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# A repair time model of a web based system including administrator working hours

### Keywords

reliability, web based system, repair time, working hours

### Abstract

The paper presents a web based system reliability analysis. We propose to model different types of faults (hardware one, software and security incidents) taking consideration only the effect of the failure, not the source of the fault. It is assumed that failure events are independent and the time to failure is exponential. Whereas the time to repair is not exponential since repair actions are taken only when administrators are at work. We assume that administrators are not working 24/7. The paper presents an algorithmic model of the repair time including administrator working hours. The model is used to estimate reliability parameters (mean time, standard deviation and 90$^{th}$ percentile of yearly down time) by a use of Monte-Carlo simulation. The numerical results are compared with results from the analytical model (Markov one) that assumes exponential distribution of all repair times. Results for two web exemplar web systems (with reliability model consisting of two and five states) show the range of error caused by the exponential distribution assumption.

## 1. Introduction

Web based systems are widely used in modern society. One of the most significant problems that web site providers have to face is how they can provide the QoS (quality-of-service) required by their clients. Therefore, it is important to have a realistic model of web system reliability that could allow calculating values of QoS parameters, like yearly down time or its guaranteed/maximum value.
Human administrators manage the web systems. They are needed to perform the repair process, but their work time is limited. For small and middle size companies administrators are not working in the three-shift system (24/7). Therefore, there are periods when no administrator is at work. During these time periods no repair could be performed [6]-[7]. Therefore, we propose a realistic model of a repair time that takes into consideration working hours of administrators. Whereas due to the popularity of the Markov approach the most common assumption done by researches is a usage of exponential distribution for system component repair times [1]. In fewer cases, researches are using other distributions [5] or compose the repair time of two periods: waiting and real repair [9].
The paper is structured as follows. In the next section, we present a fault model of Web based systems. It is followed by a detail model of realistic repair time is presented with a set of figures illustrating its distribution. Next, we analyse a simple two state system and compare results achieved from proposed model with the most widely used Markov approach [2]. In section 5, a multi-state reliability model of web system is presented and an error of Markov approximation for set of reliability statistics is analyzed.

## 2. Fault model

There are different sources of faults in web-based systems. These includes hardware malfunctions (like faults of computer equipment, network devices, power down or failures of internet connection), software bugs, exploitation of software vulnerabilities, malware proliferation, drainage type attacks on system and its infrastructure (such as DDOS). We propose to analyze faults not based on their primary source, but on the effect, they have on the system [3]. Therefore, we analyze only faults that results in system failure, i.e. situation when the system is producing no output or producing incorrect outputs [8].
The faults can affect either a host or only a service running on it. However, the effect is almost the same. Since in case of a host failure the host cannot

process services that are located on it. These in turn do not produce any responses to queries from the services located on other hosts and the system is not responding.

We assume that an administrator maintains the web based system. In addition, he or she is responsible for maintaining the continuity of business services. It includes reaction to failures and such actions as restring software/hardware components, isolation of the affected hardware and software (to prevent propagation of the problem to yet unaffected parts of the system), reinstallation of software and ordering/replacing new hardware components.

Therefore, the web system from the reliability point of view is a repairable system. Faults occur in the system randomly, usually with a predictable distribution. Then the system for some time becomes inoperational until maintenance procedures start. We assume that host/service faults are independent and that time to failure could be model by exponential distribution. However, in case of repair time we will analyze more complex model presented in the next section.

## 3. Repair time model

### 3.1 Real repair time

Let us name an administrator work time required to restore the web system to operation as a real repair time (*rrt*). Its value depends on the fault type and is a random value with a predictable distribution.

Administrators work for a given time during a day (defined by work start and finish hour) and only in given days (working days) within a week. Therafore the repair time (*rt*, time from failure occurrence to restoring the system to operation) includes not only the work time of the administrator but also a time when the administrator is not working (weekends, nights). The relation between the real repair time and the time between failure occurrence and system recovery is illustrated in *Figure 1*.

