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Integrated model of critical infrastructure accident consequences

Keywords

critical infrastructure, accident, initiating events, environment threats, environment degradation, losses

Abstract

An integrated general model of critical infrastructure accident consequences including the process of initiating events, the process of environment threats and the process of environment degradation models is presented. The model is proposed to the evaluation of losses associated with the environment degradation caused by the critical infrastructure accident.

1. Introduction

The critical infrastructure accident is understand as an event that causes changing the critical infrastructure safety state into the safety state worse than the critical safety state that is dangerous for the critical infrastructure itself and its operating environment as well [2], [16]-[17], [19]-[24], [27]-[34], [37]. Each critical infrastructure accident can generate the initiating event causing dangerous situations in the critical infrastructure operating surroundings. The process of those initiating events can result in this environment threats and lead to the environment dangerous degradations (*Figure 1*) [1], [3-4].

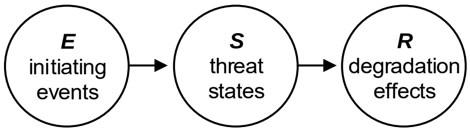

Figure 1. Interrelations of the critical infrastructure accident consequences general model.

Thus, the general model of a critical infrastructure accident consequences is constructed as a joint probabilistic model including the process of initiating events generated either by its accident or by its loss of safety critical level, the process of environment threats and the process of environment degradation.

To construct this general model of critical infrastructure accident consequences and to apply it practically, the basic notions concerned with those three particular processes it is composed of should be defined and the methods and procedures of estimating those processes unknown parameters should be developed. Under those all assumptions from the constructed model after its unknown parameters identification, the main characteristics of the process of environment degradation can be predicted. Finally, the proposed model can be applied to modelling, identification and prediction of the critical infrastructure accident consequences generated by real critical infrastructures.
The proposed approach and the methods developed will be applied in the Project Case Study 2, Scenario 2 [15] to modelling, identification and prediction of the critical infrastructure accident consequences generated by a ship operating in the Baltic Sea area, the member of Baltic Shipping Critical Infrastructure Network (BSCIN) defined in [10].

2. Process of initiating events modelling

We call a particular consequence of the critical infrastructure accident caused by the loss of its required safety critical level the initiating event that is an event initiating dangerous threats for the critical infrastructure operating environment. Next, we can define the process of all initiating events caused by the critical infrastructure accident placed in the critical infrastructure operating environment, interacting with that environment and changing in time its states.

To model the process of initiating events, we fix the time interval $t \in <0,+\infty)$, as the time of a critical infrastructure operation and we distinguish n_1, $n_1 \in N$, events initiating the dangerous situation for the critical infrastructure operating environment and mark them by $E_1, E_2, ..., E_{n_1}$. Further, we introduce the set of vectors

$$E = \{e: \quad e = [e_1, e_2, ..., e_{n_1}], e_i \in \{0,1\}\},$$

where

$$e_i = \begin{cases} 1, \text{if the initiating event } E_i \text{ occurs,} \\ 0, \text{if the initiating event } E_i \text{ does not occur,} \end{cases}$$

for $i = 1,2,\ldots,n_1$.

We may eliminate vectors that cannot occur and we number the remaining states of the set E from $l = 1$ up to ω, $\omega \in N$, where ω is the number of different elements of the set

$$E = \{e^1, e^2, ..., e^\omega\},$$

where

$$e^l = [e_1^l, e_2^l, ..., e_{n_1}^l], \; l = 1,2,\ldots,\omega,$$

and

$$e_i^l \in \{0,1\}, \; i = 1,2,\ldots,n_1.$$

Next, we can define the process of initiating events $E(t)$ on the time interval $t \in <0,+\infty)$, with its discrete states from the set

$$E = \{e^1, e^2, ..., e^\omega\}.$$

After that, we assume a semi-Markov model [9], [12], [23]-[24], [35]-[36] of the process of initiating events $E(t)$ that may be described by the following parameters:

– the number of states ω, $\omega \in N$,
– the initial probabilities $p^l(0) = P(E(0) = e^l)$, $l = 1,2,\ldots,\omega$, of the process of initiating events $E(t)$ staying at the states e^l at the moment $t = 0$,
– the probabilities of transitions p^{lj}, $l, j = 1,2,\ldots,\omega$, between the states e^l and e^j,
– the conditional distribution functions $H^{lj}(t) = P(\theta^{lj} < t)$, $t \in <0,+\infty)$, $l, j = 1,2,\ldots,\omega$, $l \neq j$, of the process of initiating events $E(t)$ conditional sojourn times θ^{lj} at the states e^l while its next transition will be done to the state e^j, $l, j = 1,2,\ldots,\omega$, $l \neq j$, and their mean values $M^{lj} = E[\theta^{lj}]$, $l, j = 1,2,\ldots,\omega$, $l \neq j$.

