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Critical infrastructure preparedness: cascading of disruptions considering vulnerability and dependency

Keywords

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Abstract

Critical Infrastructures' disruptions may result in crises of unacceptable outcomes in modern societies. Thus, it is important to develop models that allow describing CIs' disruptions and their propagation characteristics. CI disruptions depend on both the type of the threat and on the nature of the CIs' mutual dependencies. A model describing the cascade of disruptions should, then, be able to consider the CI-threat vulnerability and the CI-CI dependency. The paper presents a model where cascades are exactly described using an integral equation. The integral equation admits an analytical solution if the occurrence probability distribution functions (pdf) of the disruptions obey Stochastic Poisson Processes (SPP). The introduction of the "vulnerability to the threat" and the "CIs' (inter)dependencies" is carried out with the help of time constant factors called: "vulnerability strain factor" and "disruption strain factor", respectively. An academic case is presented in order to demonstrate the applicability of the model and illustrate some interesting features of the model. A complete set of numerical applications will be published separately.

1. Introduction

A succession of disruptions can be treated as a sequence of some ordered events. Ordered events analyses are frequently met in system safety and reliability analyses under the title "sequence analysis". Analysts may use "Event trees", "Dynamic Fault Trees" with "Priority Gates", "Markov Graphs" or "Monte-Carlo Simulation" tools in order to deal with the dynamic aspect of this problem. The problem is also known as cascading of failures modelling.

Whatever is the method used to describe the "sequence" of events, one would often like to determine occurrence probabilities. It is also of great interest to determine occurrence probability densities and occurrence rates. Other probabilistic quantities can also be of interest depending on the case.

It has been demonstrated that there exist an analytical solution to this problem when the involved disruptions are independent and if they follow Stochastic Poisson Process (SPP) [1], [4].

However, in the case of a cascade of disruptions, one should consider the dependency on the threat and on the other CIs' disruptions. That requires the introduction of new parameters, describing CI's vulnerability (threat dependency) and disruptions dependency (CIs (inter-)dependency). The resultant model may then be complex. The complexity of the resultant model will depend on the complexity of these new parameters.

Now and then, the term "dependency" will exclusively be used to describe CIs' disruption dependency and interdependency while the term "vulnerability" will be used for the disruption-threat dependency. We will distinguish later between the dependency and the interdependency, as well.

2. CI vulnerability to threats

The term "Vulnerability" is used here to describe the dependency between a well-defined threat and the disruption of a well-defined CI (with respect to a given disruption mode). A CI does not react to all threats in the same manner. The stochastic disruption of the CI is dependent on the threat specifications. In our model, a vulnerability matrix is established for each identified CI disruption mode and corresponds to a well-specified set of threats. It is obvious that the set of the involved threats depends on the location of the CI. The threat is generally specified by its: intensity, magnitude, likelihood, locality and dynamics.
The vulnerability of a given CI "*i*" to a well-defined threat "*j*"will be described using a vulnerability strain factor "υ_{ij}". The disruption rate $\lambda_i(j)$ of a given CI "*i*" under the action of the threat "*j*" will then be given by:

$$\lambda_i(j) = \lambda_i(o)(1+\upsilon_{ij})$$

Where, $\lambda_i(o)$ is the systemic (unstressed) disruption rate of the CI, "i", and υ_{ij} is its vulnerability strain factor regarding the threat, "*j*". The strain factor υ_{ij} is a positive parameter.
If the CI, "i", is acted upon by multiple *N* threats, its effective disruption rate $\lambda_i^{N,0}$ will, then, be given by:

$$\lambda_i^{N,0} = \lambda_i(o)\left[\prod_{j=1}^{N}(1+\upsilon_{ij})\right]$$

Where, $\lambda_i^{N,0}$ is the effective disruption rate.
In the presented model, threats act on the same CI independently. We have not considered the possibility of a compound damage mechanisms. Considering independently the vulnerability of each threat gives a conservative estimation of the effective disruption rate.

