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Modelling climate-weather change process including extreme weather hazards for critical infrastructure operating area

Keywords

climate-weather states, climate-weather change process, modelling, extreme weather hazards

Abstract

The climate-weather change process for the critical infrastructure operating area is considered and its states are introduced. The semi-Markov process is used to construct a general probabilistic model of the climate-weather change process for the critical infrastructure operating area. To build this model the vector of probabilities of the climate-weather change process staying at the initials climate-weather states, the matrix of probabilities of the climate-weather change process transitions between the climate-weather states, the matrix of conditional distribution functions and the matrix of conditional density functions of the climate-weather change process conditional sojourn times at the climate-weather states are defined. To describe the climate-weather change process conditional sojourn times at the particular climate-weather states the uniform distribution, the triangular distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull distribution, the chimney distribution and the Gamma distribution are suggested and introduced.

1. Introduction

The climate-weather change processes for the real critical infrastructures operating areas are very complex and it is difficult to analyse these infrastructures' safety additionally with respect to changing in time their operation conditions that often are essential in this analysis. The complexity of the climate-weather change processes and their influence on changing in time their operation processes and their components' safety parameters is often difficult to fix. Usually, the climate-weather change processes have either an explicit or an implicit strong influence on the critical infrastructures safety. As a rule, some of the extreme weather events define a set of different climate-weather states of the critical infrastructures in which the systems change their operation processes and their components safety parameters. A convenient tool for analysing this problem is semi-Markov modelling [7]-[10], [13]-[15], [19] of the climate-weather change processes proposed in this paper.

2. States of climate-weather change process

To define the climate-weather states in the fixed area, we distinguish a, $a \in N$, parameters that describe the climate-weather states in this area and mark the values they can take by $w_1, w_2, \ldots, w_a$. Further, we assume that the possible values of the i-th parameter w_i, $i = 1,2,\ldots, a$, can belong to the interval $< b_i, d_i)$, $i = 1,2,\ldots, a$. We divide each of the intervals $< b_i, d_i)$, $i = 1,2,\ldots, a$, into n_i, $n_i \in N$, disjoint subintervals

$$< b_{i1}, d_{i1}), < b_{i2}, d_{i2}), \ldots, < b_{in_j}, d_{in_j}),$$

such that

$$< b_{i1}, d_{i1}) \cup < b_{i2}, d_{i2}) \cup \ldots \cup < b_{in_i}, d_{in_i})$$

$$= < b_i, d_i), \; d_{ij_i} = b_{ij_i+1},$$

$$j_i = 1,2,\ldots, n_i - 1, \; i = 1,2,\ldots, a.$$

Thus, the vector (w_1, w_2,..., w_a) describing the climate-weather states can take values from the set of the a dimensional space points of the Descartes product

$$< b_1, d_1) \times < b_2, d_2) \times ... \times < b_a, d_a)$$

that is composed of the a dimensional space domains of the form

$$< b_{1j_1}, d_{1j_1}) \times < b_{2j_2}, d_{2j_2}) \times ... \times < b_{aj_a}, d_{aj_a}),$$

where $j_i = 1,2,\ldots n_i$, $i = 1,2,\ldots, a$.

The domains of the above form are called the climate-weather states of the climate-weather change process and numerated from 1 up to the value $w = n_1 \cdot n_2 \cdot ... \cdot n_a$ and mark by $c_1, c_2,\ldots, c_w$.

The interpretation of the states of the climate-weather change process in the case $a = 2$ is given in *Figure 1*. In this case, we have $w = n_1 \cdot n_2$ climate-weather states of the climate-weather change process represented in *Figure 1* by the squares marked by c_1, $c_2,\ldots, c_w$.