The statistical identification of the unknown parameters of the process of initiating events, i.e. estimating the probabilities of this process staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states can be performed according to the way presented in [6].

After identification of the process of initiating events, its main characteristics can by predicted [13].

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ^l, $l = 1,2,\ldots,\omega$, of the process of initiating events $E(t)$ at the states e^l, $l = 1,2,\ldots,\omega$, are determined by [27]

$$H^l(t) = \sum_{j=1}^{\omega} p^{lj} H^{lj}(t), \; l = 1,2,\ldots,\omega.$$

Hence, the mean values $E[\theta^{lj}]$ of the process of initiating events $E(t)$ unconditional sojourn times θ^l, $l = 1,2,\ldots,\omega$, at the states are given by

$$M^l = E[\theta^l] = \sum_{l=1}^{\omega} p^{lj} M^{lj}, \; l = 1,2,\ldots,\omega. \quad (1)$$

The limit values of the process of initiating events $E(t)$ transient probabilities at the particular states

$$p^l(t) = P(E(t) = e^l), \; t \in <0,+\infty), \; l = 1,2,\ldots,\omega, \quad (2)$$

are given by [27]

$$p^l = \lim_{t\to\infty} p^l(t) = \frac{\pi^l M^l}{\sum_{j=1}^{\omega} \pi^j M^j}, \; l = 1,2,\ldots,\omega, \quad (3)$$

where M^l are given by (1), while the steady probabilities π^l of the vector $[\pi^l]_{1x\omega}$ satisfy the system of equations

$$\begin{cases} [\pi^l] = [\pi^l][p^{lj}] \\ \sum_{j=1}^{\omega} \pi^j = 1, \end{cases}$$

$l = 1,2,\ldots,\omega.$

The asymptotic distribution of the sojourn total time $\hat{\theta}^l$ of the process of initiating events $E(t)$ in the time interval $\langle 0,\theta\rangle$, $\theta > 0$, at the state e^l is normal with the expected value

$$\hat{M}^l = E[\hat{\theta}^l] \cong p^l\theta,$$

where p^l are given by (3).

3. Process of environment threats modelling

To construct the general model of the environment threats caused by the process of the initiating events generated by critical infrastructure loss of required safety critical level, we distinguish the set of n_2, $n_2 \in N$, kinds of threats as the consequences of initiating events that may cause the sea environment degradation and denote them by $H_1, H_2, \ldots, H_{n_2}$ [4].

We also distinguish n_3, $n_3 \in N$, environment sub-regions $D_1, D_2, \ldots, D_{n_3}$ of the considered critical infrastructure operating environment region $D = D_1 \cup D_2 \cup \ldots \cup D_{n_3}$, that may be degraded by the environment threats H_i, $i = 1,2,\ldots,n_2$. The environment threats possibility of influence on the distinguished its operating environment sub-regions is presented in *Figure 2*.

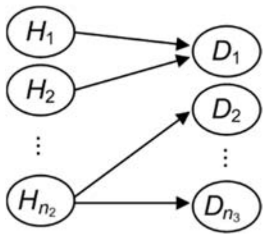

Figure 2. Illustration of environment threats possibility of influence on the critical infrastructure operating environment sub-regions.