3. CIs dependency & interdependency

In order to count for the cascading of disruptions, the possible dependencies between CIs should be analysed and considered, as well. In the presented model, a disruption dependency matrix (D-D matrix) is established describing the existing dependency between a given set of identified CIs. It is obvious that the set of considered CIs depends on the mode of the disruptions of all considered CIs.
The dependency of the disruption of a given CI "*i*" on the disruption of another CI "*j*" is described by a factor ε_{ij} that is called the CI disruption dependency strain factor. The disruption rate $\lambda_i(j)$ of a given CI "*i*" given the disruption of the CI "*j*" is then given by:

$$\lambda_i(j) = \lambda_i(o)(1+\varepsilon_{ij})$$

Where, $\lambda_i(o)$ is the systemic (unstressed) disruption rate of the CI, "*i*", and ε_{ij} is its dependency strain factor regarding the disruption of the CI, "*j*".
A disruption dependency is "directional" if the disruption of the CI "*j*" impacts on the disruption of the CI "*i*", while the inverse is not true. In that case, one has $\varepsilon_{ij} > 0$ and $\varepsilon_{ji} = 0$.
If the disruption dependency is not directional, it is called "interdependency" rather than "dependency" and have, generally, $(\varepsilon_{lk} > 0, \forall l,k)$ and $(\varepsilon_{ij} \neq \varepsilon_{ji})$.
If the CI, "*i*", is acted upon by multiple disruptions of other M CIs, its effective disruption rate $\lambda_i^{0,M}$ will, then, be given by:

$$\lambda_i^{0,M} = \lambda_i(o)\left[\prod_{j=1}^{M}(1+\varepsilon_{ij})\right]$$

In the presented model, the disruptions of many CIs act independently on the CI. We have not considered the possibility of a compound damage mechanisms. Considering independently the impact of each other disruption gives a conservative estimation of the effective disruption rate.

4. Modelling dependency & multi-threat vulnerability

In a complex case, where there are multi-threat actions and many disrupted CIs simultaneously, the overall effective disruption rate $\lambda_i^{N,M}$ will be given by:

$$\lambda_i^{N,M} = \lambda_i(o)\left[\prod_{k=1}^{N}(1+\upsilon_{ik})\right]\left[\prod_{j=1}^{M}(1+\varepsilon_{ij})\right]$$

Where N refers to the number of the simultaneous acting threats and M refers to the number of the already disrupted CIs.

5. Determination of the Strain Factors

The main difficulty of this model is to find out the strain factors (vulnerability to threats and disruptions dependency). Crud data are available in different databases but a systematic data-mining and data statistical treatment are strongly lacking.
An example of the crud vulnerability data is, e.g., the “number of electrical blackouts” each time “wind speed is higher than 50 km/h”, in a given location. This information can help in determining the vulnerability strain factor of the electrical grid to strong wind.
Another example of the crud disruptions independency data is, e.g., the “number of disruptions of water supply” each time an electrical blackout occurs. These information can help in determining the dependency strain factor of the water supply system disruptions on the electrical grid disruptions.
The screening of existing records about past crises involving CIs disruptions with and without threats action would enable us to extract the required strain factors,applying the appropriate statistical treatments.
Individually, most of the CI owners and operators have the preliminary data necessary to work out the threats vulnerability strain factors (regarding strong winds, torrential rains, earthquakes, volcano eruptions, extreme cold/hot weather, violent solar-winds etc. …) and the CI disruption dependency strain factors (energy/communication, communication/energy, communication/transport, energy/transport, energy/water-supply etc. …).

