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## **Identification of port oil piping transportation system operation process including operating environment threats**

### **Keywords**

port oil piping transport, critical infrastructure, operation process, operating environment threats

### **Abstract**

In the paper, the traditional semi-Markov approach to a complex technical system operation process modeling is proposed to modelling a critical infrastructure operation process including operating environment threats. Next the model is applied to real critical infrastructures such as the port oil piping transportation system.

### **1. Introduction**

The operation process of a critical infrastructure is very complex and often it is difficult to analyze these critical infrastructure safety with respect to changing in time its operation process states and operating environment conditions that are essential in this analysis. The complexity of the critical infrastructure operation process and its influence on changing in time the critical infrastructure structure and its components' safety parameters are essential in critical infrastructure safety analysis and protection. Usually, the critical infrastructure environment have either an explicit or an implicit strong influence on the critical infrastructure operation process. As a rule, some of the environmental events together with the infrastructure operation conditions define a set of different operation states of the critical infrastructure in which the critical infrastructure change its safety structure and its components safety parameters. In this report, we propose a convenient tool for analyzing this problem applying the semi-Markov model [14]-[16], [18], [23]-[24] of the critical infrastructure operation process, both without including critical infrastructure environment threats and with including them into this model.

### **2. Identification of critical infrastructure operation process**

#### **2.1. Estimating parameters of critical infrastructure operation process including operating environment threats identified by expert opinion – expert data only**

In the case of lack of statistical data collection, together with experienced experts operating the critical infrastructure, it is possible to estimate approximately the unknown parameters of the critical infrastructure operation process including operating environment threats performing the following steps:  
i) to determine the vector

$$[p'(0)] = [p'_1(0), p'_2(0), \dots, p'_{\nu'}(0)],$$

of expert evaluations of the probabilities  $p'_b(0)$ ,  $b=1,2,\dots,\nu'$ , of the critical infrastructure operation process staying at the operation states at the initial moment  $t=0$ , after explanation to the expert practical meaning of the formula

$$p'_b(0) = \frac{n'_b(0)}{n'(0)} \text{ for } b=1,2,\dots,\nu';$$

ii) to determine the matrix

$$[p'_{bl}] = \begin{bmatrix} p'_{11} & p'_{12} & \dots & p'_{1\nu'} \\ p'_{21} & p'_{22} & \dots & p'_{2\nu'} \\ \dots & & & \\ p'_{\nu'1} & p'_{\nu'2} & \dots & p'_{\nu'\nu'} \end{bmatrix},$$

of expert evaluations of the probabilities  $p'_{bl}$ ,  $b, l = 1, 2, \dots, \nu'$ , of the critical infrastructure operation process transitions from the operation state  $z_b$  to the operation state  $z_l$ , after explanation to the expert practical meaning of the formula

$$p'_{bl} = \frac{n'_{bl}}{n'_b} \text{ for } b, l = 1, 2, \dots, \nu', b \neq l, p_{bb} = 0$$

for  $b = 1, 2, \dots, \nu'$ ;

iii) to determine the matrix

$$[M'_{bl}] = \begin{bmatrix} M'_{11} & M'_{12} & \dots & M'_{1\nu'} \\ M'_{21} & M'_{22} & \dots & M'_{2\nu'} \\ \dots & & & \\ M'_{\nu'1} & M'_{\nu'2} & \dots & M'_{\nu'\nu'} \end{bmatrix},$$

of expert evaluations of the mean values  $M'_{bl}$ ,  $b, l = 1, 2, \dots, \nu'$ , of the critical infrastructure operation process conditional sojourn times  $\theta'_{bl}$ ,  $b, l = 1, 2, \dots, \nu'$ , at the operation state  $z_b$  when the next operation state is  $z_l$ , after explanation to the expert practical meaning of these parameters.

### 3. Applications in port transport including operating environment threats identified by expert opinion – statistical and expert data

#### 3.1. Identification of port oil piping transportation system operation process including operating environment threats identified by expert opinion – statistical and expert data

On the basis of the expert opinions concerning the operation process of the considered port oil pipeline transportation system, in [3] the number of the pipeline system operation process states  $\nu' = 28$  is fixed and the operation states  $z'_b$ ,  $b = 1, 2, \dots, 28$ , are defined as follows:

- the operation states  $z'_i$ ,  $i = 1, 2, \dots, 7$ , without including operating environment threats  $ut_1$ ,  $ut_2$ ,  $ut_3$ , marked by

$$z'_i = z_i, i = 1, 2, \dots, 7;$$

- the operation states  $z'_i$ ,  $i = 1, 2, \dots, 7$ , including the threat  $ut_1$ , respectively marked by

$$z'_i, i = 8, 9, \dots, 14;$$

- the operation states  $z'_i$ ,  $i = 1, 2, \dots, 7$ , including the threat  $ut_2$ , respectively marked by

$$z'_i, i = 15, 16, \dots, 21;$$

- the operation states  $z'_i$ ,  $i = 1, 2, \dots, 7$ , including the threat  $ut_3$ , respectively marked by

$$z'_i, i = 22, 23, \dots, 28.$$

The influence of the above system operation states changing on the changes of the pipeline system safety structure is similar to that described in Section 2.2 [3].

At the system operation states  $z'_1, z'_8, z'_{15}, z'_{22}$  and  $z'_7, z'_{14}, z'_{21}, z'_{28}$ , the system is composed of the subsystem  $S_3$ , that is a series-“2 out of 3” system containing three series subsystems with the scheme showed in *Figure 10* [3].

At the system operation state  $z'_2, z'_9, z'_{16}, z'_{23}$ , the system is composed of a series-parallel subsystem  $S_3$ , which contains three pipelines with the scheme showed in *Figure 11* [3].

At the system operation states  $z'_3, z'_{10}, z'_{17}, z'_{24}$  and  $z'_5, z'_{12}, z'_{19}, z'_{26}$ , the system is series and composed of two series-parallel subsystems  $S_1, S_2$  each containing two pipelines with the scheme showed in *Figure 12* [3].

At the operation states  $z'_4, z'_{11}, z'_{18}, z'_{25}$  and  $z'_6, z'_{13}, z'_{20}, z'_{27}$ , the system is series and composed of two series-parallel subsystems  $S_1, S_2$  each containing two pipelines and one series-“2 out of 3” subsystem  $S_3$ , with the scheme showed in *Figure 13* [3].