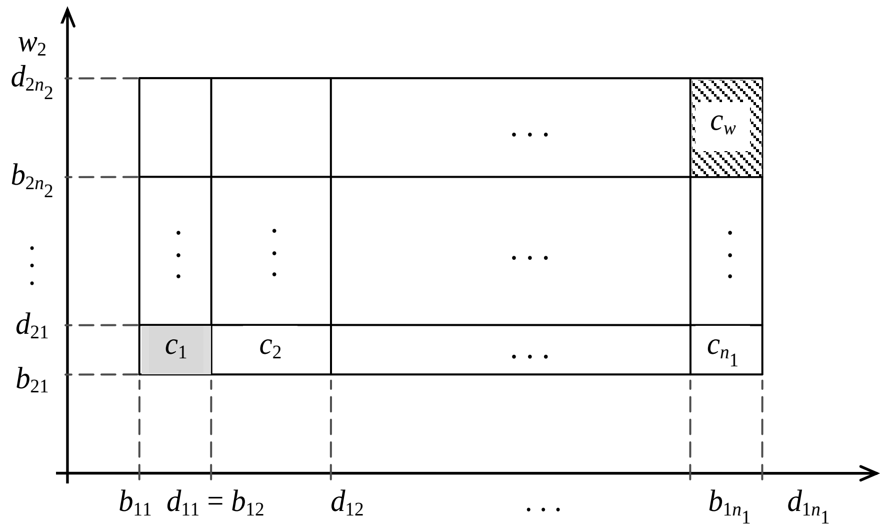

Figure 1. Interpretation of the two dimensional climate-weather states of the climate-weather change process

3. Semi-Markov model of climate-weather change process

To model the climate-weather change process for the critical infrastructure operating area we assume that the climate-weather in this area is taking w, $w \in N$, different climate-weather states $c_1, c_2,\ldots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in < 0, +\infty)$, with discrete operation states from the set $\{c_1, c_2,\ldots, c_w\}$. Assuming that the climate-weather change process $C(t)$ is a semi-Markov process it can be described by:

– the number of climate-weather states w, $w \in N$,

– the vector

$$[q_b(0)]_{1\mathrm{x}w} = [q_1(0), q_2(0),\ldots, q_w(0)] \qquad (3.1)$$

of the initial probabilities

$$q_b(0) = P(C(0) = c_b),\ b = 1,2,\ldots, w,$$

of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b at the moment $t = 0$;

– the matrix

$$[q_{bl}]_{w\mathrm{x}w} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1w} \\ q_{21} & q_{22} & \cdots & q_{2w} \\ \cdots \\ q_{w1} & q_{w2} & \cdots & q_{ww} \end{bmatrix}$$

of the probabilities of transitions q_{bl}, $b, l = 1,2,\ldots, w$, $b \neq l$, of the climate-weather change process $C(t)$ from the climate-weather states c_b to c_l, where by formal agreement

$$q_{bb} = 0 \text{ for } b = 1,2,\ldots, w;$$

– the matrix

$$[C_{bl}(t)]_{wxw} = \begin{bmatrix} C_{11}(t)\, C_{12}(t) \dots C_{1w}(t) \\ C_{21}(t)\, C_{22}(t) \dots C_{2w}(t) \\ \dots \\ C_{w1}(t)\, C_{w2}(t) \dots C_{ww}(t) \end{bmatrix}$$

of the conditional distribution functions

$C_{bl}(t) = P(C_{bl} < t)$, $b, l = 1,2,\dots, w$,

of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, $b, l = 1,2,\dots, w$, $b \neq l$, where by formal agreement

$C_{bb}(t) = 0$ for $b = 1,2,\dots, w$.

Further, we introduce the matrix

$$[c_{bl}(t)]_{wxw} = \begin{bmatrix} c_{11}(t)\, c_{12}(t) \dots c_{1w}(t) \\ c_{21}(t)\, c_{22}(t) \dots c_{2w}(t) \\ \dots \\ c_{w1}(t)\, c_{w2}(t) \dots c_{ww}(t) \end{bmatrix}$$

of the conditional density functions of the climate-weather change process $C(t)$ conditional sojourn times C_{bl} at the climate-weather states corresponding to the conditional distribution functions $C_{bl}(t)$, where

$$c_{bl}(t) = \frac{d}{dt}[c_{bl}(t)] \text{ for } b, l = 1,2,\dots, w,\ b \neq l,$$

and by formal agreement

$c_{bb}(t) = 0$ for $b = 1,2,\dots, w$.