6. Cascading of Systemic Disruptions

Let T be a well-defined top event, occurs if and only if some discrete and independent disruptions e_i happen in a well-specified chronological order $[e_1 \rightarrow e_2 \rightarrow e_3 ... \rightarrow e_n]$.
The top event T is a sever accident or a major crisis. The corresponding occurring instants of the elementary disruptions are defined by $[t_1, t_2, t_3, ..., t_n]$, where $[t_1 < t_2 < t_3 < ... < t_n]$. Each of these instances $[t_1, t_2, t_3, ..., t_n]$ has its distribution probability density function (pdf). The first disruption event is e_1 and the last is e_n.
The probability $p_n(t)$ that the major crisis T happens within the interval [0,*t*] is given by:

$$p_n(t) = \int_0^t \rho_1(\xi_1)d\xi_1 * \int_{\xi_1}^t \rho_2(\xi_2)d\xi_2 * \ldots * \int_{\xi_{n-1}}^t \rho_n(\xi_n)d\xi_n \qquad (1)$$

where ρ_i is any probability density function (pdf) characterising the occurrence instances of events e_i.
Whatever the type of the density probability functions ρ_i, the integral in (1) can hopefully be solved in many cases.
It can be solved: numerically or by Monte-Carlo Simulation (MCS).
However, there exists an analytical solution if ρ_i obeys a Poisson probability density function [1].

7. Stochastic Poisson Process

If ρ_i follows a Poisson probability density function, it would be described as following:

$$\rho_i = \lambda_i * e^{-\lambda_i t} \qquad (2)$$

where λ_i is the occurrence rate of disruption e_i.
Many authors have previously developed approached analytical solutions for equation (1) when it was a matter of limited number of ordered events obeying a Poison’s Stochastic Process, e.g, [5]-[7].
But, none gave an exact analytical general solution. Some of the solutions were asymptotic and others were approximated.
An exact solution of (1) has been developed in [1] and has the following form:

$$p_n(t) = \sum_{j=1}^{n} C_j^n * (1 - e^{-(\sum_{l=n-j+1}^{n} \lambda_l)t}) \qquad (3)$$

where each event e_i is defined by a constant occurrence rate λ_i, $\{ i \in [1,2,...,n] \}$ and the coefficient C_1^{i+1} is given by:

$$C_1^{i+1} = \sum_{j=1}^{i} C_j^i \, , C_{j+1}^{i+1} = -\frac{\lambda_{i+1}}{\sum_{l=i-j+1}^{i+1} \lambda_l} C_j^i \, , \qquad (4)$$

where $j = 1,2,..., i$, $i \in [1,2,..., n]$ and $C_1^1 = 1$.

8. Integration of the dependency & vulnerability

The integration of the CIs’ vulnerability and disruption dependency, in the model, is straight forward such as:

$$\lambda_i = \lambda_i(o)\left[\prod_{k=1}^{N}(1+\upsilon_{ik})\right]\left[\prod_{j=1}^{M}(1+\varepsilon_{ij})\right]$$

where λ_i is the effective occurrence rate of disruption 'i' under the action of N threats and M disrupted CIs.
Certainly, the solution given in (3) is valid, if and only if, vulnerability stress factors υ_{ij} and the dependency stress factors ε_{ij} are time-constant, as well.

9. Disruptions cascading assessment

As shown in (3), for each well-defined cascade of disruptions, one can determine its occurrence probability. However, some other probabilistic characteristic quantities can also be determined, such as:

- The occurrence density and the occurrence rate functions
- The mean time to occur
- The asymptotic stochastic behavior.

9.1. Occurrence Density and Occurrence rate

By definition, the corresponding occurrence density function $\Theta_i(t)$ can directly be deduced, from (3), using the first derivative as following:

$$\Theta_n(t) = \frac{dp_n(t)}{dt}.$$

The occurrence density function will then be given by:

$$\Theta_n(t) = \sum_{j=1}^{n} C_j^n * \left(\sum_{l=n-j+1}^{n} \lambda_l\right) e^{-\left(\sum_{l=n-j+1}^{n} \lambda_l\right)t}. \qquad \ldots\ldots\ldots(6)$$

The equivalent occurrence rate Λ_i of the whole sequence (cascade), is determined such as:

$$\Lambda_i(t) = \frac{1}{p_i(t)} * \frac{dp_i(t)}{dt} = \frac{\sum_{j=1}^{i} \left(\sum_{l=i-j+1}^{i} \lambda_l\right) * C_j^i e^{-\left(\sum_{l=i-j+1}^{i} \lambda_l\right)t}}{\sum_{j=1}^{i} C_j^i * \left(1 - e^{-\left(\sum_{l=i-j+1}^{i} \lambda_l\right)t}\right)}. \qquad (7)$$

As we may expect, although the ordered events are individually governed by a SPP, the sequence T is not. The occurrence rate of the sequence T is time dependent, (7).