#### 3.2. Defining parameters and data collection of port oil piping transportation system operation process including operating environment threats identified by expert opinion – statistical and expert data

The unknown parameters of the critical infrastructure operation process semi-Markov model are:

- the initial probabilities  $p'_b(0)$ ,  $b=1,2,\dots,28$ , of the pipeline system operation process staying at the particular states  $z'_b$  at the moment  $t=0$ ,
- the probabilities  $p'_{bl}$ ,  $b,l=1,2,\dots,28$ ,  $b \neq l$ , of the pipeline system operation process transitions from the operation state  $z'_b$  into the operation state  $z'_l$ ,
- the distributions of the pipeline system conditional sojourn times  $\theta'_{bl}$ ,  $b,l=1,2,\dots,28$ ,  $b \neq l$ , at the particular operation states and their mean values  $M'_{bl} = E[\theta'_{bl}]$ ,  $b,l=1,2,\dots,28$ ,  $b \neq l$ .

To identify all these parameters of the pipeline system operation process the statistical data about this process is needed.

### 3.3. Evaluating parameters of port oil piping transportation system operation process including operating environment threats identified by expert opinion– statistical and expert data

On the basis of the statistical data from Section 3.1.1 [3], using respectively the formulae (2.1)-(2.3) in [3] and (2.4)-(2.6) given in Section 2.2 [3], it is possible to evaluate the following unknown basic parameters of the port oil piping transportation system operation process including operating environment threats without their separation:

- the vector

$$[p(0)]_{1 \times 7} = [0.34, 0.05, 0, 0, 0.23, 0.19, 0.19] \quad (1)$$

of the initial probabilities  $p_b(0)$ ,  $b=1,2,\dots,7$ , of the pipeline system operation process staying at the particular states  $z_b$  at the  $t=0$ ,

- the matrix

$$[p_{bl}]_{7 \times 7} = \begin{bmatrix} 0 & 0.022 & 0.022 & 0 & 0.534 & 0.111 & 0.311 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.488 & 0.023 & 0 & 0.023 & 0 & 0.233 & 0.233 \\ 0.095 & 0 & 0 & 0 & 0.667 & 0 & 0.238 \\ 0.531 & 0.062 & 0 & 0 & 0.219 & 0.188 & 0 \end{bmatrix} \quad (2)$$

of the probabilities  $p_{bl}$ ,  $b,l=1,2,\dots,7$ , of transitions of the pipeline system operation process from the operation state  $z_b$  into the operation state  $z_l$ ;

- the matrix

$$[M_{bl}]_{7 \times 7} = \begin{bmatrix} 0 & 1920 & 480 & 0 & 1999.4 & 1250 & 1129.6 \\ 9960 & 0 & 0 & 0 & 0 & 0 & 810 \\ 575 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 380 \\ 874.7 & 480 & 0 & 300 & 0 & 436.3 & 1042.5 \\ 325 & 0 & 0 & 0 & 510.7 & 0 & 438 \\ 874.1 & 510 & 0 & 0 & 2585.7 & 2380 & 0 \end{bmatrix} \quad (3)$$

of the mean values  $M_{bl}$ ,  $b,l=1,2,\dots,7$ , of the conditional sojourn times  $\theta_{bl}$ ,  $b,l=1,2,\dots,7$ , the pipeline system operation process at the the operation state  $z_b$  when the next operation state is  $z_l$ . Some of the values of the probabilities existing in the vector  $[p(0)]_{1 \times 7}$  and in the matrix  $[p_{bl}]_{7 \times 7}$ , besides of that standing on the main diagonal, and equal to zero does not mean that the events they are concerned with, can not appear. They are evaluated on the basis of real statistical data and their values may change and become more precise if the time of the experiment is longer.

Considering expert opinion from BOTD that in all operations states of the port oil piping system, at each of the operation states if  $z_b(0) \neq 0$ ,  $b=1,2,\dots,7$ , the probability of a human error can be approximately evaluated as

$$P_b(ut_1) = P(ut_1) = 1/1158h = 0.00086,$$

the probability of a terrorist attack can be approximately evaluated as

$$P_b(ut_2) = P(ut_2) = 0,$$

and the probability of an act of vandalism and/or theft can be approximately evaluated as

$$P_b(ut_3) = P(ut_3) = 1/7896h = 0.000076,$$

We distribute the initial probabilities (1) according to the variant 1 procedure defined by (5.1)-(5.4) [3], as follows:

- if  $p_b(0) \neq 0$ ,  $b=1,2,\dots,7$ ,

we replace it by

$$\begin{aligned} p'_{4(b-1)+1}(0) &= p_b(0) - [P(ut_1) + P(ut_2) + P(ut_3)] \\ &= p_b(0) - [0.00086 + 0.0 + 0.000076] \\ &= p_b(0) - [0.000936], \\ p'_{4(b-1)+1+i}(0) &= P(ut_i), \quad i=1,2,3, \end{aligned}$$

for  $b=1,2,\dots,7$ ;

- if  $p_b(0) = 0, b = 1, 2, \dots, 7,$

we replace it by

$$p'_{4(b-l)+1}(0) = 0, \\ p'_{4(b-l)+1+i}(0) = 0, i = 1, 2, 3$$

for  $b = 1, 2, \dots, 7.$

Thus, in particular, we distribute

-  $p_1(0) = 0.34$

into the initial probabilities

$$p'_1(0) = 0.339064, p'_2(0) = 0.00086, \\ p'_3(0) = 0.0, p'_4(0) = 0.000076$$

-  $p_2(0) = 0.05$

into the initial probabilities

$$p'_5(0) = 0.0499064, p'_6(0) = 0.00086, \\ p'_7(0) = 0, p'_8(0) = 0.000076 ;$$

-  $p_3(0) = 0$

into the initial probabilities

$$p'_9(0) = 0, p'_{10}(0) = 0, p'_{11}(0) = 0, p'_{12}(0) = 0;$$

-  $p_4(0) = 0$

into the initial probabilities

$$p'_{13}(0) = 0, p'_{14}(0) = 0, p'_{15}(0) = 0, p'_{16}(0) = 0;$$

-  $p_5(0) = 0.23$

into the initial probabilities

$$p'_{17}(0) = 0.229064, p'_{18}(0) = 0.00086, \\ p'_{19}(0) = 0.0, p'_{20}(0) = 0.000076$$

