\omega!} (\frac{l_g\lambda_g(u+1)}{c_g(u+1)} - \frac{l_g\lambda_g(u)}{c_g(u)})}{\sum_{\omega=0}^{l_g-1} \frac{[\frac{l_g\lambda_g(u)}{c_g(u)} a]^{\omega}}{\omega!}}]$$
$$\cdot \exp[-(\frac{l_g\lambda_g(u+1)}{c_g(u+1)} - \frac{l_g\lambda_g(u)}{c_g(u)}) a],$$
$$u = 1,2,\ldots,z-1, \quad (73)$$

and

$$\boldsymbol{S}_{MLS}(t,z) = \prod_{i=1}^{k} [\sum_{\omega=0}^{l_i-1} \frac{[\frac{l_i\lambda_i(z)}{c_i(z)} t]^{\omega}}{\omega!}]^k$$
$$\cdot \exp[-\sum_{i=1}^{k} \frac{l_i\lambda_i(z)}{c_i(z)} t]. \quad (74)$$

The results for a multistate regular parallel-series system with dependent components are subsystems are presented in [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006].

10. Multistate "*m* out of *l*"-series networks with dependent subnetworks and dependent assets of these subnetworks

10.1. Approach description

Next, we apply a mixed load sharing model of assets and subnetworks dependency to the safety analysis of a multistate "m_i out of l_i"-series network. We consider a multistate "m_i out of l_i"-series network composed of k "m_i out of l_i" subnetworks N_i, i = 1,2,…,k, linked in series, illustrated in Figure 9. Further, similarly as for a parallel-series network, by E_{ij}, i = 1,2,…,k, j = 1,2,…,l we denote the jth asset being in the ith subnetwork N_i, and we assume all assets in the ith subnetwork have identical exponential safety functions, given by (61)-(62).

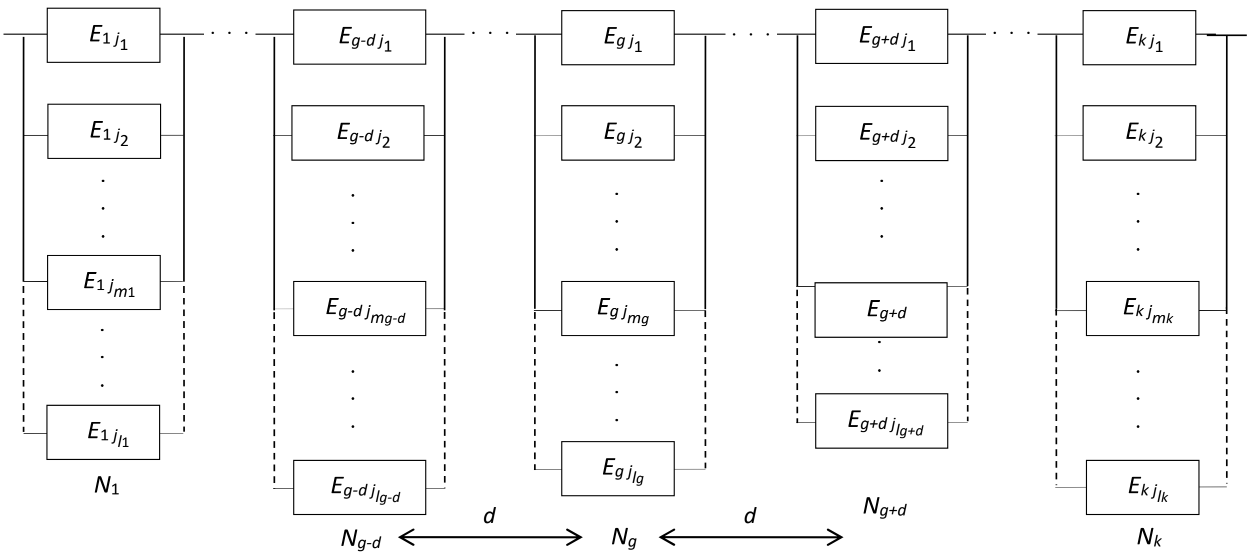

Figure 9. The scheme of a "m_i out of l_i"-series network

10.2. Safety of a multistate "m out of l"-series network with dependent subnetworks and dependent assets of these subnetworks

In the ith "m_i out of l_i" subnetwork N_i we consider, similary as in previous section, dependency of its l_i components according to the equal load sharing model, presented in Section 7.6.1. Then, after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by $\omega_i, \omega_i = 0,1,2,\ldots,l_i - m_i$, assets of the subnetwork, the intensities of departure from this safety state subset of the remaining assets in the subnetwork are given by (67).