9.2. Mean-time to occur

One may also determine the mean time to occur τ_n corresponding to a given sequence (S_n) of n-events $\{e_1, e_2, e_3, ..., e_n\}$, such as:

$$\tau_n = \int_{t=0}^{\infty} t * dp_n(t),$$

$$\tau_n = \sum_{j=1}^{n} \frac{C_j^n}{\left(\sum_{l=n-j+1}^{n} \lambda_l\right)}. \qquad (8)$$

9.3. Asymptotic occurrence probability

Having demonstrated that the occurrence probability $p_n(t)$ of a given cascade of disruptions can be described by equation (3), it is straightforward to demonstrate that the occurrence probability $p_n(t)$ has an asymptotic value equal to:

$$p_n(t \to \infty) \to \sum_{j=1}^{n} C_j^n \qquad (9)$$

10. Study case

We are considering a hypothetical major crisis occurs when four disruptions $[e_1, e_2, e_3, e_4]$ occur. The systemic occurrence rates of the elementary disruptions are constant and having the following values: 10^{-4}/h, $5 \cdot 10^{-3}$/h, $2.5 \cdot 10^{-2}$/h, $1.25 \cdot 10^{-1}$/h, respectively. Thus, they are following SPPs.
Given a threat, $Th1$ the vulnerability strain factors υ_{ij} are, respectively: 1,0,1,0. That means that only disruptions e_1 and e_3 are impacted by the threat and their vulnerability strain factors are equal to 1.
Regarding, the disruption dependency strain matrix is given below, *Table 1*.
Table 1 shows that the disruption e_2 impacts on the disruptions e_3 and e_4, with the dependency strain factors 0,3 and 0,2 respectively.
Table 1 shows also the CIs disruption dependency is directional. No interdependency is considered, then.

Table 1. DDS Matrix

		Impacting disruptions			
		e_1	e_2	e_3	e_4
Impacted disrup.	e_1	0	0	0	0
	e_2	0	0	0	0
	e_3	0	0,3	0	0
	e_4	0	0,2	0	0

Accordingly, the stressed occurrence rates will be:

$$\lambda_1^{1,1} = 10^{-4}(1+1)(1+0)$$
$$\lambda_2^{1,1} = 5*10^{-3}(1+0)(1+0)$$
$$\lambda_3^{1,1} = 2.5*10^{-2}(1+1)(1+0.3)$$
$$\lambda_4^{1,1} = 1.25*10^{-1}(1+0)(1+0.2)\,.$$

In this illustrative case, the occurrence probability of the major crisis $p_4(t)$ has been calculated and traced, *Figure 1*, in an interval of time of 80 hours.
The two situations (stressed and unstressed CI) are presented. Unstressed situations are when the CIs disruption are due to random systemic reasons. While, stressed situations are when one considers in addition the threat and the CIs' dependency.
The asymptotic occurrence probability $p_4(t \to \infty)$ is equal to 12% and to 13% in the unstressed and stressed situations, respectively. Stressing the CIs has increased by 8% the likelihood of the occurrence of the crisis.
The occurrence rate of the cascade of disruptions is also traced in *Figure 2*, for stressed and unstressed CI. Both rates are decreasing with time but that of the stressed situation decreases somehow faster after the first 20 hours. This is mainly because disruptions e_3 and e_4 occur faster than e_1 and e_2. If any of e_3 and e_4 occurs before any of e_1 and e_2, the rate of observing a cascade in the required order $e_1 \to e_2 \to e_3 \to e_4$ is evidently decreasing with time. This fact is amplified during the stress phase because the occurrence rate of e_3 is even faster.

