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## **Identification methods and procedures of climate-weather change process including extreme weather hazards for port oil piping transportation system operating at under water Baltic Sea area**

### **Keywords**

climate-weather change process, semi-Markov model, modelling, identification, transportation system

### **Abstract**

There are presented the methods of identification of the climate-weather change process. These are the methods and procedures for estimating the unknown basic parameters of the climate-weather change process semi-Markov model and identifying the distributions of the climate-weather change process conditional sojourn times at the climate-weather states. There are given the formulae estimating the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather change transitions between the climate-weather states and the parameters of the distributions suitable and typical for the description of the climate-weather change process conditional sojourn times at the particular climate-weather states. The proposed statistical methods applications for the unknown parameters identification of the climate-weather change process model determining the climate-weather change process parameters for the initial point of the port oil piping transportation system are presented.

### **1. Introduction**

The general model of the climate-weather change processes is proposed in [3] and [12]. The safety models of various multistate complex technical systems are considered in [5]. Consequently, the general joint models linking these system safety models with the model of their climate-weather processes, allowing us for the safety analysis of the complex technical systems at the variable climate-weather conditions, are constructed in [6]. To be able to apply these general models practically in the evaluation and prediction of the reliability and safety of real complex technical systems it is necessary to have the statistical methods concerned with determining the unknown parameters of the proposed models [1]-[2], [7]-[8], [10]-[11], [21]. Particularly,

concerning the climate-weather process, the probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment, the probabilities of the climate-weather process transitions between the climate-weather states and the distributions of the conditional sojourn times of the climate-weather process at the particular climate-weather states should be identified [9], [14]-[15]. It is also necessary to use the methods of testing the hypotheses concerned with the climate-weather process conditional sojourn times at the climate-weather states [14].

### **2. Identification of climate-weather change process**

We assume, as in [12], that the climate-weather

change process for the critical infrastructure operating area is taking  $w$ ,  $w \in N$ , different climate-weather states  $c_1, c_2, \dots, c_w$ . Next, we mark by  $C(t)$ ,  $t \in \langle 0, +\infty \rangle$ , the climate-weather change process, that is a function of a continuous variable  $t$ , taking discrete values in the set  $\{c_1, c_2, \dots, c_w\}$  of the climate-weather states. We assume a semi-Markov model [13]-[19], of the climate-weather change process  $C(t)$  and we mark by  $C_{bl}$  its random conditional sojourn times at the climate-weather states  $c_b$ , when its next climate-weather state is  $c_l$ ,  $b, l = 1, 2, \dots, w, b \neq l$ .

Under these assumptions, the climate-weather change process may be described by the vector  $[q_b(0)]_{1 \times w}$  of probabilities of the climate-weather change process staying at the particular climate-weather states at the initial moment  $t = 0$ , the matrix  $[q_{bl}(t)]_{w \times w}$  of the probabilities of the climate-weather change process transitions between the climate-weather states and the matrix  $[C_{bl}(t)]_{w \times w}$  of the distribution functions of the conditional sojourn times  $C_{bl}$  of the climate-weather change process at the climate-weather states or equivalently by the matrix  $[c_{bl}(t)]_{w \times w}$  of the density functions of the conditional sojourn times  $C_{bl}$ ,  $b, l = 1, 2, \dots, w, b \neq l$ , of the climate-weather change process at the climate-weather states. These all parameters of the climate-weather change process are unknown and before their use to the prognosis of this process characteristics have to be estimated on the basis of statistical data coming from practice.

## 2.1. Defining unknown parameters of climate-weather change process and data collection

To make the estimation of the unknown parameters of the climate-weather change process, the experiment delivering the necessary statistical data should be precisely planned.

First, before the experiment, we should perform the following preliminary steps:

- i) to analyze the climate-weather change process;
- ii) to fix or to define the climate-weather change process following general parameters:
  - the number of the climate-weather states of the climate-weather change process  $w$ ,
  - the climate-weather states of the climate-weather change process  $c_1, c_2, \dots, c_w$ ;
- iii) to fix the possible transitions between the climate-weather states;
- iv) to fix the set of the unknown parameters of the climate-weather change process semi-Markov model.

