Prediction of climate-weather change process for port oil piping transportation system and maritime ferry operating at Baltic Sea area

Keywords
climate-weather change process, semi-Markov model, modelling, identification, transportation system

Abstract
There are presented the methods of prediction of the climate-weather change process. These are the methods and procedures for estimating the unknown basic parameters of the climate-weather change process semi-Markov model and identifying the distributions of the climate-weather change process conditional sojourn times at the climate-weather states. There are given the formulae estimating the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather change transitions between the climate-weather states and the parameters of the distributions suitable and typical for the description of the climate-weather change process conditional sojourn times at the particular climate-weather states. The proposed statistical methods applications for the prediction of the climate-weather change process model determining the climate-weather change process parameters for for port oil piping transportation system and maritime ferry operating at Baltic Sea area are presented.

1. Introduction
The general model of the climate-weather change processes is proposed in [3] and [12]. The safety models of various multistate complex technical systems are considered in [5]. Consequently, the general joint models linking these system safety models with the model of their climate-weather processes, allowing us for the safety analysis of the complex technical systems at the variable climate-weather conditions, are constructed in [6]. To be able to apply these general models practically in the evaluation and prediction of the reliability and safety of real complex technical systems it is necessary to have the statistical methods concerned with determining the unknown parameters of the proposed models [7]-[8], [10]-[11], [21]. Particularly, concerning the climate-weather process, the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather process transitions between the climate-weather states and the distributions of the conditional sojourn times of the climate-weather process at the particular climate-weather states should be identified [9], [14]-[15]. It is also necessary to use the methods of testing the hypotheses concerned with the climate-weather process conditional sojourn times at the climate-weather states [15].

2. Prediction of climate-weather change process
Assuming that we have identified the unknown parameters of the climate-weather change process semi-Markov model:
- the initial probabilities $q_{bl}(0)$, $b = 1, 2, ... , 6$, of the climate-weather change process staying at the particular state $c_b$ at the moment $t = 0$;
- the probabilities $q_{bl}$, $b, l = 1, 2, ... , 6$, $b \neq l$, of the climate-weather change process transitions from the climate-weather state $c_b$ into the climate-weather state $c_l$;
- the distributions of the climate-weather change process conditional sojourn times $C_{bl}$, $b, l = 1, 2, ... , 6$, $b \neq l$, at the particular climate-weather states and their mean values $M_{bl} = E[C_{bl}]$, $b, l = 1, 2, ... , 6$,
we can predict this process basic characteristics.

The mean values of the conditional sojourn times $C_{bl}$ are given by [3]
\( N_{b,l} = E[C_{b,l}] = \int_0^\infty dC_{b,l}(t) = \int_0^{\infty} tc_{b,l}(t) \, dt, \quad b, l = 1,2,\ldots,w, \ b \neq l, \) 

then for the distinguished in [3] distributions, the mean values of the climate-weather change process \( C(t) \) conditional sojourn times \( C_{b,l} \), \( b, l = 1,2,\ldots,w, \ b \neq l, \) at the particular operation states are respectively given by [3]:

- for the uniform distribution

\[
N_{b,l} = E[C_{b,l}] = \frac{x_{b,l} + y_{b,l}}{2};
\]  

(2)

- for the triangular distribution

\[
N_{b,l} = E[C_{b,l}] = \frac{x_{b,l} + y_{b,l} + z_{b,l}}{3};
\]  

(3)

- for the double trapezium distribution

\[
N_{b,l} = E[C_{b,l}] = \frac{x_{b,l} + y_{b,l} + z_{b,l} + w_{b,l}(y_{b,l}) - q_{b,l}(x_{b,l})}{2} \quad + \frac{w_{b,l} + q_{b,l}}{6} [(x_{b,l}z_{b,l} - y_{b,l}z_{b,l}) + x_{b,l}y_{b,l}(x_{b,l} + y_{b,l})] \quad - \frac{(x_{b,l})^3}{3(y_{b,l} - x_{b,l})};
\]  

(4)