-  $p_6(0) = 0.19$

into the initial probabilities

$$p'_{21}(0) = 0.189064, p'_{22}(0) = 0.00086, \\ p'_{23}(0) = 0, p'_{24}(0) = 0.000076 ;$$

-  $p_7(0) = 0.19$

into the initial probabilities

$$p'_{25}(0) = 0.189064, p'_{26}(0) = 0.00086, \\ p'_{27}(0) = 0, p'_{28}(0) = 0.000076.$$

After that, we get new vector of initial probabilities of the port oil piping transportation system operation process including operating environment threats with their separation:

$$[p'(0)]_{1 \times 28} \\ = [0.339064, 0.00086, 0, 0.000076; \\ 0.049604, 0.00086, 0, 0.000076; 0, 0, 0, 0; \\ 0, 0, 0, 0; 0.229064, 0.00086, 0, 0.000076; \\ 0.189064, 0.00086, 0, 0.000076; 0.189064, \\ 0.00086, 0, 0.000076]$$

Similarly, considering expert opinions from BOTD, we distribute the probabilities of transitions between the operation states (2) according to the variant 1 procedure defined by (5.9)-(5.14) [3] as follows:

- if  $p_{bl} \neq 0, b, l = 1, 2, \dots, 7,$

we replace it by

$$p'_{4(b-l)+1 \ 4(l-l)+1} = p_{bl} - [P(ut_1) + P(ut_2) + P(ut_3)] \\ = p_{bl} - [0.00086 + 0 + 0.000076] \\ = p_{bl} - [0.000936],$$

$$p'_{4(b-l)+1 \ 4(l-l)+1+i} = P(ut_i), i = 1, 2, 3,$$

for  $b, l = 1, 2, \dots, 7,$

and we additionally assume that

$$p'_{4(b-l)+1+i \ 4(b-l)+1} = 1, i = 1, 2, 3, \\ p'_{4(b-l)+1+i \ j} = 0, i = 1, 2, 3, j = 1, 2, \dots, 28,$$

$$j \neq 4(b-l)+1$$

- if  $p_{bl} = 0, b, l = 1, 2, \dots, 7,$

we replace it by

$$p'_{4(b-l)+1 \ 4(l-l)+1} = 0, \\ p'_{4(b-l)+1 \ 4(l-l)+1+i}(0) = 0, i = 1, 2, 3,$$

for  $b, l = 1, 2, \dots, 7.$

Thus, in particular, we distribute:

-  $p_{11} = 0$

into the probabilities of transitions

$$p'_{11} = 0, p'_{12} = 0, p'_{13} = 0, p'_{14} = 0,$$

$$- p_{12} = 0.022$$

into the probabilities of transitions

$$p'_{15} = 0.021064, p'_{16} = 0.00086,$$

$$p'_{17} = 0, p'_{18} = 0.000076;$$

$$- p_{13} = 0.022$$

into the probabilities of transitions

$$p'_{19} = 0.021064, p'_{110} = 0.00086,$$

$$p'_{11} = 0, p'_{12} = 0.000076;$$

$$- p_{14} = 0$$

into the probabilities of transitions

$$p'_{113} = 0, p'_{114} = 0, p'_{115} = 0, p'_{116} = 0;$$

$$- p_{15} = 0.534$$

into the probabilities of transitions

$$p'_{117} = 0.533064, p'_{118} = 0.00086,$$

$$p'_{119} = 0, p'_{120} = 0.000076;$$

$$- p_{16} = 0.111$$

into the probabilities of transitions

$$p'_{121} = 0.110064, p'_{122} = 0.00086,$$

$$p'_{123} = 0, p'_{124} = 0.000076;$$

$$- p_{17} = 0.311$$

into the probabilities of transitions

$$p'_{125} = 0.310064, p'_{126} = 0.00086,$$

$$p'_{127} = 0, p'_{128} = 0.000076;$$

and additionally, we assume

$$p'_{21} = 1, p'_{2j} = 0, j = 2, 3, \dots, 28,$$

$$p'_{31} = 1, p'_{3j} = 0, j = 2, 3, \dots, 28,$$

$$p'_{41} = 1, p'_{4j} = 0, j = 2, 3, \dots, 28;$$

to replace the 1<sup>st</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0 \ 0 \ 0 \ 0; 0.021064 \ 0.00086 \ 0 \ 0.000076; 0.021064$$

$$0.00086 \ 0 \ 0.000076; 0 \ 0 \ 0 \ 0; 0.533064 \ 0.00086 \ 0.0$$

$$0.000076; 0.110064 \ 0.00086 \ 0 \ 0.000076; 0.310064$$

$$0.00086 \ 0 \ 0.000076]$$

$$[1 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0]$$

$$[1 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0]$$

$$[1 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 0].$$

We distribute:

$$- p_{21} = 0.2$$

into the probabilities of transitions

$$p'_{51} = 0.199064, p'_{52} = 0.00086,$$

$$p'_{53} = 0, p'_{54} = 0.000076;$$

$$- p_{22} = 0$$

into the probabilities of transitions

$$p'_{55} = 0, p'_{56} = 0, p'_{57} = 0, p'_{58} = 0,$$

$$- p_{23} = 0$$

into the probabilities of transitions

$$p'_{59} = 0, p'_{510} = 0, p'_{511} = 0, p'_{512} = 0;$$

$$- p_{24} = 0$$

into the probabilities of transitions

$$p'_{513} = 0, p'_{514} = 0, p'_{515} = 0, p'_{516} = 0;$$

$$- p_{25} = 0$$

into the probabilities of transitions

$$p'_{517} = 0, p'_{518} = 0, p'_{519} = 0, p'_{520} = 0;$$

$$- p_{26} = 0$$

into the probabilities of transitions

$$p'_{521} = 0, p'_{522} = 0, p'_{523} = 0, p'_{524} = 0;$$

$$- p_{27} = 0.8$$

into the probabilities of transitions

$$p'_{525} = 0.799064, p'_{126} = 0.00086,$$

$$p'_{127} = 0, p'_{128} = 0.000076;$$

and additionally, we assume

$$p'_{65} = 1, p'_{6j} = 0, j = 1, 2, \dots, 4, 6, \dots, 28,$$

$$p'_{75} = 1, p'_{7j} = 0, j = 1, 2, \dots, 4, 6, \dots, 28,$$

$$p'_{85} = 1, p'_{8j} = 0, j = 1, 2, \dots, 4, 6, \dots, 28;$$

to replace the 2<sup>nd</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0.199064 \ 0.00086 \ 0 \ 0.000076; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \\ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0.799064 \ 0.00086 \ 0 \ 0.000076] \\ [0 \ 0 \ 0 \ 0; \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0] \\ [0 \ 0 \ 0 \ 0; \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0] \\ [0 \ 0 \ 0 \ 0; \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0].$$