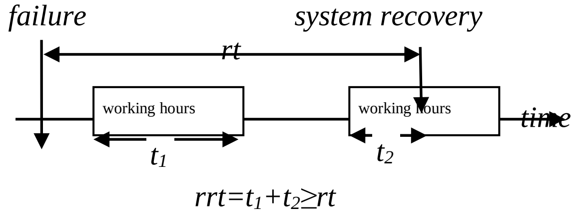


*Figure 1*. Relation between real repair time (*rrt*) and repair time (*rt*)

Since, working hours depends on a time therefore the real repair time is a function of two random values:

- $t_f$ – time when failure occurs
- *rrt* – work time required to repair the system, i.e.:

$$rt\,(t_f, rrt). \qquad (1)$$

### 3.2. Repair time function

The value of function (1) is defined by following algorithm:

Input: $t_f$, *rrt*
Steps:

1. $t = t_f$; $rt=0$;
2. while $rrt>0$
   i. if not *workinghours*($t$) then $t$=*nextworkingday*($t$)
   ii. $t_e$=*endofworkingday*($t$)
   iii. *delta*=min($t_e$-$t$,*rrt*)
   iv. *rrt*=*rrt*-*delta*
   v. $t$=$t$+*delta*
3. *return* $t-t_f$

Where:

- *workinghours*($t$) – function that returns true if time $t$ is within administrator working hours
- *nextworkinday*($t$) – function that returns start of next working period
- *endofworkingday*($t$) – function that returns the time of when working day finishes (where $t$ is a time moment during working hours).

During numerical experiments, the results of which are presented within this paper, we have assumed that working hours of administrator are 8 am to 16 pm, Monday to Friday.

### 3.3. Repair time distribution

To analyze the properties of repair time defined by algorithm from the previous section we have performed a set of numerical experiments using Monte-Carlo simulation [4]. For a set of pseudo-randomly generated values of $t_f$ and *rrt*, we have calculated values of function (1) using the proposed algorithm.

Assumed working hours are periodic over a week, so values of $t_f$ were generated using uniform distribution (with a week duration). For preliminary experiments, we have assumed that real repair time is exponential.

Histograms of the repair time for different values of the mean real repair time are presented in *Figure 2*. It could be noticed that distribution shape changes in a function of mean real repair time. However, the relation between the mean of repair time and the mean of real repair time is almost linear (*Figure 3*).

The repair time is almost five times larger than the real repair time.

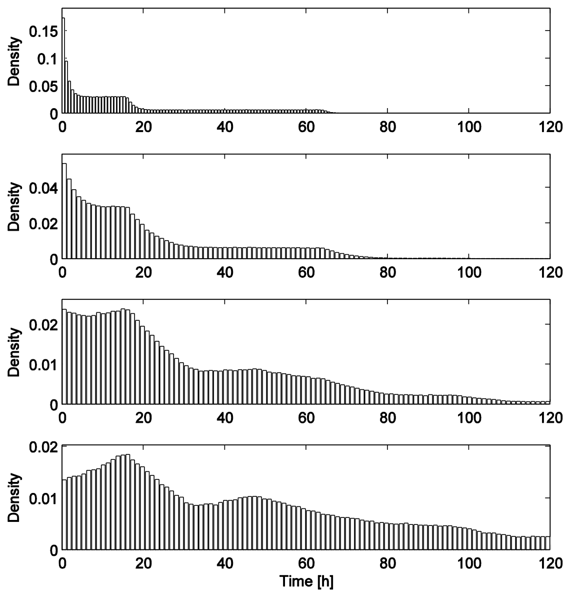


*Figure* 2. Histograms of repair time for different values of the mean real repair time (0.5, 2, 5 and 10h)