Next, to estimate the unknown parameters of the climate-weather change process, based on the experiment, we should collect necessary statistical data performing the following steps:

- i) to fix and to collect the following statistical data necessary to evaluate the probabilities  $q_b(0)$  of the climate-weather change process staying at the climate-weather states at the initial moment  $t = 0$ :
  - the duration time of the experiment  $\Theta$ ,
  - the number of the investigated realizations of the climate-weather change process  $n(0)$ ,
  - the vector of the realizations  $q_b(0)$ ,  $b = 1, 2, \dots, w$ , of the numbers of staying of the climate-weather change process respectively at the climate-weather states  $c_1, c_2, \dots, c_w$ , at the initial moments  $t = 0$  of all  $n(0)$  observed realizations of the climate-weather change process

$$[n_b(0)] = [n_1(0), n_2(0), \dots, n_w(0)],$$

where

$$n_1(0) + n_2(0) + \dots + n_w(0) = n(0);$$

- ii) to fix and to collect the following statistical data necessary to evaluate the probabilities  $q_{bl}$  of the climate-weather change process transitions between the climate-weather states:

- the matrix of the realizations of the numbers  $n_{bl}$ ,  $b, l = 1, 2, \dots, w, b \neq l$ , of the transitions of the climate-weather change process from the climate-weather state  $c_b$  into the climate-weather state  $c_l$  at all observed realizations of the climate-weather change process

$$[n_{bl}] = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1w} \\ n_{21} & n_{22} & \dots & n_{2w} \\ \dots & & & \\ n_{w1} & n_{w2} & \dots & n_{ww} \end{bmatrix},$$

where  $n_{bb} = 0$ , for  $b = 1, 2, \dots, w$ ,

- the vector of the realizations of the numbers  $n_b$ ,  $b = 1, 2, \dots, w$ , of departures of the climate-weather change process from the climate-weather states  $c_b$

$$[n_b] = [n_1, n_2, \dots, n_w],$$

where

$$\begin{aligned} n_1 &= n_{11} + n_{12} + \dots + n_{1w}, \\ n_2 &= n_{21} + n_{22} + \dots + n_{2w}, \\ &\dots \\ n_w &= n_{w1} + n_{w2} + \dots + n_{ww}; \end{aligned}$$

iii) to fix and to collect the following statistical data necessary to evaluate the unknown parameters of the distributions  $C_{bl}(t)$  of the conditional sojourn times  $C_{bl}$  of the climate-weather change process at the particular climate-weather states:

- the numbers  $n_{bl}$ ,  $b, l = 1, 2, \dots, w$ ,  $b \neq l$ , of realizations of the conditional sojourn times  $C_{bl}$ ,  $b, l = 1, 2, \dots, w$ ,  $b \neq l$ , of the climate-weather change process at the climate-weather state  $c_b$  when the next transition is to the climate-weather state  $c_l$  during the observation time  $\Theta$ ,
- the realizations  $C_{bl}^k$ ,  $k = 1, 2, \dots, n_{bl}$ , of the conditional sojourn times  $C_{bl}$  of the climate-weather change process at the climate-weather state  $c_b$  when the next transition is to the climate-weather state  $c_l$  during the observation time  $\Theta$  for each  $b, l = 1, 2, \dots, w$ ,  $b \neq l$ .

## 2.2. Estimating basic parameters of climate-weather change process

After collecting the statistical data, it is possible to estimate the unknown parameters of the climate-weather change process performing the following steps:

i) to determine the vector

$$[q(0)] = [q_1(0), q_2(0), \dots, q_w(0)], \quad (1)$$

of the realizations of the probabilities  $q_b(0)$ ,  $b = 1, 2, \dots, w$ , of the climate-weather change process staying at the climate-weather states at the initial moment  $t = 0$ , according to the formula

$$q_b(0) = \frac{n_b(0)}{n(0)}, \text{ for } b = 1, 2, \dots, w, \quad (2)$$

where

$$n(0) = \sum_{b=1}^w n_b(0), \quad (3)$$

is the number of the realizations of the climate-weather change process starting at the initial moment  $t = 0$ ;