We assume that the operating environment region D can be affected by some of threats H_i, $i = 1,2,\ldots,n_2$, and that a particular environment threat H_i, $i = 1,2,\ldots,n_2$, can be characterised by the parameter f^i, $i = 1,2,\ldots,n_2$. Moreover, we assume that the scale of the threat H_i, $i = 1,2,\ldots,n_2$, influence on region D depends on the range of its parameter value and for particular parameter f^i, $i = 1,2,\ldots,n_2$, we distinguish l_i ranges $f^{i1}, f^{i2}, \ldots, f^{il_i}$ of its values.
After that, we introduce the set of vectors

$$s_{(k)} = [f^1_{(k)}, f^1_{(k)}, \ldots, f^{n_2}_{(k)}], \; k = 1,2,\ldots,n_3, \tag{4}$$

where

$$f^i_{(k)} = \begin{cases} 0, & \text{if a threat } H_i \text{ does not appear at the sub-region } D_k, \\ f^{ij}_{(k)}, & \text{if a threat } H_i \text{ appears at the sub-region } D_k \text{ and its parameter is in the range } f^{ij}_{(k)},\ j = 1,2,\ldots,l_i, \end{cases} \tag{5}$$

for $i = 1,2,\ldots,n_2$, $k = 1,2,\ldots,n_3$,

is called the environment threat state of the sub-region D_k. From the above definition, the maximum number of the environment threat states for the sub-region D_k, $k = 1,2,\ldots,n_3$, is equalled to

$$\upsilon_k = (l^1_{(k)} + 1), (l^2_{(k)} + 1), \ldots, (l^{n_2}_{(k)} + 1), \; k = 1,2,\ldots,n_3.$$

Further, we number the sub-region environment threat states defined by (4) and (5) and mark them by

$$s^{\upsilon}_{(k)} \text{ for } \upsilon = 1,2,\ldots,\upsilon_k,\ k = 1,2,\ldots,n_3,$$

and form the set

$$S_{(k)} = \{s^{\upsilon}_{(k)},\ \upsilon = 1,2,\ldots,\upsilon_k\},\ k = 1,2,\ldots,n_3,$$

where

$$s^i_{(k)} \neq s^j_{(k)} \text{ for } i \neq j,\ i,j \in \{1,2,\ldots,\upsilon_k\}.$$

The set $S_{(k)}$, $k = 1,2,\ldots,n_3$, is called the set of the environment threat states of the sub-region D_k, $k = 1,2,\ldots,n_3$, while a number υ_k is called the number of the environment threat states of this sub-region.
A function

$$S_{(k)}(t),\ k = 1,2,\ldots,n_3,$$

defined on the time interval $t \in \langle 0,+\infty)$, and having values in the environment threat states set

$$S_{(k)},\ k = 1,2,\ldots,n_3,$$

is called the sub-process of the environment threats of the sub-region D_k, $k = 1,2,\ldots,n_3$.
Next, to involve the sub-process of environment threats of the sub-region with the process of initiating events, we introduced the function

$S_{(k/l)}(t)$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$,

defined on the time interval $t \in <0,+\infty)$, depending on the states of the process of initiating events $E(t)$ and taking its values in the set of the environment threat states set $S_{(k)}$, $k = 1,2,\dots,n_3$. This function is called the conditional sub-process of the environment threats in the sub-region D_k, $k = 1,2,\dots,n_3$, while the process of initiating events $E(t)$ is at the state e^l, $l = 1,2,\dots,\omega$.

We assume a semi-Markov model of the sub-process $S_{(k/l)}(t)$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, that may be described by the following parameters:

– the number of states υ_k, $\upsilon_k \in N$,
– the initial probabilities $p^i_{(k/l)}(0) = P(S_{(k/l)}(0) = s^i_{(k)})$, $i = 1,2,\dots,\upsilon_k$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, of the conditional sub-process of environment threats $S_{(k/l)}(t)$, staying at the states $s^i_{(k)}$ at the moment $t = 0$,
– the probabilities of transitions $p^{ij}_{(k/l)}$, $i,j = 1,2,\dots,\upsilon_k$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, of the conditional sub-process of environment threats $S_{(k/l)}(t)$ between the states $s^i_{(k)}$ and $s^j_{(k)}$,
– the conditional distribution functions $H^{ij}_{(k/l)}(t) = P(\eta^{ij}_{(k/l)} < t)$, $t \in <0,+\infty)$, $i,j = 1,2,\dots,\upsilon_k$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, of the conditional sub-process of environment threats $S_{(k/l)}(t)$, conditional sojourn times $\eta^{ij}_{(k/l)}$ at the states $s^i_{(k)}$, while its next transition will be done to the state $s^j_{(k)}$, $i, j = 1,2,\dots,\upsilon_k$, $i \neq j$, and their mean values $M^{ij}_{(k/l)} = E[\eta^{ij}_{(k/l)}]$, $i, j = 1,2,\dots,\upsilon_k$, $i \neq j$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$.