We distribute:

$$- p_{31} = 1$$

into the probabilities of transitions

$$p'_{91} = 0.999064, p'_{92} = 0.00086,$$

$$p'_{93} = 0, p'_{94} = 0.000076;$$

$$- p_{32} = 0$$

into the probabilities of transitions

$$p'_{95} = 0, p'_{96} = 0, p'_{97} = 0, p'_{98} = 0;$$

$$- p_{33} = 0$$

into the probabilities of transitions

$$p'_{99} = 0, p'_{910} = 0, p'_{911} = 0, p'_{912} = 0;$$

$$- p_{34} = 0$$

into the probabilities of transitions

$$p'_{913} = 0, p'_{914} = 0, p'_{915} = 0, p'_{916} = 0;$$

$$- p_{35} = 0$$

into the probabilities of transitions

$$p'_{917} = 0, p'_{918} = 0, p'_{919} = 0, p'_{920} = 0;$$

$$- p_{36} = 0$$

into the probabilities of transitions

$$p'_{921} = 0, p'_{922} = 0, p'_{923} = 0, p'_{924} = 0;$$

$$- p_{37} = 0$$

into the probabilities of transitions

$$p'_{925} = 0, p'_{926} = 0, p'_{927} = 0, p'_{928} = 0;$$

and additionally, we assume

$$p'_{109} = 1, p'_{10j} = 0, j = 1, 2, \dots, 8, 10, \dots, 28,$$

$$p'_{119} = 1, p'_{11j} = 0, j = 1, 2, \dots, 8, 10, \dots, 28,$$

$$p'_{129} = 1, p'_{12j} = 0, j = 1, 2, \dots, 8, 10, \dots, 28;$$

to replace the 3<sup>rd</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0.999064 \ 0.00086 \ 0 \ 0.000076; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \\ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0] \\ [0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0] \\ [0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0] \\ [0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0].$$

We distribute:

$$- p_{41} = 0$$

into the probabilities of transitions

$$p'_{131} = 0, p'_{132} = 0, p'_{133} = 0, p'_{134} = 0;$$

$$- p_{42} = 0$$

into the probabilities of transitions

$$p'_{135} = 0, p'_{136} = 0, p'_{137} = 0, p'_{138} = 0;$$

$$- p_{43} = 0$$

into the probabilities of transitions

$$p'_{139} = 0, p'_{1310} = 0, p'_{1311} = 0, p'_{1312} = 0;$$

$$- p_{44} = 0$$

into the probabilities of transitions

$$p'_{1313} = 0, p'_{1314} = 0, p'_{1315} = 0, p'_{1316} = 0;$$

$$- p_{45} = 0$$

into the probabilities of transitions

$$p'_{1317} = 0, p'_{1318} = 0, p'_{1319} = 0, p'_{1320} = 0;$$

$$- p_{46} = 0$$

into the probabilities of transitions

$$p'_{1321} = 0, p'_{1322} = 0, p'_{1323} = 0, p'_{1324} = 0;$$

$$- p_{47} = 1$$

into the probabilities of transitions

$$p'_{13\ 25} = 0.999064, p'_{13\ 26} = 0.00086, \\ p'_{13\ 27} = 0, p'_{13\ 28} = 0.000076;$$

and additionally, we assume

$$p'_{14\ 13} = 1, p'_{14\ j} = 0, j = 1, 2, \dots, 12, 14, \dots, 28, \\ p'_{15\ 13} = 1, p'_{15\ j} = 0, j = 1, 2, \dots, 12, 14, \dots, 28, \\ p'_{16\ 13} = 1, p'_{16\ j} = 0, j = 1, 2, \dots, 12, 14, \dots, 28;$$

to replace the 4<sup>th</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; \\ 0.999064\ 0.00086\ 0.0\ 0.000076] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0].$$

We distribute:

$$- p_{5\ 1} = 0.488$$

into the probabilities of transitions

$$p'_{17\ 1} = 0.487064, p'_{17\ 2} = 0.00086, \\ p'_{13\ 3} = 0, p'_{13\ 4} = 0.000076;$$

$$- p_{5\ 2} = 0.023$$

into the probabilities of transitions

$$p'_{17\ 5} = 0.022064, p'_{17\ 6} = 0.00086, \\ p'_{17\ 7} = 0, p'_{17\ 8} = 0.000076;$$

$$- p_{5\ 3} = 0$$

into the probabilities of transitions

$$p'_{17\ 9} = 0, p'_{17\ 10} = 0, p'_{17\ 11} = 0, p'_{17\ 12} = 0;$$

$$- p_{5\ 4} = 0.023$$

into the probabilities of transitions

$$p'_{17\ 13} = 0.022064, p'_{17\ 14} = 0.00086, \\ p'_{17\ 15} = 0, p'_{17\ 16} = 0.000076;$$

$$- p_{5\ 5} = 0$$

into the probabilities of transitions

$$p'_{17\ 17} = 0, p'_{17\ 18} = 0, p'_{17\ 19} = 0, p'_{17\ 20} = 0;$$

$$- p_{5\ 6} = 0.233$$

into the probabilities of transitions

$$p'_{17\ 21} = 0.232064, p'_{17\ 22} = 0.00086, \\ p'_{17\ 23} = 0, p'_{17\ 24} = 0.000076;$$