ii) to determine the matrix

$$[q_{bl}] = \begin{bmatrix} q_{11} & q_{12} & \dots & q_{1w} \\ q_{21} & q_{22} & \dots & q_{2w} \\ \dots & \dots & \dots & \dots \\ q_{w1} & q_{w2} & \dots & q_{ww} \end{bmatrix}, \quad (4)$$

of the realizations of the probabilities  $q_{bl}$ ,  $b, l = 1, 2, \dots, w$ , of the climate-weather change process transitions from the climate-weather state  $c_b$  to the climate-weather state  $c_l$  according to the formula

$$\begin{aligned} q_{bl} &= \frac{n_{bl}}{n_b}, \text{ for } b, l = 1, 2, \dots, w, b \neq l, \\ q_{bb} &= 0, \text{ for } b = 1, 2, \dots, w, \end{aligned} \quad (5)$$

where

$$n_b = \sum_{b \neq l}^w n_{bl}, \text{ } b = 1, 2, \dots, w, \quad (6)$$

is the realization of the total number of the climate-weather change process departures from the climate-weather state  $c_b$ ,  $b = 1, 2, \dots, w$ , during the experiment time  $\Theta$ .

## 2.3. Estimating parameters of distributions of climate-weather change process conditional sojourn times at climate-weather states

Prior to estimating the parameters of the distributions of the climate-weather change process conditional sojourn times at the particular climate-weather states, we have to determine the following empirical characteristics of the realizations of the conditional sojourn time of the climate-weather change process at the particular climate-weather states:

- the realizations of the empirical mean values  $\bar{C}_{bl}$  of the conditional sojourn times  $C_{bl}$  of the climate-weather change process at the climate-weather state  $c_b$ ,  $b = 1, 2, \dots, w$ , when the next transition is to the climate-weather state  $c_l$ ,  $l = 1, 2, \dots, w$ , according to the formula

$$\bar{C}_{bl} = \frac{1}{n_{bl}} \sum_{k=1}^{n_{bl}} C_{bl}^k, \text{ } b, l = 1, 2, \dots, w, b \neq l, \quad (7)$$

- the number  $\bar{r}_{bl}$  of the disjoint intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots, \bar{r}_{bl}$ , that include the realizations  $C_{bl}^k$ ,  $k = 1, 2, \dots, n_{bl}$ , of the conditional sojourn times  $C_{bl}$  at the climate-weather state  $c_b$

when the next transition is to the climate-weather state  $c_l$ , according to the formula

$$\bar{r}_{bl} \cong \sqrt{n_{bl}},$$

- the length  $d_{bl}$  of the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots, \bar{r}_{bl}$ , according to the formula

$$d_{bl} = \frac{\bar{R}_{bl}}{\bar{r}_{bl} - 1},$$

where  $\bar{R}_{bl} = \max_{1 \leq k \leq n_{bl}} \theta_{bl}^k - \min_{1 \leq k \leq n_{bl}} \theta_{bl}^k$ ,

- the ends  $a_{bl}^j, b_{bl}^j$ , of the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots, \bar{r}_{bl}$ , according to the formulae

$$a_{bl}^1 = \max\left\{ \min_{1 \leq k \leq n_{bl}} \theta_{bl}^k - \frac{d_{bl}}{2}, 0 \right\},$$

$$b_{bl}^j = a_{bl}^1 + j d_{bl}, \quad j = 1, 2, \dots, \bar{r}_{bl},$$

$$a_{bl}^j = b_{bl}^{j-1}, \quad j = 2, 3, \dots, \bar{r}_{bl},$$

in such a way that

$$I_1 \cup I_2 \cup \dots \cup I_{\bar{r}_{bl}} = \langle a_{bl}^1, b_{bl}^{\bar{r}_{bl}} \rangle,$$

$$I_i \cap I_j = \emptyset \text{ for all } i \neq j, \quad i, j \in \{1, 2, \dots, \bar{r}_{bl}\},$$

- the numbers  $n_{bl}^j$  of the realizations  $C_{bl}^k$  in the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots, \bar{r}_{bl}$ , according to the formula

$$n_{bl}^j = \#\{k : C_{bl}^k \in I_j, k \in \{1, 2, \dots, n_{bl}\}\}, \quad j = 1, 2, \dots, \bar{r}_{bl},$$

where  $\sum_{j=1}^{\bar{r}_{bl}} n_{bl}^j = n_{bl}$ ,

whereas the symbol # means the number of elements of the set.