The statistical identification of the unknown parameters of the process of environment threats i.e. estimating the probabilities of this process of staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states can be performed according to the way presented in [7].

After identification of the process of environment treats, it can be predicted by finding its main characteristics like ones listed below and other [13].

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $\eta^i_{(k/l)}$, $i = 1,2,\dots,\upsilon_k$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, of the sub-process of environment threats $S_{(k/l)}(t)$ at the states $s^i_{(k/l)}$ $i = 1,2,\dots,\upsilon_k$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, are determined by [27]

$$H^i_{(k/l)}(t) = \sum_{j=1}^{\upsilon_k} p^{ij}_{(k/l)} H^{ij}_{(k/l)}(t),\ i = 1,2,\dots,\upsilon_k,$$

$$k = 1,2,\dots,n_3,\ l = 1,2,\dots,\omega,$$

Hence, the mean values $E[\eta^{ij}_{(k/l)}]$ of the sub-process of environment threats $S_{(k/l)}(t)$ unconditional sojourn times $\eta^i_{(k/l)}$, $i = 1,2,\dots,\upsilon_k$, $k = 1,2,\dots,n_3$, $l = 1,2,\dots,\omega$, at the states are given by

$$M^i_{(k/l)} = E[\eta^i_{(k/l)}] = \sum_{i=1}^{\upsilon_k} p^{ij}_{(k/l)} M^{ij}_{(k/l)},\ i = 1,2,\dots,\upsilon_k,$$

$$k = 1,2,\dots,n_3,\ l = 1,2,\dots,\omega. \tag{6}$$

The limit values of the sub-process of environment threats $S_{(k/l)}(t)$ transient probabilities at the particular states

$$p^i_{(k/l)}(t) = P(S_{(k/l)}(t) = s^i_{(k/l)}),\ t \in <0,+\infty),$$

$$i = 1,2,\dots,\upsilon_k,\ k = 1,2,\dots,n_3,\ l = 1,2,\dots,\omega, \tag{7}$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p^i_{(k/l)} = \lim_{t\to\infty} p^i_{(k/l)}(t) = \frac{\pi^i_{(k/l)} M^i_{(k/l)}}{\sum_{j=1}^{\upsilon_k} \pi^j_{(k/l)} M^j_{(k/l)}},$$

$$i = 1,2,\dots,\upsilon_k,\ k = 1,2,\dots,n_3,\ l = 1,2,\dots,\omega, \tag{8}$$

where $M^i_{(k/l)}$ are given by (6), while the steady probabilities $\pi^i_{(k/l)}$ of the vector $[\pi^i_{(k/l)}]_{1\mathrm{x}\upsilon_k}$ satisfy the system of equations

$$\begin{cases} [\pi^i_{(k/l)}] = [\pi^i_{(k/l)}][p^{ij}_{(k/l)}] \\ \sum_{j=1}^{\upsilon_k} \pi^j_{(k/l)} = 1, \end{cases}$$

$$i = 1,2,\dots,\upsilon_k,\ k = 1,2,\dots,n_3,\ l = 1,2,\dots,\omega.$$

The asymptotic distribution of the sojourn total time $\hat{\eta}^i_{(k/l)}$ of the sub-process of environment threats $S_{(k/l)}(t)$ in the time interval $\langle 0,\eta\rangle$, $\eta > 0$, at the state $s^i_{(k/l)}$ is normal with the expected value

$$\hat{M}^{i}_{(k/l)} = E[\hat{\eta}^{i}_{(k/l)}] \cong p^{i}_{(k/l)}\eta,$$

$$i = 1,2,\dots,\upsilon_k,\ k = 1,2,\dots,n_3,\ l = 1,2,\dots,\omega.$$

Thus, according to the formula for total probability and (2) and (7), the probabilities

$$p^{i}_{(k)}(t) = P(S(t) = s^{i}_{(k)}),\ \ t \in <0,+\infty),\ \ i = 1,2,\dots,\upsilon_k,$$

$$k = 1,2,\dots,n_3, \tag{9}$$

are defined by

$$p^{i}_{(k)}(t) = \sum_{l=1}^{\omega} P\left(E(t) = e^{l}\right)\cdot P\left(S_{(k)}(t) = s^{i}_{(k)} \middle| E(t) = e^{l}\right)$$