Further, between these subnetworks, linked in series, we assume the local load sharing model of dependency, presented in Section 7.2.1. Then, we assume that after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by the subnetwork N_g, $g = 1,2,\ldots,k$, the safety parameters of assets of remaining subnetworks are changing dependently of the distance from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, expressed by index d. The mean values of assets lifetimes of remaining subnetworks in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, are decreasing according to (70).

Considering results, given in [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006], concerned with Erlang distribution of network lifetime in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in case assets of "m out of l" network are dependent according to the equal load sharing rule, and linking this result with the safety function of a series network with assets dependent according to the local load sharing rule and having Erlang safety functions, presented in [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006], we can obtain the safety function of a multistate "m_i out of l_i"-series network with mixed model of dependency. Then, applying Proposition 7.1 for series network composed of k subnetworks N_i, $i = 1,2,\ldots,k$, and using fact that the ith subnetwork has Erlang safety functions with the shape parameter $l_i - m_i + 1$ and with the intensity parameter $l_i\lambda_i(u)/c_i(u)$, $u = 1,2,\ldots,z$, we immediately get the following result.

Proposition 7.12. If in a multistate "m_i out of l_i"-series network, there are k "m_i out of l_i" subnetworks N_i, $i = 1,2,\ldots,k$, dependent according to the local load sharing rule and assets of these subnetworks are dependent according to the equal load sharing rule and have exponential safety functions (61)-(62), then its safety function is given by the vector

$$\boldsymbol{S}_{MLS}(t,\cdot) = [1, \boldsymbol{S}_{MLS}(t,1),\ldots, \boldsymbol{S}_{MLS}(t,z)],\ t \geq 0, \quad (75)$$

with the coordinates

$$\boldsymbol{S}_{MLS}(t,u) = \prod_{i=1}^{k}\Big[\sum_{\omega=0}^{l_i-m_i} \frac{\big[\frac{l_i\lambda_i(u+1)}{c_i(u+1)}t\big]^{\omega}}{\omega!}\Big] \cdot \exp\Big[-\frac{l_i\lambda_i(u+1)}{c_i(u+1)}t\Big]$$

$$+ \sum_{g=1}^{k}\int_0^t \tilde{f}_g(a,u+1)\cdot \prod_{\substack{i=1\\ i\neq g}}^{k}\Big[\sum_{\omega=0}^{l_i-m_i} \frac{\big[\frac{l_i\lambda_i(u+1)}{c_i(u+1)}a\big]^{\omega}}{\omega!}\Big]$$

$$\cdot \sum_{\omega=0}^{l_g-m_g} \frac{\big[\frac{l_g\lambda_g(u)}{c_g(u)}a\big]^{\omega}}{\omega!}$$

$$\cdot \exp[-(\sum_{\substack{i=1\\i\neq g}}^{k}\frac{l_i\lambda_i(u+1)}{c_i(u+1)}+\frac{l_g\lambda_g(u)}{c_g(u)})a]$$

$$\cdot [\prod_{i=1}^{k}\frac{\sum_{\omega=0}^{l_i-m_i}\frac{[\frac{l_i\lambda_i(u)}{c_i(u)q(u,d_{ig})}t]^{\omega}}{\omega!}}{\sum_{\omega=0}^{l_i-m_i}\frac{[\frac{l_i\lambda_i(u)}{c_i(u)}a]^{\omega}}{\omega!}}$$

$$\cdot \exp[-\frac{l_i\lambda_i(u)}{c_i(u)q(u,d_{ig})}t+\frac{l_i\lambda_i(u)}{c_i(u)}a]]da,$$