The estimates of the unknown parameters for the distribution functions of the conditional sojourn times of the climate-weather change process at the particular climate-weather states distinguished in [12] are presented in [6], [15].

To formulate and next to verify the non-parametric hypothesis concerning the form of the distribution of the climate-weather change process conditional sojourn time  $C_{bl}$  at the climate-weather state  $c_b$  when

the next transition is to the climate-weather state  $c_l$ , on the basis of at least 30 its realizations  $C_{bl}^k$ ,  $k = 1, 2, \dots, n_{bl}$ , it is due to proceed according to the scheme presented in [4], [15].

### 3. Statistical identification of climate-weather change process for initial point of port oil piping transportation system

#### 3.1. Defining parameters and data collection of climate-weather change process for port oil piping transportation system

The unknown parameters of the climate-weather change process semi-Markov model are:

- the initial probabilities  $q_b(0)$ ,  $b = 1, 2, \dots, 6$ , of the climate-weather change process staying at the particular states  $c_b$  at the moment  $t = 0$ ,

- the probabilities  $q_{bl}$ ,  $b, l = 1, 2, \dots, 6$ ,  $b \neq l$ , of the climate-weather change process transitions from the climate-weather state  $c_b$  into the climate-weather state  $c_l$ ,

- the distributions of the climate-weather change conditional sojourn times  $C_{bl}$ ,  $b, l = 1, 2, \dots, 6$ ,  $b \neq l$ , at the particular climate-weather states and their mean values  $M_{bl} = E[C_{bl}]$ ,  $b, l = 1, 2, \dots, 6$ ,  $b \neq l$ .

To identify all these parameters of the climate-weather change process the statistical data about this process is needed.

The collected by the system operators statistical data necessary to evaluating the initial transient probabilities of the climate-weather change process at the particular states are:

- the climate-weather change process observation / experiment time  $\Theta = 6$  years (1988-1993),

- the number of the climate-weather change process realizations  $n(0) = 170$ ,

- the vector of realizations of the numbers of the climate-weather change process staying at the particular climate-weather state  $c_b$  at the initial moment  $t = 0$

$$[n_b(0)] = [162, 1, 0, 5, 2, 0].$$

The collected statistical data necessary to evaluate the probabilities of transitions of the climate-weather state change process  $C(t)$  between the climate-weather states are:

- the matrix of realizations of the numbers of climate-weather change process transitions from the state  $c_b$  into the state  $c_l$  during the experiment time

$$[n_{bl}] = \begin{bmatrix} 0 & 44 & 0 & 129 & 34 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 13 & 3 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

- the vector of realizations of the total numbers of the climate-weather change process transitions from the climate-weather state  $c_b$  during the experiment time

$$[n_b] = [207, 8, 0, 23, 9, 0].$$

The statistical data for the conditional sojourn times  $C_{bl}$  at the climate-weather states  $c_b$  when the next climate-weather state is  $c_l$ ,  $b, l = 1, 2, \dots, 6$ ,  $b \neq l$ , are as follows:

- the realizations  $C_{12}$ : 588, 564, 540, 516, 492, 468, 444, 420, 396, 372, 348, 324, 300, 276, 252, 228, 204, 180, 156, 132, 108, 84, 60, 36, 12, 3, 3, 288, 285, 237, 213, 189, 165, 141, 117, 93, 69, 45, 21, 3, 3, 3, 1212, 1206;
- the realizations  $C_{14}$ : 1071, 1050, 1026, 1002, 474, 456, 432, 408, 384, 360, 336, 312, 288, 264, 240, 216, 192, 168, 144, 120, 96, 72, 48, 24, 3, 12, 6, 210, 207, 183, 159, 135, 111, 87, 63, 39, 15, 882, 876, 852, 828, 804, 780, 756, 732, 708, 684, 660, 636, 612, 588, 198, 177, 153, 129, 105, 81, 57, 33, 9, 75, 63, 39, 15, 378, 360, 336, 312, 288, 264, 240, 216, 192, 168, 144, 120, 96, 72, 48, 24, 27, 9, 3, 3, 87, 873, 867, 843, 819, 795, 771, 747, 723, 699, 675, 651, 627, 603, 579, 555, 531, 507, 483, 459, 435, 411, 387, 363, 339, 315, 291, 267, 243, 219, 195, 171, 147, 123, 99, 75, 51, 27, 3, 183, 165, 141, 117, 93, 69;
- the realizations  $C_{15}$ : 816, 795, 771, 747, 723, 699, 675, 651, 627, 603, 579, 555, 531, 507, 483, 459, 435, 411, 387, 363, 339, 315, 291, 267, 243, 219, 195, 171, 147, 123, 99, 75, 51, 27;
- the realizations  $C_{21}$ : 3, 3, 3, 3, 3, 3, 3, 3;
- the realizations  $C_{41}$ : 9, 6, 3, 6, 3, 3, 6, 3, 6, 3, 15, 9, 9;
- the realizations  $C_{42}$ : 3;
- the realizations  $C_{45}$ : 9, 9, 3, 3, 12, 3;
- the realizations  $C_{54}$ : 6, 3, 3, 18, 6, 3, 3, 3, 3.

### 3.2. Evaluating basic parameters of climate-weather change process for port oil piping transportation system

On the basis of the statistical data from Section 3.1, it is possible to evaluate the following unknown basic parameters of the climate-weather change process:

- the vector

$$[q_b(0)] = [0.953, 0.006, 0, 0.029, 0.012, 0],$$

of the initial probabilities  $q_b(0)$ ,  $b = 1, 2, \dots, 6$ , of the climate-weather change process staying at the particular states  $c_b$  at the  $t = 0$ ,

- the matrix

$$[q_{bl}] = \begin{bmatrix} 0 & 0.21 & 0 & 0.62 & 0.16 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.57 & 0.13 & 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

of the probabilities  $q_{bl}$ ,  $b, l = 1, 2, \dots, 6$ , of transitions of the climate-weather change process from the climate-weather state  $c_b$  into the climate-weather state  $c_l$ .

### 3.3. Evaluating parameters of distributions of climate-weather change process for port oil piping transportation system

On the basis of the statistical data partly presented in Section 3.1 using the procedure and the formulae given in Section 2.3, it is possible to determine the empirical parameters of the conditional sojourn times of the climate-weather change process at the particular climate-weather states. To illustrate the application of this procedure and these formulae, we perform it for the conditional sojourn time  $C_{12}$  and the results are:

- the realization  $\bar{C}_{12}$  of the defined by (7) mean value of the conditional sojourn time  $C_{12}$  of the climate-weather change process at the climate-weather state  $c_1$  when the next transition is to the climate-weather state  $c_2$

$$\bar{C}_{12} = \frac{1}{44} \sum_{k=1}^{44} C_{12}^k = \frac{11796}{44} \cong 268.09,$$

- the number  $\bar{r}_{12}$  of the disjoint intervals  $I_j = \langle a_{12}^j, b_{12}^j \rangle$ ,  $j=1,2,\dots,\bar{r}_{12}$ , that include the realizations  $C_{12}^k$ ,  $k=1,2,\dots,44$ , of the conditional sojourn times  $C_{12}$  at the climate-weather state  $c_1$  when the next transition is to the climate-weather state  $c_2$

$$\bar{r}_{12} \cong \sqrt{44} \cong 7,$$

- the length  $d_{12}$  of the intervals  $I_j = \langle a_{12}^j, b_{12}^j \rangle$ ,  $j=1,2,\dots,7$ , that after considering

$$\bar{R}_{12} = \max_{1 \leq k \leq 44} C_{12}^k - \min_{1 \leq k \leq 44} C_{12}^k = 1212 - 3 = 1209,$$

is

$$d_{12} = \frac{\bar{R}_{12}}{\bar{r}_{12} - 1} = \frac{1209}{6} = 201.50,$$

- the ends  $a_{12}^j$ ,  $b_{12}^j$ , of the intervals  $I_j = \langle a_{12}^j, b_{12}^j \rangle$ ,  $j=1,2,\dots,7$ , that after considering