$$= \sum_{l=1}^{\omega} p^{l}(t)\cdot p^{i}_{(k/l)}(t),$$

$$i = 1,2,\dots,\upsilon_k,\ \ k = 1,2,\dots,n_3,$$

and according to (3) and (8) their limit forms are

$$p^{i}_{(k)} = \sum_{l=1}^{\omega} p^{l}\cdot p^{i}_{(k/l)},$$

$$i = 1,2,\dots,\upsilon_k,\ \ k = 1,2,\dots,n_3. \tag{10}$$

4. Process of environment degradation modelling

The particular states of the process of the environment threats $S_{(k)}(t)$ of the sub-region D_k, $k = 1,2,\dots,n_3$, may lead to dangerous effects degrading the environment at this sub-region. Thus, we assume that there are m_k different dangerous degradation effects for the environment sub-region D_k, $k = 1,2,\dots,n_3$, and we mark them by

$$R^{1}_{(k)}, R^{2}_{(k)},\dots, R^{m_k}_{(k)}.$$

This way the set

$$R_{(k)} = \{R^{1}_{(k)}, R^{2}_{(k)},\dots, R^{m_k}_{(k)}\},\ \ k = 1,2,\dots,n_3,$$

is the set of degradation effects for the environment of the sub-region D_k.
These degradation effects may attain different levels. Namely, the degradation effect

$$R^{m}_{(k)},\ \ m = 1,2,\dots,m_k,$$

may reach $\nu^{m}_{(k)}$ levels

$$R^{m1}_{(k)}, R^{m2}_{(k)},\dots, R^{m\nu^{m}_{(k)}}_{(k)},\ \ m = 1,2,\dots,m_k,$$

that are called the states of this degradation effect.
The set

$$R^{m}_{(k)} = \{R^{m1}_{(k)}, R^{m2}_{(k)},\dots, R^{m\nu^{m}_{(k)}}_{(k)}\},\ \ m = 1,2,\dots, m_k,$$

is called the set of states of the degradation effect $R^{m}_{(k)}$, $m = 1,2,\dots,m_k$, $k = 1,2,\dots,n_3$ for the environment of the sub-region D_k, $k = 1,2,\dots,n_3$.
Under the above assumptions, we can introduce the environment sub-region degradation process as a vector

$$R_{(k)}(t) = [R^{1}_{(k)}(t), R^{2}_{(k)}(t),\dots, R^{m_k}_{(k)}(t)],$$

$$t \in <0,+\infty),$$

where

$$R^{m}_{(k)}(t),\ \ t \in <0,+\infty),\ m = 1,2,\dots,m_k,\ k = 1,2,\dots,n_3,$$

are the processes of degradation effects for the environment of the sub-region D_k, defined on the time interval $t \in <0,+\infty)$, and having their values in the degradation effect state sets

$$m = 1,2,\dots,m_k,\ k = 1,2,\dots,n_3,$$

is called the degradation process of the environment of the sub-region D_k.
The vector

$$r^{m}_{(k)} = [d^{1}_{(k)}, d^{2}_{(k)},\dots, d^{m_k}_{(k)}],\ \ k = 1,2,\dots,n_3, \tag{11}$$

where

$$d^{m}_{(k)} = \begin{cases} 0, & \text{if a degradation effect } R^{m}_{(k)} \text{ does not appear at the sub-region } D_k, \\ R^{m\,j}_{(k)}, & \text{if a degradation effect } R^{m}_{(k)} \text{ appears at the sub-region } D_k \text{ and its level is equal to } R^{m\,j}_{(k)},\ j = 1,2,\dots,\nu^{m}_{(k)}, \end{cases} \tag{12}$$

for $m = 1,2,\dots,m_k$, $k = 1,2,\dots,n_3$,

is called the degredation state of the sub-region D_k.