$$u=1,2,\ldots,z-1, \tag{76}$$

where $\tilde{f}_g(a,u+1)$ is given by

$$\tilde{f}_g(a,u+1)$$

$$=[\frac{\sum_{\omega=0}^{l_g-m_g}\frac{[\frac{l_g\lambda_g(u+1)}{c_g(u+1)}a]^{\omega}}{\omega!}\cdot\sum_{\omega=0}^{l_g-m_g-1}\frac{[\frac{l_g\lambda_g(u)}{c_g(u)}]^{\omega+1}a^{\omega}}{\omega!}}{[\sum_{\omega=0}^{l_g-m_g}\frac{[\frac{l_g\lambda_g(u)}{c_g(u)}a]^{\omega}}{\omega!}]^2}$$

$$-\frac{\sum_{\omega=0}^{l_g-m_g-1}\frac{[\frac{l_g\lambda_g(u+1)}{c_g(u+1)}]^{\omega+1}a^{\omega}}{\omega!}\cdot\sum_{\omega=0}^{l_g-m_g}\frac{[\frac{l_g\lambda_g(u)}{c_g(u)}a]^{\omega}}{\omega!}}{[\sum_{\omega=0}^{l_g-m_g}\frac{[\frac{l_g\lambda_g(u)}{c_g(u)}a]^{\omega}}{\omega!}]^2}$$

$$+\frac{\sum_{\omega=0}^{l_g-m_g}\frac{[\frac{l_g\lambda_g(u+1)}{c_g(u+1)}a]^{\omega}}{\omega!}(\frac{l_g\lambda_g(u+1)}{c_g(u+1)}-\frac{l_g\lambda_g(u)}{c_g(u)})}{\sum_{\omega=0}^{l_g-m_g}\frac{[\frac{l_g\lambda_g(u)}{c_g(u)}a]^{\omega}}{\omega!}}]$$

$$\cdot \exp[-(\frac{l_g\lambda_g(u+1)}{c_g(u+1)}-\frac{l_g\lambda_g(u)}{c_g(u)})a],$$

$$u=1,2,\ldots,z-1, \tag{77}$$

and

$$\boldsymbol{S}_{MLS}(t,z)=\prod_{i=1}^{k}[\sum_{\omega=0}^{l_i-m_i}\frac{[\frac{l_i\lambda_i(z)}{c_i(z)}t]^{\omega}}{\omega!}]^{k}$$

$$\cdot \exp[-\sum_{i=1}^{k}\frac{l_i\lambda_i(z)}{c_i(z)}t]. \tag{78}$$

The results for a multistate regular "*m* out of *l*"-series system with dependent components are subsystems are presented in [Blokus-Roszkowska, Kołowrocki, Soszyńska-Budny, 2006].

11. Conclusions

In this report, a local load sharing (LLS) model of dependency for a multistate series CI network has been proposed. Considering the network composed of multistate subnetworks, the influence of subnetworks' inside-dependences on their safety as well as the impact of subnetworks' degradation on other subnetworks safety have been analyzed. LLS model of dependency has been also applied to the safety analysis of the multistate series-parallel network with dependent assets of its subnetworks and multistate series-"*m* out of *k*" network with dependent assets of its subnetworks. Next, equal load sharing (ELS) model of dependency has been introduced and used for the analysis of the multistate parallel and "*m* out of n" networks with dependent assets. ELS model has been used for determination of the safety function of multistate parallel-series and "*m* out of *l*"-series networks with dependent assets of its subnetworks. Finally, mixed load sharing (MLS) model has been described and applied to safety analysis of multistate parallel-series and "*m* out of *l*"-series networks with dependent subnetworks and dependent assets of these subnetworks.

Proposed theoretical results are applied to the safety analysis of the exemplary electricity network. Since components of transmission and distribution networks require constant maintenance and degrading causes their insulation properties deterioration over time, multistate approach to the safety analysis of electricity systems seems to be reasonable. In critical and overload states the insulation uses much faster. The multistate safety analysis of the electricity network is performed regarding its assets and subnetworks interdependencies. The voltage instability in some subnetworks or load assets can cause voltage collapse of the whole system. Further, such approach to analysis of interconnections and interdependencies can help to capture the critical points and critical operations that can affect the whole network functioning.

Acknowledgements

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/

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