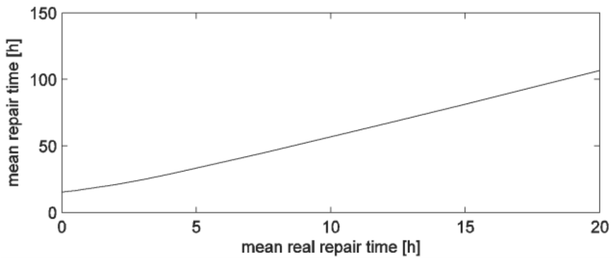


*Figure* 3. The mean repair time in function of the mean real repair time

## 4. Two-state system

### 4.1 Reliability parameters

Let us analyze a simple system consisting of one host. We propose to model host and service failures by the same process. It gives a simple two reliability state system. The system could be in an operating or failure state. The repair time is modeled as described in section 3. In presented numerical results, we have assumed that the intensity of failures is equal to two per year. Let us analyze two random variables:

- time to repair (RT) - a time from failure occurrence to the system recovery;
- yearly down time (DT) - sum of the RT during a year;

and their statistics:

- mean value (marked as *mrt* and *mdt* respectively),
- standard deviation (marked as *srt* and *sdt*),
- $90^{th}$ percentile (marked as *90prt* and *90pdt* respectively) i.e.:

$$P(\mathrm{RT}>90prt) = 0.9 \qquad (2)$$

calculated over a period of two years.

Web based systems has a respectively short live time, not more than several years. Moreover systems are upgraded quite often what causes changes in a system structure and functional/reliability parameters. That is why we are not analyzing the system in the stationary state (even so in case of Markov model, it could be very close to the stationary state) but we have assumed a two-year period of analysis. The $90^{th}$ percentile could be understood as a guaranteed (with 0.9 probability), maximum failure time (*90prt*) or yearly down time (*90pdt*). Such statistics could be used for economic decisions concerning a web system management (for example for service level agreement definition).

### 4.2 Markov model approximation

As earlier mentioned we analyze the level of inaccuracy caused by Markov model assumptions. For reason of clarity, we will use name realistic for repair time model presented in section 3 and approximated for model assuming Markov properties [2].

Since one host system is a simple two state system, only two parameters, the failure and repair rate, are required to define Markov model. The failure rate for Markov model is the same as for realistic model from section 3 (since the realistic failure model assumes exponential distribution of time to failure and independence of failures). In case of the repair time, we have to calculate the rate as an inverse of mean repair time defined by function (1). Since we do not know the analytical formula for the mean time of repair time distribution, we used results presented in *Figure 3*.

Next, we performed a set of numerical experiments and calculated standard deviation and $90^{th}$ percentile of time to repair for realistic model and Markov approximation. Their results are presented in *Figure 5* and *Figure 6*. It could be noticed that the standard deviation of time to repair in approximated model, for real repair times smaller than 1 h, is smaller than the value for realistic model, and is higher for values larger than 1h.

The dependence is similar for $90^{th}$ percentile however; the change point is now around 3h (of mean real repair time).

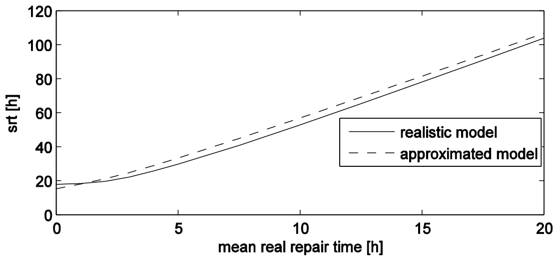


*Figure 5*. Standard deviation of time to repair for real value (solid line) and Markov approximation (dashed line)

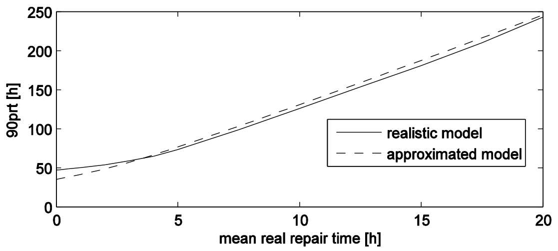


*Figure 6*. Standard deviation of time to repair for real value (solid line) and Markov approximation (dashed line)