- for the quasi-trapezium distribution

\[
N_{b,l} = E[C_{b,l}] = \frac{q_{b,l}}{2}[(z_{b,l})^2 - (x_{b,l})^2] \quad - \frac{A_{b,l} - q_{b,l}}{6} (2(z_{b,l})^2 - 5x_{b,l}z_{b,l} - (x_{b,l})^2) \quad + \frac{A_{b,l}}{2} [(z_{b,l})^3 - (z_{b,l})^2] \quad + \frac{w_{b,l}}{2} [(y_{b,l})^3 - (z_{b,l})^2] \quad - \frac{w_{b,l} - A_{b,l}}{6} (2(z_{b,l})^2 - 5y_{b,l}z_{b,l} - (y_{b,l})^2); \]

\[
N_{b,l} = E[C_{b,l}] = \frac{q_{b,l}}{2}[(z_{b,l})^2 - (x_{b,l})^2] - \frac{A_{b,l} - q_{b,l}}{6} (2(z_{b,l})^2 - 5x_{b,l}z_{b,l} - (x_{b,l})^2) \quad + \frac{A_{b,l}}{2} [(z_{b,l})^3 - (z_{b,l})^2] \quad + \frac{w_{b,l}}{2} [(y_{b,l})^3 - (z_{b,l})^2] \quad - \frac{w_{b,l} - A_{b,l}}{6} (2(z_{b,l})^2 - 5y_{b,l}z_{b,l} - (y_{b,l})^2); \]

(5)

- for the exponential distribution

\[
N_{b,l} = E[C_{b,l}] = x_{b,l} + \frac{1}{\alpha_{b,l}}; \quad \alpha_{b,l} = \int_0^{\infty} \frac{1}{t} \, dt, \quad \alpha > 0;
\]  

(6)

- for the Weibull distribution

\[
N_{b,l} = E[C_{b,l}] = x_{b,l} + \frac{1}{\beta_{b,l}} \Gamma(1 + \frac{1}{\beta_{b,l}}), \quad \beta_{b,l} = \int_0^{\infty} t^{\frac{1}{\beta_{b,l}}} \, dt, \quad \beta > 0;
\]

(7)

where

\[
\Gamma(u) = \int_0^\infty t^{u-1} \, dt, \quad u > 0,
\]

(8)

is the gamma function;

- for the chimney distribution

\[
N_{b,l} = E[C_{b,l}] = \frac{1}{2} [A_{b,l} (x_{b,l} + z_{b,l}) + C_{b,l} (z_{b,l} + z_{b,l}) + D_{b,l} (z_{b,l}^2 + y_{b,l})];
\]

(9)

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times \( C_{b,l} \), \( b, l = 1,2,\ldots,w, \) of the climate-weather change process \( C(t) \) at the climate-weather states \( c_b, \ b = 1,2,\ldots,w, \) are given by [3], [4], [6], [12]

\[
C_b(t) = \sum_{i=1}^{w} q_{b,i} C_{b,i}(t), \ b = 1,2,\ldots,w.
\]

(9)

Hence, the mean values \( E[C_{b,l}] \) of the climate-weather change process \( C(t) \) unconditional sojourn times \( C_{b,l} \), \( b, l = 1,2,\ldots,w, \) at the climate-weather states are given by

\[
N_{b,l} = \sum_{i=1}^{w} q_{b,i} N_{b,l}, \ b = 1,2,\ldots,w,
\]

(10)

where \( N_{b,l} \) are defined by the formula (1) in a case of any distribution of sojourn times \( C_{b,l} \) and by the formulae (2)-(8) in the cases of particular defined respectively by (2.5)-(2.11) [3] distributions of these sojourn times.

The limit values of the climate-weather change process \( C(t) \) transient probabilities at the particular operation states

\[
q_{b}(t) = P(C(t) = c_b) \ , \ t \in 0, +\infty, \ b = 1,2,\ldots,w,
\]

are given by [3], [6], [12].
\[ q_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b N_b}{\sum_{i=1}^{w} \pi_i N_i}, \quad b = 1, 2, \ldots, w, \quad (11) \]

where \( N_b, \quad b = 1, 2, \ldots, w, \) are given by (10), while the steady probabilities \( \pi_b \) of the vector \([\pi_b]_{1w}\] satisfy the system of equations

\[
\begin{align*}
\sum_{b=1}^{w} \pi_b &= 1, \\
[\pi_b] &= [\pi_b] [p_{bw}] 
\end{align*}
\] (12)

In the case of a periodic climate-weather change process, the limit transient probabilities \( q_b, \quad b = 1, 2, \ldots, w, \) at the climate-weather states defined by (11), are the long term proportions of the climate-weather change process \( C(t) \) sojourn times at the particular climate-weather states \( c_b, \quad b = 1, 2, \ldots, w. \)

Other interesting characteristics of the system climate-weather change process \( C(t) \) possible to obtain are its total sojourn times \( \hat{C}_b \) at the particular climate-weather states \( c_b, \quad b = 1, 2, \ldots, w, \) during the fixed time. It is well known [3], [6], [12] that the climate-weather change process total sojourn times \( \hat{C}_b \) at the particular climate-weather states \( c_b, \) for sufficiently large time \( \theta, \) have approximately normal distributions with the expected value given by

\[ \hat{N}_b[\hat{C}_b] = q_b \theta, \quad b = 1, 2, \ldots, w; \quad (13) \]

where \( q_b \) are given by (11).