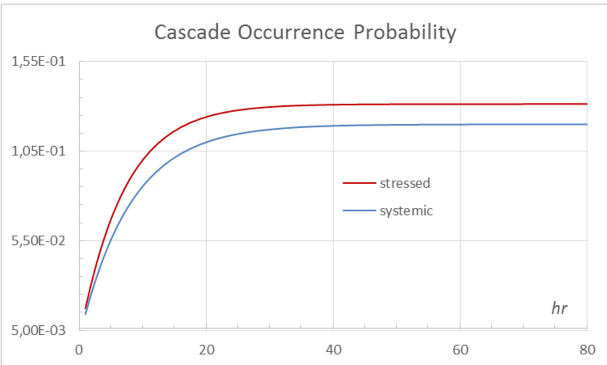

Figure 1. Cascad occurrence probability (stressed and unstressed CIs)

Many other quantities relevant to the resilience and recovery characteristic of the CIs can be determined using the same model after the introduction of both the vulnerability and the disruption dependency strain factors. These quantities for resilience and recovery characteristic have already been assessed, in [2]-[3] in the case of unstressed CIs.

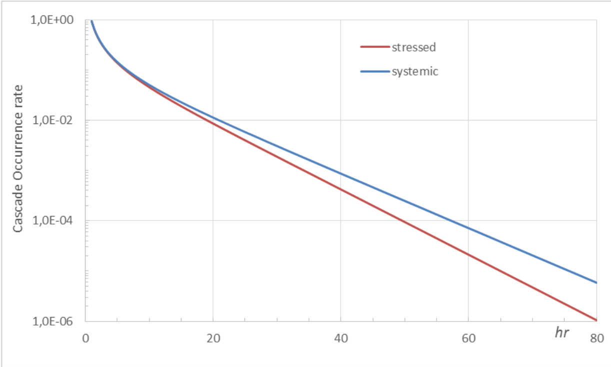

Figure 2. Cascad occurrence pdf (stressed and unstressed CIs)

11. Conclusion

An original model describing the cascading of CI's disruptions analytically, thanks to three simplifications: the disruptions obey stochastic Poisson's processes, the dependence of the CI's disruption and the threat is described by a constant vulnerability stress factor and the (inter-) dependencies between the CIs are described using a CI's disruption stress factor.
The model suits the law statistical quality of the available data on vulnerability and CIs' (inter-) dependency. It, also, suits the end-users' persisting requirements of avoiding complex models.

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References

[1] Eid, M. (2011). A General Analytical Solution for the Occurrence Probability of a Sequence of Ordered Events following Poison Stochastic Processes. *Journal of Reliability Theory & Applications, RT&A*. ISSN 1932-2321. 03, 22, 2.

[2] Eid, M. et al. (2015). A resilience model based on Stochastic Poison Process. *48th ESReDA Seminar*.

[3] Eid, M. et al. (2015). Critical Infrastructures Protection (CIP) – Coupled Modelling for Threats and Resilience. *SSARS 2015, Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars* 6, 2.

[4] Fussell, J.B., Aber E.F. & Rahl, R.G. (1976). On the Quantitative Analysis of Priority-AND Failure Logic. *IEEE Transactions on Reliability* 25, 5.

[5] Kohda, T. & Inoue, K. (2003). A simple method to evaluate system failure occurrence probability using minimal cut sets. *Proceedings of ESRel"03, Safety and Reliability*. *Bedford & van Gelder (Ed.). Swets & Zeitlinger, Lisse*. ISBN 90 5809 551 7. 923-926.
[6] Long, W., Sato, Y. & Horigome, M. (2000). Quantification of sequential failure logic for fault tree analysis. *Reliability Engineering and System Safety* 67, 269-274.
[7] Yuge, T. & Yanagi, S. (2008). Quantitative analysis of a fault tree with priority AND gates. *Reliability Engineering & System Safety* 93, 1577-1583.