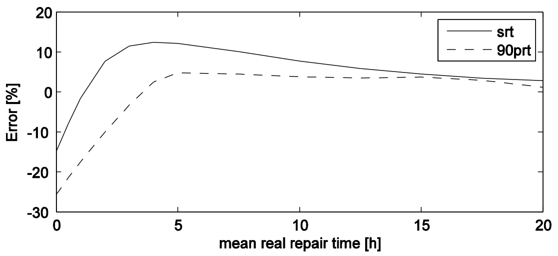


*Figure 7*. Difference between standard deviation Mean repair time in function of real repair time for exponential, Gaussian and hyperexponential model of real repair time

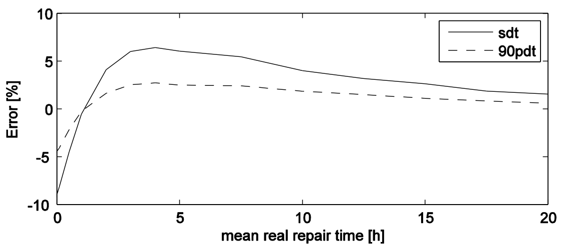


*Figure 8*. Error of standard deviation approximation of repair time by exponential distribution

## 4.3 Comparative analysis

To compare results of reliability parameters mentioned in section 4.1 for realistic model and approximated one we propose to use a relative error, defined as:

$$Error = \left(x_e - x_r\right)/x_r \cdot 100\% \tag{3}$$

where $x_r$ is the value of a statistic achieved for the realistic model (presented in 3), and $x_e$ is the value for Markov model (assuming exponential distributions).

Results for time to repair are presented in *Figure 7*. It could be noticed that for small values of real repair time the exponential model gives smaller values of standard deviation and the 90$^{th}$ percentile of repair time then realistic model. The largest value (in amplitude) of error (-25%) is for minimal real repair time. *Figure 7* is not showing the error for mean failure time since it is equal to zero (both random values has the same mean from the assumption).

*Figure 8* presents errors values between both models for yearly down time. Similarly to results from *Figure 7*, the error of standard deviation and 90$^{th}$ percentile differs (from around -9% to 7%) in a function of the mean real repair time. The mean value statistic is not shown since the archived approximation error was very close to the numerical error of the mean value calculation. The statistics for realistic model were calculated using Monte-Carlo approach. In addition, the number of algorithm repetition was set up to achieve results with numerical error smaller than 0.1%.

## 4.4 Non-exponential real repair time

The real repair time analyzed so far was assumed to be driven by exponential distribution (section 3.3). We would like to check the influence of distribution dispersion, measured by the coefficient of variation CV), on the repair time distribution. For experiments, we have chosen two other distributions with coefficient of variation different from one (as it is for exponential distribution). One is truncated Gaussian distribution with CV = 0.1 and other is hyperexponential distribution with CV = 3.1. As it could be noticed in *Figure 9*, the mean of resulting failure time distribution slightly differ for assumed three real time distribution.

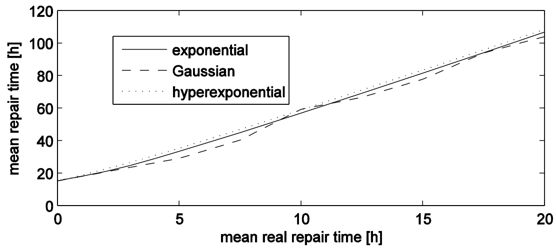


*Figure 9*. Mean repair time in function of real repair time for exponential, Gaussian and hyperexponential model of real repair time

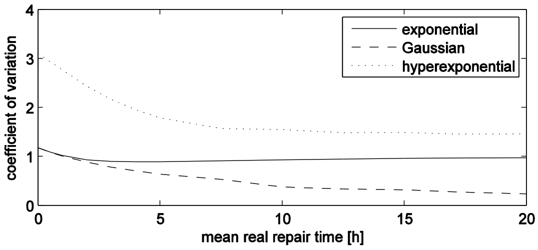


*Figure 10*. Coefficient of variation of repair time in a function of real repair time for exponential, Gaussian and hyperexponential model of real repair time

As it could expected the coefficient of variation of real repair time influence the coefficient of variance of resulting repair time (*Figure 10*). Moreover, the coefficient of variance changes in function of mean real repair time even so it is constant for real repair time.