$$\min_{1 \leq k \leq 44} C_{12}^k - \frac{d_{12}}{2} = 3 - \frac{201.50}{2} = -97.75,$$

are

$$\begin{aligned} a_{12}^1 &= \max\{-97.75, 0\} = 0, \\ b_{12}^1 &= a_{12}^1 + 201.5 = 0 + 201.5 = 201.5, \\ a_{12}^2 &= b_{12}^1 = 201.5, \\ b_{12}^2 &= a_{12}^1 + 2 \cdot 201.5 = 0 + 403 = 403.0, \\ a_{12}^3 &= b_{12}^2 = 403.0, \\ b_{12}^3 &= a_{12}^1 + 3 \cdot 201.5 = 0 + 604.5 = 604.5, \\ a_{12}^4 &= b_{12}^3 = 604.5, \\ b_{12}^4 &= a_{12}^1 + 4 \cdot 201.5 = 0 + 806 = 806.0, \\ a_{12}^5 &= b_{12}^4 = 806.0, \\ b_{12}^5 &= a_{12}^1 + 5 \cdot 201.5 = 0 + 1007.5 = 1007.5, \\ a_{12}^6 &= b_{12}^5 = 1007.5, \\ b_{12}^6 &= a_{12}^1 + 6 \cdot 201.5 = 0 + 1209.0 = 1209.0, \\ a_{12}^7 &= b_{12}^6 = 1209.0, \\ b_{12}^7 &= a_{12}^1 + 7 \cdot 201.5 = 0 + 1410.5 = 1410.5, \end{aligned}$$

- the numbers  $n_{12}^j$  of the realizations  $C_{12}^k$  in particular intervals  $I_j = \langle a_{12}^j, b_{12}^j \rangle$ ,  $j=1,2,\dots,7$ ,

$$\begin{aligned} n_{12}^1 &= 21, \quad n_{12}^2 = 13, \quad n_{12}^3 = 8, \quad n_{12}^4 = 0, \quad n_{12}^5 = 0, \\ n_{12}^6 &= 1, \quad n_{12}^7 = 1. \end{aligned}$$

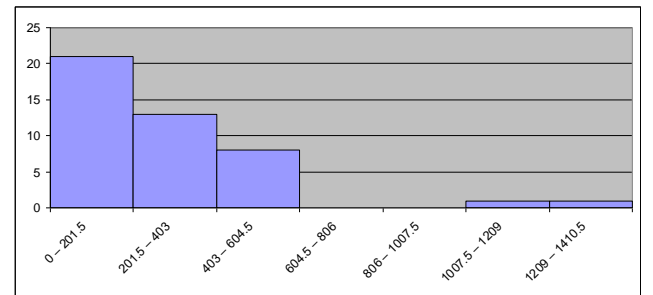
### 3.4. Identification of distribution functions of climate-weather change process for initial point of port oil piping transportation system

Using the procedure given in [4], [15] and the statistical data from Section 3.1 and the results from Section 3.3, we may verify the hypotheses on the distributions of the climate-weather change process conditional sojourn times  $C_{bl}$ ,  $b,l=1,2,\dots,6$ ,  $b \neq l$ , at the particular states. To do this, we need a sufficient number of realizations of these variables [2], [8], [20], [22]-[23], namely, the sets of their realizations should contain at least 30 realizations coming from the experiment. This condition is not satisfied for the statistical data we have in disposal and that are presented in Section 3.1. However, to make the procedure familiar to the reader, we perform it for the conditional sojourn time  $C_{12}$  having sufficiently numerous set of realizations and preliminarily analyzed in Section 3.3.

The realization  $\bar{c}_{12}(t)$  of the histogram of the climate-weather change process conditional sojourn time  $C_{12}$ , is presented in *Table 1* and illustrated in *Figure 1*.