$$- p_{5\ 7} = 0.233$$

into the probabilities of transitions

$$p'_{17\ 25} = 0.232064, p'_{17\ 26} = 0.00086, \\ p'_{17\ 27} = 0, p'_{17\ 28} = 0.000076;$$

and additionally, we assume

$$p'_{18\ 17} = 1, p'_{18\ j} = 0, j = 1, 2, \dots, 16, 18, \dots, 28, \\ p'_{19\ 17} = 1, p'_{19\ j} = 0, j = 1, 2, \dots, 16, 18, \dots, 28, \\ p'_{20\ 17} = 1, p'_{20\ j} = 0, j = 1, 2, \dots, 16, 18, \dots, 28;$$

to replace the 5<sup>th</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0.487064\ 0.00086\ 0\ 0.000076; 0.022064\ 0.00086\ 0 \\ 0.000076; 0\ 0\ 0\ 0; 0.022064\ 0.00086\ 0\ 0.000076; 0\ 0\ 0 \\ 0; 0.232064\ 0.00086\ 0\ 0.000076; 0.232064\ 0.00086\ 0 \\ 0.000076] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0].$$

We distribute:

$$- p_{6\ 1} = 0.095$$

into the probabilities of transitions

$$p'_{21\ 1} = 0.094064, p'_{17\ 2} = 0.00086, \\ p'_{13\ 3} = 0, p'_{13\ 4} = 0.000076;$$

$$- p_{6\ 2} = 0$$

into the probabilities of transitions

$$p'_{21\ 5} = 0, p'_{21\ 6} = 0, p'_{21\ 7} = 0, p'_{21\ 8} = 0;$$

$$- p_{6\ 3} = 0$$

into the probabilities of transitions

$$p'_{21\ 9} = 0, p'_{21\ 10} = 0, p'_{21\ 11} = 0, p'_{21\ 12} = 0;$$

$$- p_{6\ 4} = 0$$

into the probabilities of transitions

$$p'_{21\ 13} = 0, p'_{21\ 14} = 0, p'_{21\ 15} = 0, p'_{21\ 16} = 0;$$

$$- p_{6\ 5} = 0.667$$

into the probabilities of transitions

$$p'_{21\ 17} = 0.666064, p'_{21\ 18} = 0.00086, \\ p'_{21\ 19} = 0, p'_{21\ 20} = 0.000076;$$

$$- p_{6\ 6} = 0$$

into the probabilities of transitions

$$p'_{21\ 21} = 0, p'_{21\ 22} = 0, p'_{21\ 23} = 0, p'_{21\ 24} = 0;$$

$$- p_{6\ 7} = 0.238$$

into the probabilities of transitions

$$p'_{21\ 25} = 0.237064, p'_{17\ 26} = 0.00086, \\ p'_{17\ 27} = 0, p'_{17\ 28} = 0.000076;$$

and additionally, we assume

$$p'_{22\ 21} = 1, p'_{22\ j} = 0, j = 1, 2, \dots, 20, 22, \dots, 28, \\ p'_{23\ 21} = 1, p'_{23\ j} = 0, j = 1, 2, \dots, 20, 22, \dots, 28, \\ p'_{23\ 21} = 1, p'_{24\ j} = 0, j = 1, 2, \dots, 20, 22, \dots, 28;$$

to replace the 6<sup>th</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0.094064\ 0.00086\ 0\ 0.000076; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; \\ 0\ 0\ 0\ 0; 0.666064\ 0.00086\ 0.0\ 0.000076; 0\ 0\ 0\ 0; \\ 0.237064\ 0.00086\ 0\ 0.000076] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0; 0\ 0\ 0\ 0].$$

We distribute:

$$- p_{7\ 1} = 0.531$$

into the probabilities of transitions

$$p'_{25\ 1} = 0.530064, p'_{17\ 2} = 0.00086, \\ p'_{13\ 3} = 0, p'_{13\ 4} = 0.000076;$$

$$- p_{7\ 2} = 0.062$$

into the probabilities of transitions

$$p'_{25\ 5} = 0.0610064, p'_{21\ 6} = 0.00086, \\ p'_{21\ 7} = 0, p'_{21\ 8} = 0.000076;$$

$$- p_{7\ 3} = 0$$

into the probabilities of transitions

$$p'_{25\ 9} = 0, p'_{25\ 10} = 0, p'_{25\ 11} = 0, p'_{25\ 12} = 0;$$

$$- p_{7\ 4} = 0$$

into the probabilities of transitions

$$p'_{25\ 13} = 0, p'_{25\ 14} = 0, p'_{25\ 15} = 0, p'_{25\ 16} = 0;$$

$$- p_{7\ 5} = 0.219$$

into the sum of probabilities of transitions

$$p'_{25\ 17} = 0.218064, p'_{21\ 18} = 0.00086, \\ p'_{21\ 19} = 0, p'_{21\ 20} = 0.000076;$$

$$- p_{7\ 6} = 0.188$$

into the probabilities of transitions

$$p'_{21\ 21} = 0.187064, p'_{21\ 22} = 0.00086, \\ p'_{21\ 23} = 0, p'_{21\ 24} = 0.000076;$$

$$- p_{7\ 7} = 0$$

into the probabilities of transitions

$$p'_{25\ 25} = 0, p'_{17\ 26} = 0, p'_{17\ 27} = 0, p'_{17\ 28} = 0;$$

and additionally, we assume

$$p'_{26\ 25} = 1, p'_{26\ j} = 0, j = 1, 2, \dots, 24, 26, \dots, 28, \\ p'_{27\ 25} = 1, p'_{27\ j} = 0, j = 1, 2, \dots, 24, 26, \dots, 28, \\ p'_{28\ 25} = 1, p'_{28\ j} = 0, j = 1, 2, \dots, 24, 26, \dots, 28;$$

to replace the 7<sup>th</sup> row of the matrix  $[p_{bl}]_{7 \times 7}$  given by (2) by the following 4 rows of the matrix  $[p'_{bl}]_{28 \times 28}$

$$[0.530064\ 0.00086\ 0\ 0.000076; 0.061064\ 0.00086\ 0 \\ 0.000076; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0.218064\ 0.00086\ 0 \\ 0.000076; 0.187064\ 0.00086\ 0\ 0.000076; 0\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0] \\ [0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 1\ 0\ 0\ 0].$$