## 4.5 Comparative analysis for non-exponential real repair time

Following the idea presented in 4.3, we have calculated the approximation error (3) for Gaussian and hyperexponential model of real repair time. The results for statistics of repair time are presented in *Figure 11* and *Figure 13*, whereas statistics for down time in *Figure 12* and *Figure 14*. The largest approximation error in analyzed range of mean repair time (0-20h) is for standard deviation of real repair time, achieving more than 300%. For hyperexponential real repair time the approximation error is in the range from -58% to 50%. The results shows that even for simple two state system the exponential approximation of repair time gives value of guaranteed (with 0.9 probability) yearly down time larger than 90% or smaller than 20% depending on distribution of real repair time. It shows that the usage of realistic model presented in chapter 3 is important in reliability analysis of web system with administrators that works within time limits.

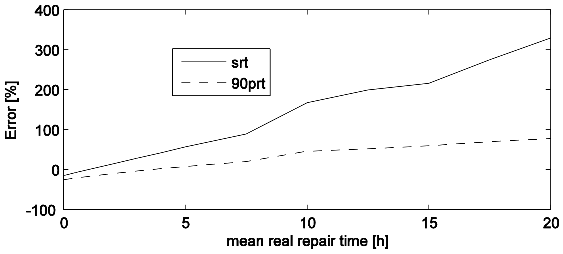


*Figure 11*. Standard deviation and 90th percentile of repair time approximation error for Gaussian model of real repair time

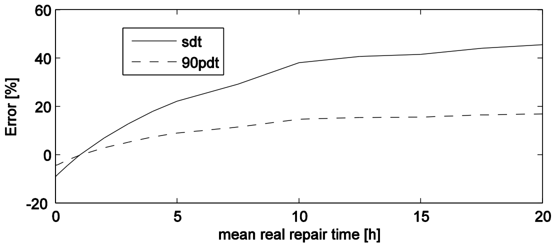


*Figure 12*. Standard deviation and 90th percentile of down time approximation error for Gaussian real repair time

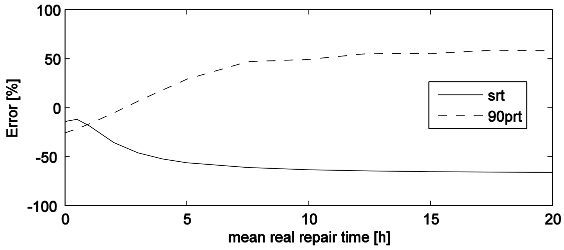


*Figure 13*. Standard deviation and 90th percentile of repair time approximation error for hyperexponential real repair time

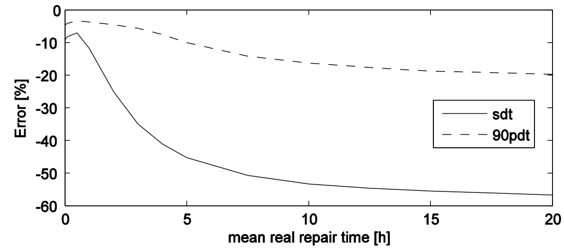


*Figure 14*. Standard deviation and 90th percentile of down time approximation error for hyperexponential real repair time

Next, we would like to analyse the approximation error for a multistate system.