3. Climate-weather change process for initial point of port oil piping transportation system operating under water Baltic Sea area characteristics prediction

On the basis of the statistical data from Section 3.1.1 [3], it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

\[ [q_b(0)] = [0.953, 0.006, 0.0, 0.029, 0.012, 0] \quad (14) \]

of the initial probabilities \( q_b(0), \quad b = 1, 2, \ldots, 6, \) of the climate-weather change process staying at the particular states \( c_b \) at the \( t = 0, \)

- the matrix

\[ [q_{w}] = \begin{bmatrix} 0 & 0.21 & 0 & 0.62 & 0.17 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.57 & 0.13 & 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (15) \]

of the probabilities \( q_{w}, \quad b = 1, 2, \ldots, 6, \) of transitions of the climate-weather change process from the climate-weather state \( c_b \) into the climate-weather state \( c_i. \)

Thus, after applying the formula (1), it is possible to find the mean value:

\[ N_{12} \approx 270.27, \quad N_{13} \approx 333.33, \quad N_{15} \approx 493.04, \quad (16) \]

and to assume

\[ N_{bl} = 0 \quad for \quad the \quad remaining \quad b, \quad and \quad l. \quad (17) \]

This way, the climate-weather change process for the initial point of port oil piping transportation system operating under water Baltic Sea area is approximately identified and we may predict its main characteristics. Namely, considering (16)-(17) and applying (10), the unconditional mean sojourn times of the considered climate-weather change process at the particular climate-weather states are:

\[ N_1 = E[C_1] = p_{12} N_{12} + p_{13} N_{13} + p_{15} N_{15} = 0.21 \times 270.27 + 0.62 \times 333.33 + 0.17 \times 493.04 = 347.24, \]

\[ N_2 = E[C_2] = p_{21} N_{21} = 13 = 3, \]

\[ N_3 = E[C_3] = 0, \]

\[ N_4 = E[C_4] = p_{41} N_{41} + p_{42} N_{42} + p_{45} N_{45} = 0.57 \times 6.23 + 0.13 \times 3 + 0.30 \times 5.47 = 5.47, \]

\[ N_5 = E[C_5] + p_{54} N_{54} = 1 \times 5.33 = 5.33, \]

\[ N_6 = E[C_6] = 0. \quad (18) \]

Considering (12), we get the following system of equations

\[
[\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6] = [\pi_1, \pi_2, \ldots, \pi_6] [p_{bl}]_{6 \times 6}
\]

\[
\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 = 1,
\]

and its solution
\( \pi_1 \approx 0.25, \pi_2 \approx 0.25, \pi_3 \approx 0, \pi_4 \approx 0.25, \pi_5 \approx 0.25, \pi_6 \approx 0. \) (19)

Hence and from (18), after applying (11), it follows that the limit values of the climate-weather change process for the considered point of the port oil piping system operation area the transient probabilities \( q_b(t) \) at the particular climate-weather states \( c_b, \ b=1,2,\ldots,6, \) are:

\[
q_1 \approx 0.962, \ q_2 \approx 0.008, \ q_3 \approx 0, \ q_4 \approx 0.015, \\
q_5 \approx 0.015, \ q_6 \approx 0.
\]

Considering the above result, after applying (13), the expected values of the total sojourn times \( \hat{C}_b, \ b=1,2,\ldots,6, \) of the climate-weather change process at the particular climate-weather states \( c_b, \ b=1,2,\ldots,6, \) during the fixed operation time \( \theta=1 \) year\( =365 \) days, amount:

\[
\hat{N}_1 = E[\hat{C}_1] \approx 0.962 \text{ year } = 351.13 \text{ days}, \\
\hat{N}_2 = E[\hat{C}_2] \approx 0.008 \text{ year } = 2.92 \text{ days}, \\
\hat{N}_3 = E[\hat{C}_3] \approx 0 \text{ year } = 0 \text{ days}, \\
\hat{N}_4 = E[\hat{C}_4] \approx 0.015 \text{ year } = 5.48 \text{ days}, \\
\hat{N}_5 = E[\hat{C}_5] \approx 0.015 \text{ year } = 5.47 \text{ days}, \\
\hat{N}_6 = E[\hat{C}_6] \approx 0 \text{ year } = 0 \text{ days}.
\]

4. Climate-weather change process for maritime ferry operating at Gdynia port area characteristics prediction

On the basis of the statistical data from Section 3.3.1 [4], it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

\[
[q_b(0)] = [0.569, 0.426, 0, 0, 0, 0, 0.005, 0],
\]

of the initial probabilities \( q_b(0), \ b=1,2,\ldots,8, \) of the climate-weather change process staying at the particular states \( c_b \) at the \( t=0, \)