Finally, we transform the corresponding matrix  $[M_{bl}]_{7 \times 7}$  of the mean values of the conditional sojourn times  $\theta_{bl}$ ,  $b, l = 1, 2, \dots, 7$ , into the the matrix  $[M'_{bl}]_{28 \times 28}$  of the mean values of the conditional sojourn times  $\theta'_{bl}$ ,  $b, l = 1, 2, \dots, 28$ , according to the procedure (5.21)-(5.24) [3], in the following way:

$$- \text{if } M_{bl}(0) \neq 0, b, l = 1, 2, \dots, 7,$$

we fix the mean values

$$M'_{4(b-l)+1+i\ 4(b-l)+1} \quad i = 1, 2, 3, \quad b = 1, 2, \dots, 7,$$

on the basis of expert opinions and assume

$$M'_{4(b-l)+1+i\ j} = 0, \quad i = 1, 2, 3, \quad j = 1, 2, \dots, 28,$$

and  $j \neq 4(b-l)+1$ ,



and

$$M'_{4(b-l)+1\ 4(l-l)+1} = M_{bl} - \sum_{i=1}^3 M'_{4(b-l)+1+i\ 4(b-l)+1},$$

- if  $M_{bl}(0) = 0$ ,  $b, l = 1, 2, \dots, 7$ ,

we replace it by

$$M'_{4(b-l)+1\ 4(l-l)+1} = 0,$$

$$M'_{4(b-l)+1\ 4(l-l)+1+i} = 0, \quad i = 1, 2, 3,$$

for  $b, l = 1, 2, \dots, 7$ .

Considering expert opinions [BOTD] that in all operations states of the port oil piping system the mean value of the time needed to eliminate a human error is approximately equal to 2h, the mean value of the time needed to eliminate a terrorist attack is equal to 0h (it is assumed that the system does not work under that threats), the mean value of the time needed to eliminate an act of vandalism and/or theft is approximately equal to 8h, we distribute the mean values of the piping system operation process conditional sojourn times at the operation states (3) in the following way:

We fix the mean values:

$$M'_{21} = 2, M'_{31} = 0, M'_{41} = 8.$$

We distribute:

$$- M_{11} = 0$$

into the mean values

$$M'_{11} = 0, M'_{12} = 0, M'_{13} = 0, M'_{14} = 0;$$

$$- M_{12} = 1920$$

into the mean values

$$M'_{15} = 1910, M'_{16} = 1910,$$

$$M'_{17} = 1910, M'_{18} = 1910;$$

$$- M_{13} = 480$$

into the mean values

$$M'_{19} = 470, M'_{110} = 470, M'_{111} = 470, M'_{112} = 470;$$

$$- M_{14} = 0$$

into the mean values

$$M'_{113} = 0, M'_{114} = 0, M'_{115} = 0, M'_{116} = 0;$$

$$- M_{15} = 1999.4$$

into the mean values

$$M'_{117} = 1989.4, M'_{118} = 1989.4,$$

$$M'_{119} = 1989.4, M'_{120} = 1989.4,$$

$$- M_{16} = 1250$$

into the mean values

$$M'_{121} = 1240, M'_{122} = 1240,$$

$$M'_{123} = 1240, M'_{124} = 1240;$$

$$- M_{17} = 1129.6$$

into the mean values

$$M'_{125} = 1119.6, M'_{126} = 1119.6$$

$$M'_{127} = 1119.6, M'_{128} = 1119.6;$$

and additionally we assume

$$M'_{2l} = 0, \quad l = 2, 3, \dots, 28,$$

$$M'_{3l} = 0, \quad l = 2, 3, \dots, 28,$$

$$M'_{4l} = 0, \quad l = 2, 3, \dots, 28;$$

to replace the 1<sup>st</sup> row of the matrix  $[M_{bl}]_{7 \times 7}$  given by (3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

$$[0\ 0\ 0\ 0; 1910\ 1910\ 0.0\ 1910; 470\ 470\ 0\ 470;$$

$$0\ 0\ 0\ 0; 1989.4\ 1998.4\ 1989.4\ 1989.4; 1240\ 1240\ 1240$$

$$1240; 119.6\ 1119.6\ 1119.6\ 1119.66]$$

$$[2\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0]$$

$$[0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0]$$

$$[8\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0].$$

We fix the mean values:

$$M'_{65} = 2, M'_{75} = 0, M'_{85} = 8.$$

We distribute:

$$- M_{21} = 9960$$

into the mean values

$$M'_{51} = 9950, M'_{52} = 9950,$$

$$M'_{53} = 9950, M'_{54} = 9950;$$

$$- M_{22} = 0$$

into the mean values

$$M'_{55} = 0, M'_{56} = 0, M'_{57} = 0, M'_{58} = 0;$$

$$- M_{23} = 0$$

into the mean values

$$M'_{59} = 0, M'_{510} = 0, M'_{511} = 0, M'_{512} = 0;$$

$$- M_{24} = 0$$

into the mean values

$$M'_{513} = 0, M'_{514} = 0, M'_{515} = 0, M'_{516} = 0;$$

$$- M_{25} = 0$$

into the mean values

$$M'_{517} = 0, M'_{518} = 0, M'_{519} = 0, M'_{520} = 0;$$

$$- M_{26} = 0$$

into the mean values

$$M'_{521} = 0, M'_{522} = 0, M'_{523} = 0, M'_{524} = 0;$$

$$- M_{27} = 810$$

into the mean values

$$M'_{525} = 800, M'_{526} = 800, M'_{527} = 800, M'_{528} = 800;$$

and additionally we assume

$$M'_{6l} = 0, l = 1,2, \dots, 4,6, \dots, 28,$$

$$M'_{7l} = 0, l = 1,2, \dots, 4,6, \dots, 28,$$

$$M'_{8l} = 0, l = 1,2, \dots, 4,6, \dots, 28;$$

to replace the 2<sup>nd</sup> row of the matrix  $[M_{bl}]_{7 \times 7}$  given by (3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

$$[9950 \ 9950 \ 9950 \ 9950; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \\ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 800 \ 800 \ 800 \ 800] \\ [0000; 2000; 0000; 0000; 0000; 0000; 0000] \\ [0000; 0000; 0000; 0000; 0000; 0000; 0000] \\ [0000; 8000; 0000; 0000; 0000; 0000; 0000].$$