*Table 1.* The realization of the histogram of the climate-weather change process conditional sojourn time  $C_{12}$

Histogram of the conditional sojourn time $C_{12}$							
$I_j = \langle a_{12}^j, b_{12}^j \rangle$	0 – 201.5	201.5 – 403.0	403.0 – 604.5	604.5 – 806.0	806.0 – 1007.5	1007.5 – 1209.0	1209.0 – 1410.5
$n_{12}^j$	21	13	8	0	0	1	1
$\bar{h}_{12}(t) = n_{12}^j / n_{12}$	21/44	13/44	8/44	0/44	0/44	1/44	1/44



*Figure 1.* The graph of the histogram of the climate-weather change process conditional sojourn time  $C_{12}$

After analyzing and comparing the realization  $\bar{c}_{12}(t)$  of the histogram with the graphs of the density functions  $c_{bl}(t)$  of the previously distinguished in [3] distributions, we formulate the null hypothesis  $H_0$  in the following form:

$H_0$ : The climate-weather change process conditional sojourn time  $C_{12}$  at the climate-weather state  $c_1$  when the next transition is to the climate-weather state  $c_2$ , has the exponential distribution with the density function defined by (4.5) in [3] of the form

$$c_{12}(t) = \begin{cases} 0, & t < x_{12} \\ \alpha_{12} \exp[-\alpha_{12}(t - x_{12})], & t \geq x_{12}. \end{cases} \quad (8)$$

We estimate the unknown parameters of the density function of the hypothetical exponential distribution using the formulae (2.13) in [4] we obtain the following results

$$x_{12} = a_{12}^1 = 0, \\ \alpha_{12} = \frac{1}{\bar{C}_{12} - x_{12}} = \frac{1}{268.09 - 0} \cong 0.0037.$$

Next, we join the intervals defined in the realization of the histogram  $\bar{h}_{12}(t)$  that have the numbers  $n_{12}^j$ , of realizations less than 4 into new intervals and we perform the following steps:

- we fix the new number of intervals  $\bar{r}_{12} = 3$ ,
- we determine the new intervals and we fix the numbers of realizations in the new intervals

Table 2. The numbers of the conditional sojourn time  $C_{12}$  realizations in the intervals  $\bar{I}_j$

$\bar{I}_j = \langle a_{12}^j, b_{12}^j \rangle$	0 – 201.5	201.5 – 403.5	403.5 – $+\infty$
$\frac{\bar{r}_j}{n_{12}}$	21	13	10

- we calculate the hypothetical probabilities that the variable  $C_{12}$  takes values from the new intervals

$$p_1 = P(C_{12} \in \bar{I}_1) = P(0 \leq C_{12} < 201.5) \\ = C_{12}(201.5) - C_{12}(0) = 0.53 - 0 = 0.53, \\ p_2 = P(C_{12} \in \bar{I}_2) = P(201.5 \leq C_{12} < 403.0) \\ = C_{12}(403.0) - C_{12}(201.5) = 0.77 - 0.53 \\ = 0.24, \\ p_3 = P(C_{12} \in \bar{I}_3) = P(403.0 \leq C_{12} < +\infty) \\ = C_{12}(+\infty) - C_{12}(403.0) = 1 - 0.77 = 0.23,$$

- we calculate the realization of the  $\chi^2$  (chi-square)-Pearson's statistics

$$u_{12} = \sum_{j=1}^3 \frac{(\bar{n}_{12}^j - n_{12} p_j)^2}{n_{12} p_j} = \frac{(21 - 44 \cdot 0.53)^2}{44 \cdot 0.53} \\ + \frac{(13 - 44 \cdot 0.24)^2}{44 \cdot 0.24} + \frac{(10 - 44 \cdot 0.23)^2}{44 \cdot 0.23} \\ \cong 0.2308 + 0.5638 + 0.0014 = 0.7960 \cong 0.80,$$

- we assume the significance level  $\alpha = 0.05$ ,
- we fix the number of degrees of freedom

$$\bar{r}_{12} - l - 1 = 3 - 1 - 1 = 1,$$

- we read from the tables of the  $\chi^2$ -Pearson's distribution the value  $u_\alpha$  for the fixed values of the significance level  $\alpha = 0.05$  and the number of degrees of freedom  $\bar{r}_{12} - l - 1 = 1$ , such that, according to (2.50), the following equality holds

$$P(U_{12} > u_\alpha) = \alpha = 0.05$$

that amounts  $u_\alpha = 3.84$  and we determine the critical domain in the form of the interval  $(3.84, +\infty)$  and the acceptance domain in the form of the interval  $\langle 0, 3.84 \rangle$ .