- the matrix

\[
[q_{bl}]=
\begin{bmatrix}
0 & 1.00 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.06 & 0 & 0.94 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.86 & 0 & 0.14 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.92 & 0 & 0 & 0 & 0.08 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.00 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]

of the probabilities \( q_{bl}, \ b,l=1,2,\ldots,8, \) of transitions of the climate-weather change process from the climate-weather state \( c_b \) into the climate-weather state \( c_l \). Thus, after applying the formula (1), it is possible to find the mean value:

\[
N_{12} = 6.50, N_{21} \approx 78.18, N_{23} = 72.46, \\
N_{32} = 38.79, N_{34} \approx 23.68, N_{43} \approx 4.58, \\
N_{47} = 11.00, N_{74} = 1.00
\]

and to assume

\[
N_{bl} = 0
\]

for the remaining \( b, \) and \( l. \)

This way, the climate-weather change process for the point of maritime ferry operating at Gdynia Port area is approximately identified and we may predict its main characteristics. Namely, considering (21)-(22) and applying (10), the unconditional mean sojourn times of the considered climate-weather change process at the particular climate-weather states are:

\[
N_1 = E[C_1] = p_{12}N_{12} = 1.650 = 6.50, \\
N_2 = E[C_2] = p_{21}N_{21} + p_{23}N_{23} \\
= 0.06 \times 78.18 + 0.94 \times 72.46 = 72.80, \\
N_3 = E[C_3] = p_{32}N_{32} + p_{34}N_{34} \\
= 0.86 \times 38.79 + 0.14 \times 23.68 = 36.67, \\
N_4 = E[C_4] = p_{43}N_{43} + p_{47}N_{47} \\
= 0.92 \times 4.58 + 0.08 \times 4.29 = 4.29, \\
N_5 = E[C_5] = 0, \\
N_6 = E[C_6] = 0, \\
N_7 = E[C_7] = p_{74}N_{74} + 11 = 1, \\
N_8 = E[C_8] = 0.
\]

Considering (12), we get the following system of equations
\[\left[\pi_1, \pi_2, \ldots, \pi_8\right] = \left[\pi_1, \pi_2, \ldots, \pi_8\right]\left[p_{hi}\right]_{8 \times 8}
\]

and its solution

\[\pi_1 \cong 0.2, \pi_2 \cong 0.2, \pi_3 \cong 0.2, \pi_4 \cong 0.2, \pi_5 \cong 0, \pi_6 \cong 0, \pi_7 \cong 0.2, \pi_8 \cong 0.\]

Hence and from (23), after applying (11), it follows that the limit values of the climate-weather change process for the considered point of the maritime ferry operation area the transient probabilities \(q_i(t)\) at the particular climate-weather states \(c_b, b = 1, 2, \ldots, 8\), are:

\[q_1 \cong 0.055, q_2 \cong 0.602, q_3 \cong 0.303, q_4 \cong 0.031, q_5 \cong 0, q_6 \cong 0, q_7 \cong 0.009, q_8 \cong 0.\]

Considering the above result, after applying (13), the expected values of the total sojourn times \(\hat{\tau}_b\), \(b = 1, 2, \ldots, 8\), of the climate-weather change process at the particular climate-weather states \(c_b, c_b = 1, 2, \ldots, 8\), during the fixed operation time \(\theta = 1\) year = 365 days, amount:

\[\hat{N}_1 = E[\hat{\tau}_1] \cong 20.08\text{ days},
\]
\[\hat{N}_2 = E[\hat{\tau}_2] \cong 219.72\text{ days},
\]
\[\hat{N}_3 = E[\hat{\tau}_3] \cong 110.60\text{ days},
\]
\[\hat{N}_4 = E[\hat{\tau}_4] \cong 11.31\text{ days},
\]
\[\hat{N}_5 = E[\hat{\tau}_5] \cong 0\text{ year} = 0\text{ days},
\]
\[\hat{N}_6 = E[\hat{\tau}_6] \cong 0\text{ year} = 0\text{ days},
\]
\[\hat{N}_7 = E[\hat{\tau}_7] \cong 3.29\text{ days},
\]
\[\hat{N}_8 = E[\hat{\tau}_8] \cong 0\text{ year} = 0\text{ days}.
\]

5. Conclusions

The proposed statistical methods of identification of the unknown parameters of the climate-weather change processes allow us for the identification of the models discussed in [6] and next their practical applications in evaluation, prediction and optimization of reliability, availability and safety of real complex critical infrastructures.

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References