4. Conditional sojourn times at climate-weather states

We assume that the suitable and typical distributions suitable to describe the climate-weather change process $C(t)$ conditional sojourn times C_{bl}, $b, l = 1,2,\dots, w$, $b \neq l$, at the particular climate-weather states are [15]:

– the uniform distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{1}{y_{bl} - x_{bl}}, & x_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where $0 \le x_{bl} < y_{bl} < +\infty$;

– the triangular distribution with a density function

(3.3)

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ \dfrac{2}{y_{bl} - x_{bl}} \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where $0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty$;

– the double trapezium distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ \quad - q_{bl}] \dfrac{t - x_{bl}}{z_{bl} - x_{bl}}, & x_{bl} \le t \le z_{bl} \\ w_{bl} + [\dfrac{2 - q_{bl}(z_{bl} - x_{bl}) - w_{bl}(y_{bl} - z_{bl})}{y_{bl} - x_{bl}} \\ \quad - w_{bl}] \dfrac{y_{bl} - t}{y_{bl} - z_{bl}}, & z_{bl} \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where $0 \le x_{bl} \le z_{bl} \le y_{bl} < +\infty$, $0 \le q_{bl} < +\infty$,
$0 \le w_{bl} < +\infty$, $0 \le q_{bl}(z_{bl} - x_{bl}) + w_{bl}(y_{bl} - z_{bl}) \le 2$;

– the quasi-trapezium distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ q_{bl} + \dfrac{A_{bl} - q_{bl}}{z_{bl}^1 - x_{bl}}(t - x_{bl}), & x_{bl} \le t \le z_{bl}^1 \\ A_{bl}, & z_{bl}^1 \le t \le z_{bl}^2 \\ w_{bl} + \dfrac{A_{bl} - w_{bl}}{y_{bl} - z_{bl}^2}(y_{bl} - t), & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$$A_{bl} = \frac{2 - q_{bl}(z_{bl}^1 - x_{bl}) - w_{bl}(y_{bl} - z_{bl}^2)}{z_{bl}^2 - z_{bl}^1 + y_{bl} - x_{bl}},$$

$0 \le x_{bl} \le z_{bl}^1 \le z_{bl}^2 \le y_{bl} < +\infty$, $0 \le q_{bl} < +\infty$,

$0 \le w_{bl} < +\infty$;

– the exponential distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \alpha_{bl} \exp[-\alpha_{bl}(t - x_{bl})], & t \ge x_{bl}, \end{cases}$$

where

$0 \le \alpha_{bl} < +\infty$;

– the Weibull distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \alpha_{bl}\beta_{bl}(t - x_{bl})^{\beta_{bl}-1} \exp[-\alpha_{bl}(t - x_{bl})^{\beta_{bl}}], & t \ge x_{bl}, \end{cases}$$

where

$0 \le \alpha_{bl} < +\infty, \; 0 \le \beta_{bl} < +\infty$;

– the chimney distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{A_{bl}}{z_{bl}^1 - x_{bl}}, & x_{bl} \le t \le z_{bl}^1 \\ \dfrac{K_{bl}}{z_{bl}^2 - z_{bl}^1}, & z_{bl}^1 \le t \le z_{bl}^2 \\ \dfrac{D_{bl}}{y_{bl} - z_{bl}^2}, & z_{bl}^2 \le t \le y_{bl} \\ 0, & t > y_{bl}, \end{cases}$$

where

$$0 \le x_{bl} \le z_{bl}^1 \le z_{bl}^2 \le y_{bl} < +\infty, \; A_{bl} \ge 0, \; K_{bl} \ge 0, \; D_{bl} \ge 0, \; A_{bl} + K_{bl} + D_{bl} = 1.$$