From the above definition, the maximum number of the environment degradation states for the sub-region D_k, $k = 1,2,\ldots,n_3$, is equalled to

$$\ell_k = (\nu_{(k)}^1 + 1),(\nu_{(k)}^2 + 1),\ldots,(\nu_{(k)}^{m_k} + 1), \ k = 1,2,\ldots,n_3.$$

Further, we number the sub-region D_k, $k = 1,2,\ldots,n_3$, degradation states defined by (11) and (12) and mark them by

$$r_{(k)}^{\ell} \text{ for } \ell = 1,2,\ldots,\ell_k, \ k = 1,2,\ldots,n_3,$$

and form the set of degradation states

$$R_{(k)} = \{r_{(k)}^{\ell}, \ \ell = 1,2,\ldots,\ell_k\}, \ k = 1,2,\ldots,n_3,$$

where

$$r_{(k)}^i \neq r_{(k)}^j \text{ for } i \neq j, \ i,j \in \{1,2,\ldots,\ell_k\}.$$

The set $R_{(k)}$, $k = 1,2,\ldots,n_3$, is called the set of the environment degradation states of the sub-region D_k, $k = 1,2,\ldots,n_3$, while a number ℓ_k is called the number of the environment degradation states of this sub-region.
A function

$$R_{(k)}(t), \ k = 1,2,\ldots,n_3,$$

defined on the time interval $t \in <0,+\infty)$, and having values in the environment degradation states set

$$R_{(k)}, \ k = 1,2,\ldots,n_3,$$

is called the sub-process of the environment degradation of the sub-region D_k, $k = 1,2,\ldots,n_3$.
Next, to involve the environment sub-region D_k, $k = 1,2,\ldots,n_3$, degradation process with the process of the environment threats, we define the conditional environment sub-region degradation process, while the process of the environment threats $S_{(k)}(t)$ of the sub-region D_k, is at the state $s_{(k)}^{\upsilon}$, $\upsilon = 1,2,\ldots,\upsilon_k$, as a vector

$$R_{(k/\upsilon)}(t) = [R_{(k/\upsilon)}^1(t), R_{(k/\upsilon)}^2(t),\ldots, R_{(k/\upsilon)}^{m_k}(t)], \quad (13)$$

$$t \in <0,+\infty),$$

where

$$R_{(k/\upsilon)}^m(t), \ t \in <0,+\infty), \ m = 1,2,\ldots,m_k, \ k = 1,2,\ldots,n_3,$$

$$\upsilon = 1,2,\ldots,\upsilon_k,$$

defined on the time interval $t \in <0,+\infty)$, and having values in the degradation effect states set $R_{(k)}^m$, $m = 1,2,\ldots,m_k$, $k = 1,2,\ldots,n_3$.
The above definition means that the conditional environment sub-region degradation process $R_{(k/\upsilon)}(t)$, $t \in <0,+\infty)$, also takes the degradation states from the set $R_{(k)}$ of the unconditional sub-region degradation process $R_{(k)}(t)$, $t \in <0,+\infty)$, defined by (13).
We assume a semi-Markov model of the sub-process $R_{(k/\upsilon)}(t)$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, that may be described by the following parameters:

– the number of states ℓ_k, $\ell_k \in N$,
– the initial probabilities $q_{(k/\upsilon)}^i(0) = P(R_{(k/\upsilon)}(0) = r_{(k)}^i)$, $i = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, of the conditional sub-process of environment degradation $R_{(k/\upsilon)}(t)$, staying at the states $r_{(k)}^i$ at the moment $t = 0$,
– the probabilities of transitions $q_{(k/\upsilon)}^{ij}$, $i,j = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$, of the conditional sub-process of environment degradation $R_{(k/\upsilon)}(t)$ between the states $r_{(k)}^i$ and $r_{(k)}^j$,
– the conditional distribution functions $G_{(k/\upsilon)}^{ij}(t) = P(\zeta_{(k/\upsilon)}^{ij} < t)$, $t \in <0,+\infty)$, $i,j = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$. of the conditional sub-process of environment degradation $R_{(k/\upsilon)}(t)$, conditional sojourn times $\zeta_{(k/\upsilon)}^{ij}$ at the states $r_{(k)}^i$ while its next transition will be done to the state $r_{(k)}^j$, $i,j = 1,2,\ldots,\ell_k$, $i \neq j$, and their mean values $M_{(k/\upsilon)}^{ij} = E[\zeta_{(k/\upsilon)}^{ij}]$, $i,j = 1,2,\ldots,\ell_k$, $i \neq j$, $k = 1,2,\ldots,n_3$, $\upsilon = 1,2,\ldots,\upsilon_k$.