We fix the mean values:

$$M'_{109} = 2, M'_{119} = 0, M'_{129} = 8;$$

We distribute:

$$- M_{31} = 575$$

into the mean values

$$M'_{91} = 565, M'_{92} = 565, M'_{93} = 565, M'_{94} = 565,$$

$$- M_{32} = 0$$

into the mean values

$$M'_{95} = 0, M'_{96} = 0, M'_{97} = 0, M'_{98} = 0;$$

$$- M_{33} = 0$$

into the mean values

$$M'_{99} = 0, M'_{910} = 0, M'_{911} = 0, M'_{912} = 0,$$

$$- M_{34} = 0$$

into the mean values

$$M'_{913} = 0, M'_{914} = 0, M'_{915} = 0, M'_{916} = 0,$$

$$- M_{35} = 0$$

into the mean values

$$M'_{917} = 0, M'_{918} = 0, M'_{919} = 0, M'_{920} = 0,$$

$$- M_{36} = 0$$

into the mean values

$$M'_{921} = 0, M'_{922} = 0, M'_{923} = 0, M'_{924} = 0,$$

$$- M_{37} = 0$$

into the mean values

$$M'_{925} = 0, M'_{926} = 0, M'_{927} = 0, M'_{928} = 0,$$

and additionally we assume

$$M'_{10l} = 0, l = 1,2, \dots, 8,10, \dots, 28,$$

$$M'_{11l} = 0, l = 1,2, \dots, 8,10, \dots, 28,$$

$$M'_{12l} = 0, l = 1,2, \dots, 8,10, \dots, 28;$$

to replace the 3<sup>rd</sup> row of the matrix  $[M_{bl}]_{7 \times 7}$  given by (3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

$$[565 \ 565 \ 565 \ 565; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0; \\ 0; \ 0 \ 0 \ 0 \ 0] \\ [0000; 0000; 2000; 0000; 0000; 0000; 0000] \\ [0000; 0000; 0000; 0000; 0000; 0000; 0000] \\ [0000; 0000; 8000; 0000; 0000; 0000; 0000].$$

We fix the mean values:

$$M'_{1413} = 2, M'_{1513} = 0, M'_{1613} = 8.$$

We distribute:

$$- M_{41} = 0$$

into the mean values

$$M'_{131} = 0, M'_{132} = 0, M'_{133} = 0, M'_{134} = 0;$$

$$- M_{42} = 0$$

into the mean values

$$M'_{135} = 0, M'_{136} = 0, M'_{137} = 0, M'_{138} = 0;$$

$$- M_{43} = 0$$

into the mean values

$$M'_{139} = 0, M'_{1310} = 0, M'_{1311} = 0, M'_{1312} = 0;$$

$$- M_{44} = 0$$

into the mean values

$$M'_{1313} = 0, M'_{1314} = 0, M'_{1315} = 0, M'_{1316} = 0;$$

$$- M_{45} = 0$$

into the mean values

$$M'_{1317} = 0, M'_{1318} = 0, M'_{1319} = 0, M'_{1320} = 0;$$

$$- M_{46} = 0$$

into the mean values

$$M'_{1321} = 0, M'_{1322} = 0, M'_{1323} = 0, M'_{1324} = 0;$$

$$- M_{47} = 380$$

into the mean values

$$M'_{1325} = 370, M'_{1326} = 370, M'_{1327} = 370, M'_{1328} = 370;$$

and additionally we assume

$$M'_{14l} = 0, l = 1, 2, \dots, 12, 14, \dots, 28,$$

$$M'_{15l} = 0, l = 1, 2, \dots, 12, 14, \dots, 28,$$

$$M'_{16l} = 0, l = 1, 2, \dots, 12, 14, \dots, 28;$$

to replace the 4<sup>th</sup> row of the matrix  $[M_{bl}]_{7 \times 7}$  given by (3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

$$[0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 370\ 370\ 370\ 370]$$

$$[0000; 0000; 0000; 2000; 0000; 0000; 0000]$$

$$[0000; 0000; 0000; 0000; 0000; 0000; 0000]$$

$$[0000; 0000; 0000; 8000; 0000; 0000; 0000].$$

We fix the mean values:

$$M'_{1817} = 2, M'_{1917} = 0, M'_{2017} = 8.$$

We distribute:

$$- M_{51} = 874.7$$

into the mean values

$$M'_{171} = 864.7, M'_{172} = 864.7,$$

$$M'_{173} = 864.7, M'_{174} = 864.7;$$

$$- M_{52} = 480$$

into the mean values

$$M'_{175} = 470, M'_{176} = 470, M'_{177} = 470, M'_{178} = 470;$$

$$- M_{53} = 0$$

into the mean values

$$M'_{179} = 0, M'_{1710} = 0, M'_{1711} = 0, M'_{1712} = 0;$$

$$M_{54} = 300$$

into the mean values

$$M'_{1713} = 290, M'_{1714} = 290, M'_{1715} = 290, M'_{1716} = 290;$$

$$- M_{55} = 0$$

into the mean values

$$M'_{1717} = 0, M'_{1318} = 0, M'_{1319} = 0, M'_{1320} = 0;$$

$$- M_{56} = 436.3$$

into the mean values

$$M'_{1721} = 426.3, M'_{1722} = 426.3,$$

$$M'_{1723} = 426.3, M'_{1724} = 426.3;$$

$$- M_{57} = 1042.5$$

into the mean values

$$M'_{1725} = 1032.5, M'_{1726} = 1032.5,$$

$$M'_{1727} = 1032.5, M'_{1728} = 1032.5;$$

and additionally we assume

$$M'_{18l} = 0, l = 1, 2, \dots, 16, 18, \dots, 28,$$

$$M'_{19l} = 0, l = 1, 2, \dots, 16, 18, \dots, 28,$$

$$M'_{20l} = 0, l = 1, 2, \dots, 12, 14, \dots, 28;$$

to replace the 5<sup>th</sup> row of the matrix  $[M_{bl}]_{7 \times 7}$  given by (3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

[864.7 864.7 864.7 864.7; 470 470 470 470; 0 0 0 0; 290 290 290; 0 0 0 0; 426.3 426.3 426.3 426.3; 1032.5 1032.5 1032.5]  
[0000;0000;0000;0000;2000;0000;0000]  
[0000;0000;0000;0000;0000;0000;0000]  
[0000;0000;0000;0000;8000;0000;0000].