## 5. Multi-state system

### 5.1. Reliability structure

Let us analyze reliability structure of exemplar web system. Following reliability states were defined:

- normal operation ($S_0$),
- power down ($S_1$) – modern system has UPS but they have a limited capacity so there is a probability that a cut off of electricity will cause the system down; the intensity of power downs is marked by $\lambda_P$, the repair intensity is marked by $\mu_P$, power repair is not performed by web system administrator with limited working hours so it will be modeled by exponential distribution,

- after power down ($S_2$) – the system is inoperational till some administrators actions will be taken, intensities of repair is equal to: $\delta_S$ ,
- failure of software resulting in system inoperation ($S_3$) – with failure intensities $\lambda_S$ and intensities of repair: $\delta_S$ ,
- hardware failure ($S_4$) – the hardware failure results in system inoperation, it occurs with intensity: $\lambda_H$ ; after the failure administrator actions are performed (intensity of repairs $\delta_H$ ).

All repair processes with intensities marked by $\delta$ are driven by model described in section 3, whereas all other transitions between reliability states are exponential. The S-T model is presented in *Figure 15*.

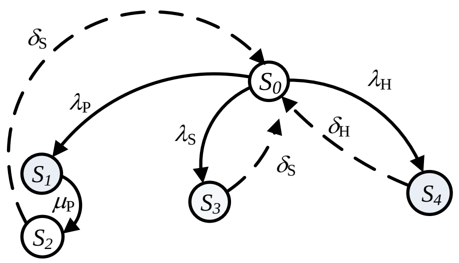


*Figure 15*. The S-T model of exemplar web based system

## 5.2 Numerical experiment results

We have assumed following values of reliability parameters:

- power failure intensity $\lambda_P$ = two failures per year,
- power down repair intensity $\mu_P$ = one per 4 h,
- hardware failure intensity $\lambda_H$ = one failure per 2 years,
- software failure intensity $\lambda_S$ = four failures per year,
- intensity of software repair $\delta_S$ = one per 2 h,
- hardware repair intensity $\delta_H$ = one per 6 h.

The reliability statistics proposed in 4.1 were calculated for realistic model with exponential, Gaussian and hyperexponential model of real repair time. The system repair time is defined as time when system is not in state $S_0$.

Next, Markov model was used to calculate the same statistics, where intensity of repairs marked by $\delta$ were assigned using relation presented in *Figure 9* (mean of real repair time was converted to mean of repair time).

The values of relative error between realistic model and approximated one are given in *Table 1* and *Table 2*.

*Table 1*. Approximation error of system repair time for different models of real repair time

| Model of real repair time | *mrt* | *srt* | *90prt* |
|---|---|---|---|
| Exponential | -11% | -14% | 6% |
| Gaussian | -11% | -19% | 6% |
| hyperexponential | -44% | 1% | -30% |

*Table 2*. Approximation error of system down time for different models of real repair time

| Model of real repair time | *mdt* | *sdt* | *90pdt* |
|---|---|---|---|
| exponential | -11% | -13% | -11% |
| Gaussian | -11% | 15% | -12% |
| hyperexponential | -43% | 10% | -36% |

The results shows that the largest error in the basic statistic, mean yearly down time (*mdt*), could be as high as 43%.

## 6. Conclusion

The paper presents a realistic model of repair time. The main assumptions of the model are that repair could be taken only when administrators are working, and that working time is limited to some ranges (8am-4pm, Monday-Friday). The presented model is not described by mathematical equations but could be solved using Monte-Carlo simulation.

Moreover, we have analyzed two exemplar web systems described by the S-T model. One with two states and one with five states. We have calculated reliability statistics such as the mean value, standard deviation an the 90$^{th}$ percentile for repair and yearly down time for three different models of real repair time (the working time of administrator) distributions. Obtained results were compared with results obtained by Markov model showing that results differs very much what could justify usage of the proposed method.

The repair model contains some arbitrary assumptions, such as working hours of administrator. So the results could not be easily generalized as it is in case of analytical methods. However, the proposed method allows changing the assumptions and thus different maintenance scenarios could be simulated and compared. However, it could require changes in a source code of the simulator.

The presented method and developed tool allow to explore the impact of changes in the system maintenance (such as number of shifts or working hours of administrator) on the web application. This is why the proposed solution may become the essential tool for operators of web systems.

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