The statistical identification of the unknown parameters of the process of environment degradation i.e. estimating the probabilities of this process of staying at the states at the initial moment, the probabilities of this processes transitions between its states and the parameters and forms of the distributions fixed for the description of this process conditional sojourn times at their states can be performed according to the way presented in [8].
After identification of the process of environment degradation, it can be predicted by finding its main characteristics like ones listed below and other [13].
From the formula for total probability, it follows that the unconditional distribution functions of the

sojourn times $\zeta^i_{(k/\upsilon)}$, $i = 1,2,...,\ell_k$, k = 1,2,…,n_3, υ = 1,2,…,υ_k, of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ at the states $r^i_{(k)}$, $i = 1,2,...,\ell_k$, k = 1,2,…,n_3, υ = 1,2,…,υ_k, are determined by [27]

$$H^i_{(k/\upsilon)}(t) = \sum_{j=1}^{\ell_k} q^{ij}_{(k/\upsilon)} H^{ij}_{(k/\upsilon)}(t),\ i = 1,2,...,\ell_k,$$

$$k = 1,2,\ldots,n_3,\ \upsilon = 1,2,\ldots,\upsilon_k,$$

Hence, the mean values $E[\zeta^{ij}_{(k/\upsilon)}]$ of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ unconditional sojourn times $\zeta^i_{(k/\upsilon)}$, $i = 1,2,...,\ell_k$, k = 1,2,…,n_3, υ = 1,2,…,υ_k, at the states are given by

$$M^i_{(k/\upsilon)} = E[\zeta^i_{(k/\upsilon)}] = \sum_{i=1}^{\ell_k} q^{ij}_{(k/\upsilon)} M^{ij}_{(k/\upsilon)},\ i = 1,2,...,\ell_k,$$

$$k = 1,2,\ldots,n_3,\ \upsilon = 1,2,\ldots,\upsilon_k. \quad (14)$$

The limit values of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ transient probabilities at the particular states

$$q^i_{(k/\upsilon)}(t) = P(R_{(k/\upsilon)}(t) = r^i_{(k/\upsilon)}),\ t \in <0,+\infty),$$

$$i = 1,2,...,\ell_k,\ k = 1,2,\ldots,n_3,\ \upsilon = 1,2,\ldots,\upsilon_k, \quad (15)$$

are given by [27]

$$q^i_{(k/\upsilon)} = \lim_{t\to\infty} q^i_{(k/\upsilon)}(t) = \frac{\pi^i_{(k/\upsilon)} M^i_{(k/\upsilon)}}{\sum_{j=1}^{\ell_k} \pi^j_{(k/\upsilon)} M^j_{(k/\upsilon)}},$$

$$i = 1,2,...,\ell_k,\ k = 1,2,\ldots,n_3,\ \upsilon = 1,2,\ldots,\upsilon_k, \quad (16)$$

where $M^i_{(k/\upsilon)}$ are given by (14), while the steady probabilities $\pi^i_{(k/\upsilon)}$ of the vector $[\pi^i_{(k/\upsilon)}]_{1\mathrm{x}\ell_k}$ satisfy the system of equations

$$\begin{cases} [\pi^i_{(k/\upsilon)}] = [\pi^i_{(k/\upsilon)}][q^{ij}_{(k/\upsilon)}] \\ \sum_{j=1}^{\ell_k} \pi^j_{(k/\upsilon)} = 1, \end{cases}$$

$$i = 1,2,...,\ell_k,\ k = 1,2,\ldots,n_3,\ \upsilon = 1,2,\ldots,\upsilon_k.$$

The asymptotic distribution of the sojourn total time $\hat{\zeta}^i_{(k/\upsilon)}$ of the sub-process of environment degradation $R_{(k/\upsilon)}(t)$ in the time interval $\langle 0,\zeta\rangle$, $\zeta > 0$, at the state $r^i_{(k/l)}$ is normal with the expected value

$$\hat{M}^i_{(k/\upsilon)} = E[\hat{\zeta}^i_{(k/\upsilon)}] \cong q^i_{(k/\upsilon)}\zeta,$$

where $q^i_{(k/\upsilon)}$ are given by (16).