– the Gamma distribution with a density function

$$c_{bl}(t) = \begin{cases} 0, & t < x_{bl} \\ \dfrac{(t - x_{bl})^{\alpha_{bl}-1} \exp[-(t - x_{bl})/\beta_{bl}]}{\beta_{bl}^{\alpha_{bl}} \cdot \Gamma(\alpha_{bl})}, & t \ge x_{bl}, \end{cases}$$

where $0 < \alpha_{bl} < +\infty, \; 0 < \beta_{bl} < +\infty$.

5. Extreme weather hazard states of climate-weather change process

To define the climate-weather states in the fixed area, we distinguished a, $a \in N$, parameters that describe them were distinguished. The values the parameters can take were marked by $w_1, w_2, ..., w_a$. Further, it was assumed that the possible values of the i-th parameter w_i, $i = 1,2,\ldots, a$, can belong to the interval $< b_i, d_i)$, $i = 1,2,\ldots, a$. Each of the intervals $< b_i, d_i)$, $i = 1,2,\ldots, a$, were divided n_i, $n_i \in N$, disjoint subintervals

$$< b_{i1}, d_{i1}), < b_{i2}, d_{i2}), \ldots, < b_{in_j}, d_{in_j}), \quad i = 1,2,\ldots, a.$$

These intervals can be called the weather parameter w_i, $i = 1,2,\ldots, a$, states. Some of those states of the weather parameters can change the critical infrastructure operation process and they also can have dengerous influence on the critical infrastructure safety.

Thus, each of the states of the weather parameter w_i, $i = 1,2,\ldots, a$, that have most negative influence no the critical infrastructure operation and safety can be called the 1st category extreme weather hazard state of the wheather parameter w_i, $i = 1,2,\ldots, a$.

Further, according to Section 2, the climate-weather change process states are defined by the vectors

$$(w_1, w_2, ..., w_a)$$

and marked by

$$c_1, c_2, \ldots, c_w, \; w = n_1 \cdot n_2 \cdot \ldots \cdot n_a,$$

then we can call each of the climate-weather change process state c_j, $j = 1,2,\ldots, w$, of the vector form $(w_1, w_2, ..., w_a)$:

- the a^{th} category extreme weather hazard state of the climate-weather change process if all a wheather parameters w_i, $i = 1,2,\ldots, a$, are at the 1st category extreme weather hazard state;
- the $(a\text{-}1)^{\text{th}}$ category extreme weather hazard state of the climate-weather change process if a-1 of wheather parameters w_i, $i = 1,2,\ldots, a$, are at the 1st category extreme weather hazard state;
- the $(a\text{-}2)^{\text{th}}$ category extreme weather hazard state of the climate-weather change process if a-2 of wheather parameters w_i, $i = 1,2,\ldots, a$, are at the 1st category extreme weather hazard state; (4.8)
- …
- the 1st category extreme weather hazard state of the climate-weather change process if 1 of wheather parameters w_i, $i = 1,2,\ldots, a$, are at the 1st category extreme weather hazard state;

– the 0^{os} category extreme weather hazard state of the climate-weather change process if none of wheather parameters w_i, $i = 1,2,\ldots, a$, are at the 1^{st} category extreme weather hazard state.

Thus, the a^{th} category extreme weather hazard state of the climate-weather change process is the most denderous for the critical infrastructure operation and safety.

6. Conclusions

The probabilistic model of the climate-weather change process for critical infrastructure operating area which is proposed in this paper is the basis for the considerations in the next tasks of the EU-CIRCLE project. This model, together with the critical infrastructure operation process model [3] will be used to construct a general joint climate-weather and operation critical infrastructure model and finally to construct the integrated general safety probabilistic model of critical infrastructure related to its operation process and its operating area climate-change process [6]. The methods of estimation of this model unknown parameters will be given in another project report [4].

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