- we compare the obtained value  $u_{12} = 0.80$  of the realization of the statistics  $U_{12}$  with the read from the tables critical value  $u_\alpha = 3.84$  of the chi-square random variable and since the value  $u_{12} = 0.80$  does not belong to the critical domain, i.e.

$$u_{12} = 0.80 \leq u_\alpha = 3.84,$$

then we do not reject the hypothesis  $H_0$ , that the sojourn time  $C_{12}$  has the exponential distribution with the density function given by (8).

For the remaining cases, when the realizations of conditional sojourn times  $C_{bl}$ ,  $b, l = 1, 2, \dots, 6$ ,  $b \neq l$ , at the particular climate-weather states are more than 30, proceeding afterwards in an analogous way as in the case of the conditional sojourn time  $C_{12}$ , we can get the following results:

- the climate-weather change process conditional sojourn time  $C_{14}$  has the exponential distribution with the density function defined by (4.5) in [3] of the form

$$c_{14}(t) = \begin{cases} 0, & t < x_{14} \\ \alpha_{14} \exp[-\alpha_{14}(t - x_{14})], & t \geq x_{14}. \end{cases}$$

with the parameters

$$x_{14} = a_{14}^1 = 0,$$

$$\alpha_{14} = \frac{1}{\bar{C}_{14} - x_{14}} = \frac{1}{333.86 - 0} \cong 0.0030.$$

- the climate-weather change process conditional sojourn time  $C_{15}$  has the uniform distribution with the density function defined by (4.1) in [3] of the form

$$c_{15}(t) = \begin{cases} 0, & t < x_{15} \\ \frac{1}{y_{15} - x_{15}} & x_{15} \leq t \leq y_{15} \\ 0, & t > y_{15}, \end{cases}$$

where  $0 \leq x_{15} < y_{15} < +\infty$ ,

with the parameters:

$$x_{15} = a_{15}^1 = 0,$$

$$y_{15} = x_{15} + \bar{r}_{15}d_{15} = 0 + 6 \cdot 157.8 = 946.8.$$

For the distributions identified in this section, by application either the general formulae for the mean value given by (2.12) or the particular formulae (2.13)-(2.19) in [15], the mean values  $M_{bl} = E[\theta_{bl}]$ ,  $b, l = 1, 2, \dots, 6$ ,  $b \neq l$ , of the port oil piping transportation climate-weather change process conditional sojourn times at the particular climate-weather states can be determined and they amount:

$$M_{12} \cong 270.27, M_{14} \cong 333.33, M_{15} \cong 493.04.$$

Because of the lack of sufficient numbers of realizations of the climate-weather change process conditional sojourn times at the climate-weather states, it is not possible to identify statistically their distributions. In those cases of not identified distributions it is possible to find the approximate empirical values of the mean values  $M_{bl} = E[C_{bl}]$  of the conditional sojourn times at the particular climate-weather states that are as follows:

$$M_{21} = 3.00, M_{41} \cong 6.23, M_{42} = 3.00, M_{45} = 6.00,$$

$$M_{54} \cong 5.33.$$

As there are no realizations of the port oil piping transportation climate-weather change process conditional sojourn times at the climate-weather states

$$C_{13}, C_{16}, C_{23}, C_{24}, C_{25}, C_{26}, C_{31}, C_{32}, C_{34}, C_{35}, C_{36},$$

$$C_{43}, C_{46}, C_{51}, C_{52}, C_{53}, C_{56}, C_{61}, C_{62}, C_{63}, C_{64}, C_{65},$$

then it is impossible to estimate their empirical conditional mean values

$$M_{13}, M_{16}, M_{23}, M_{24}, M_{25}, M_{26}, M_{31}, M_{32}, M_{34}, M_{35},$$

$$M_{36}, M_{43}, M_{46}, M_{51}, M_{52}, M_{53}, M_{56}, M_{61}, M_{62}, M_{63},$$

$$M_{64}, M_{65}.$$

## 4. Conclusions

The proposed statistical methods of identification of the unknown parameters of the climate-weather change processes allow us for the identification of the models discussed in [6] and next their practical applications in evaluation, prediction and optimization of reliability, availability and safety of real complex critical infrastructures.

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