Thus, according to the formula for total probability and (9) and (15), the probabilities

$$q^i_{(k)}(t) = P(R(t) = r^i_{(k)}),\ t \in <0,+\infty),\ i = 1,2,...,\ell_k,$$

$$k = 1,2,\ldots,n_3,$$

are defined by

$$q^i_{(k)}(t) = \sum_{\upsilon=1}^{\upsilon_k} P\left(S(t) = s^\upsilon_{(k)}\right) \cdot P\left(R_{(k)}(t) = r^i_{(k)} \middle| S(t) = s^\upsilon_{(k)}\right)$$

$$= \sum_{\upsilon=1}^{\upsilon_k} p^\upsilon_{(k)}(t) \cdot q^i_{(k/\upsilon)}(t),$$

$$i = 1,2,...,\ell_k,\ k = 1,2,\ldots,n_3.$$

Hence, according to (10) and (16), for sufficiently large t, the boundary probabilities of the process of the environment degradation $R_{(k/\upsilon)}(t)$ at its particular states are given by

$$q^i_{(k)} \cong \sum_{\upsilon=1}^{\upsilon_k} p^\upsilon_{(k)} \cdot q^i_{(k/\upsilon)} = \sum_{\upsilon=1}^{\upsilon_k} [\sum_{l=1}^{\omega} p^l \cdot p^\upsilon_{(k/l)}] q^i_{(k/\upsilon)}$$

$$i = 1,2,...,\ell_k,\ k = 1,2,\ldots,n_3, \quad (17)$$

where p^l, $p^\upsilon_{(k/l)}$ and $q^i_{(k/\upsilon)}$ are defined respectively by (3), (8) and (16).

Hence, the sojourn total time $\zeta^i_{(k)}$ of the process of the environment degradation $R_{(k)}(t)$, k = 1,2,…,n_3, in the time interval $\langle 0,\theta\rangle$, $\theta > 0$, at the state $r^i_{(k)}$ has normal distribution with the expected value

$$E[\zeta^i_{(k)}] \cong q^i_{(k)}\theta,\ i = 1,2,...,\ell_k,$$

where $q^i_{(k)}$ are given by (17).

5. Critical infrastructure accident area losses

We denote by

$$C^i_{(k)}(t),\ i = 1,2,...,\ell_k,\ k = 1,2,\ldots,n_3, \quad (18)$$

the losses associated with the process of the environment degradation as a result of critical infrastructure accident

$$R_{(k)}(t),\ t \in \langle 0,+\infty \rangle,\ k = 1,2,\ldots,n_3,$$

in the sub-region D_k, $k = 1,2,\ldots,n_3$, at the environment degradation state $r^i_{(k)}$, $i = 1,2,\ldots,\ell_k$, $k = 1,2,\ldots,n_3$, in the time interval $\langle 0,t \rangle$.

Thus, the approximate expected value of the losses in the time interval $\langle 0,\theta \rangle$, associated with the process of the environment degradation $R_{(k)}(t)$, of the sub-region D_k can be defined by

$$C_{(k)}(\theta) \cong \sum_{i=1}^{\ell_k} q^i_{(k)} \cdot C^i_{(k)}(\theta) \text{ for } k = 1,2,\ldots,n_3, \qquad (19)$$

where $q^i_{(k)}$, $i = 1,2,\ldots,\ell_k$, are given by (17) and $C^i_{(k)}(\theta)$, $k = 1,2,\ldots,n_3$, are defined by (18).

The total expected value of the losses in the time interval $\langle 0,\theta \rangle$, associated with the process of the environment degradation $R(t)$, in all sub-regions of the considered critical infrastructure operating environment region D, can be evalueted by

$$C(\theta) \cong \sum_{k=1}^{n_3} C_{(k)}(\theta),$$

where $C_{(k)}(\theta)$ are given by (19).

6. Conclusions

Modelling critical infrastructure accident consequences through designing the General Model of Critical Infrastructure Accident Consequences (GMCIAC) was performed in [5], [11]. The identification methods of its unknown parameters were proposed in [12]. Moreover, the GMCIAC adaptation to the prediction of critical infrastructure accident consequences were done in [13] and its practical applications is performed in [14] to the chemical spill consequences generated by the accident of one of the ships of the shipping critical infrastructure network operating at the Baltic Sea waters as the preparatory approach to the Case Study 2: Sea Surge and Extreme Winds at Baltic Sea Area, Scenario 2, Chemical Spill due to Extreme Surges [15]. Having presented here the integrated models of critical infrastructure consequences and losses it is possible to optimize the losses by their minimization and to investigate the climate-weather influence on these lossess what is expected to be done in further steps of EU-CIRCLE project activity.

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