We fix the mean values:

$$M'_{22\ 21} = 2, M'_{23\ 21} = 0, M'_{24\ 21} = 8.$$

We distribute:

$$- M_{6\ 1} = 325$$

into the mean values

$$M'_{21\ 1} = 315, M'_{21\ 2} = 315, M'_{21\ 4} = 355;$$

$$- M_{6\ 2} = 0$$

into the mean values

$$M'_{21\ 5} = 0, M'_{21\ 6} = 0, M'_{21\ 7} = 0, M'_{21\ 8} = 0;$$

$$- M_{6\ 3} = 0$$

into the mean values

$$M'_{21\ 9} = 0, M'_{21\ 10} = 0, M'_{21\ 11} = 0, M'_{21\ 12} = 0;$$

$$- M_{6\ 4} = 510.7$$

into the mean values

$$M'_{21\ 13} = 500.7, M'_{21\ 14} = 500.7, \\ M'_{21\ 15} = 500.7, M'_{21\ 16} = 500.7;$$

$$- M_{6\ 5} = 0$$

into the mean values

$$M'_{21\ 17} = 0, M'_{21\ 18} = 0, M'_{21\ 19} = 0, M'_{21\ 20} = 0;$$

$$- M_{6\ 6} = 0$$

into the mean values

$$M'_{21\ 21} = 0, M'_{21\ 22} = 0, M'_{21\ 23} = 0, M'_{21\ 24} = 0;$$

$$- M_{6\ 7} = 438$$

into the mean values

$$M'_{21\ 25} = 428, M'_{21\ 26} = 428, \\ M'_{21\ 27} = 428, M'_{21\ 28} = 428;$$

and additionally we assume

$$M'_{22\ l} = 0, l = 1, 2, \dots, 20, 22, \dots, 28,$$

$$M'_{23\ l} = 0, l = 1, 2, \dots, 20, 22, \dots, 28,$$

$$M'_{24\ l} = 0, l = 1, 2, \dots, 20, 22, \dots, 28;$$

to replace the 6<sup>th</sup> row of the matrix  $[M_{bl}]_{7 \times 7}$  given by (6.3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

[315 315 315 315; 0 0 0 0; 0 0 0 0; 500.7 500.7 500.7; 0 0 0 0; 0 0 0 0; 428 428 428 428]  
[0000;0000;0000;0000;0000;2000;0000]  
[0000;0000;0000;0000;0000;0000;0000]  
[0000;0000;0000;0000;0000;8000;0000].

We fix the mean values:

$$M'_{26\ 25} = 2, M'_{27\ 25} = 0, M'_{28\ 25} = 8.$$

We distribute:

$$- M_{7\ 1} = 874.1$$

into the mean values

$$M'_{25\ 1} = 864.1, M'_{25\ 2} = 864.1, \\ M'_{25\ 3} = 864.1, M'_{25\ 4} = 864.1;$$

$$- M_{7\ 2} = 510$$

into the mean values

$$M'_{25\ 5} = 500, M'_{25\ 6} = 500, M'_{25\ 8} = 500;$$

$$- M_{7\ 3} = 0$$

into the mean values

$$M'_{25\ 9} = 0, M'_{25\ 10} = 0, M'_{25\ 11} = 0, M'_{25\ 12} = 0;$$

$$- M_{7\ 4} = 0$$

into the mean values

$$M'_{25\ 13} = 0, M'_{25\ 14} = 0, M'_{25\ 15} = 0, M'_{25\ 16} = 0;$$

$$- M_{7\ 5} = 2585.7$$

into the the mean values

$$M'_{25\ 17} = 2575.7, M'_{25\ 18} = 2575.7, \\ M'_{25\ 19} = 2575.7, M'_{25\ 20} = 2575.7;$$

$$- M_{7\ 6} = 2380$$

into the mean values

$$M'_{25\ 21} = 2370, M'_{25\ 22} = 2370,$$

$$M'_{25\ 23} = 2370, M'_{25\ 24} = 2370;$$

$$- M'_{7\ 7} = 0$$

into the mean values

$$M'_{25\ 25} = 0, M'_{25\ 26} = 0, M'_{25\ 26} = 0, M'_{25\ 26} = 0;$$

and additionally we assume

$$M'_{26\ l} = 0, l = 1, 2, \dots, 24, 26, \dots, 28,$$

$$M'_{27\ l} = 0, l = 1, 2, \dots, 24, 26, \dots, 28,$$

$$M'_{28\ l} = 0, l = 1, 2, \dots, 24, 26, \dots, 28;$$

to replace the 7<sup>th</sup> row of the matrix  $[M'_{bl}]_{7 \times 7}$  given by (3) by the following 4 rows of the matrix  $[M'_{bl}]_{28 \times 28}$

$$[840.9\ 840.9\ 840.9\ 840.9; 500\ 500\ 500\ 500; 0\ 0\ 0\ 0; 0\ 0\ 0\ 0; 2575.7\ 2575.7\ 2575.7\ 2575.7; 2370\ 2370\ 2370\ 2370; 0\ 0\ 0\ 0]$$

$$[0000; 0000; 0000; 0000; 0000; 0000; 2000]$$

$$[0000; 0000; 0000; 0000; 0000; 0000; 0000]$$

$$[0\ 000; 0000; 0000; 0000; 0000; 0000; 8000].$$

#### 4. Conclusion

In the paper there is presented the probabilistic model of the critical infrastructure operation process including operating environment threats. Presented model is the basis for further considerations in particular tasks of the EU-CIRCLE project. The model is applied to real critical infrastructures such as the port oil piping transportation system. Next this model will be used to construct the integrated general safety probabilistic model of the critical infrastructure related to its operation process and climate-weather process [3].

The model further development will be done in the following EU-CIRCLE project reports: [4]-[6], [9]-[13].

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