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# Fuzzy logic expert system for supply chain resilience modelling and simulation

## Keywords

resilience, supply chain, uncertainty, fuzzy logic, modelling, simulation

## Abstract

The aim of this paper is to present the concept of the supply chain resilience assessment in the case of disruptive events occurrence. Firstly, the methods for modelling uncertainty in terms of their application to assess this type of risk will be discussed, and then the concept of a fuzzy logic expert model enabling a quantitative assessment of supply chain resilience will be presented. Finally the structure of the simulation model has been proposed, which consists of the partial resilience models, namely: security, survivability and recovery ones. In the course of the simulation process, it is possible to identify the rules involved in system output as well as changes in resilience level which account for changes in inputs values.

## 1. Introduction

The issue of assessing the risk of disruption in supply chains, in particular global ones, has been becoming more and more important in recent years. It results from the efforts to minimize costs by continuous processes of Lean Manufacturing, Lean Logistics, and Lean Management and the increasing number and intensity of external threats. Reducing the number of suppliers according to the "4S" principle (a Single Source Supply Strategy), introducing Just In Time Manufacturing system to a greater extent, minimizing the level of minimal buffers, and configuring tightly connected supply chains have resulted in a significant increase in the level of risk of process continuity. This risk is greater when the information about the possible risks that may interfere with the supply process and their possible impact is burdened with greater uncertainty.

In the case of predictable, recurring threats, it is possible to "tame" uncertainty, and at the same time limit risk, by means of statistical methods, based on the probability calculus, the theory of stochastic processes, and mathematical statistics. By contrast, so far there have been no effective methods and tools useful to assess the risk associated with a new type of threat of unique character, related to the forces of nature, or intentional, hostile interference of the human factor. Untypical risks are especially dangerous, and their impact can be adverse for all people participating in the implementation of the undertaking. These risks are sometimes called "Black Swan" events and have the following characteristics [14]:

- Uniqueness, going beyond expectations based on previous experiences and a priori unpredictability
- Extremely large impact of the event and
- A possibility to explain the reasons and the method of forecasting after its occurrence (a posteriori).

The effect of the "Black Swan" is also described in the literature as LSLIRE (Large Scale, Large Impact Rare Event). Modelling of such as phenomena requires other uncertainty description methods than in the case of common and easily identifiable threats. Thus, in the first place on the basis of information theory, the formalized languages and monotonic measures will be analyzed, to provide adequate means of the uncertainty description for occurrence of this untypical phenomena.

## 2. Methods for uncertainty modelling

### 2.1. Theories of uncertainty

The starting point for considering uncertainty may be GIT – the Generalized Information Theory, proposed by G.J. Klir [6]. Compared to the classical information theory created by C.E. Shannon [11], based on concepts of probability and entropy, it has a much more universal character. GIT is the result of two significant mathematical generalizations:

- The classical theory of additive measures to the theory of monotone measures and
- The classical theory of crisp sets to a more general theory of fuzzy sets.

The first generalization, which started in the early 1950s, extends additive measures to less restrictive monotonic measures, characterized by more diverse features. The second one, introduced in the 1960s, expands the language of the classical set theory into a more universal language of fuzzy sets, allowing the use of vague linguistic terms. The theory of uncertainty of a given type is formed by choosing the appropriate language (e.g. based on the theory of fuzzy sets) and expressing uncertainty by means of specific monotone measures (e.g. based on the theory of probability).

The classical information theory is based on the theory of probability or alternatively, the theory of possibility, applied to classical sets. And applying the probability function or possibility function to standard fuzzy sets enables the creation of new, more general theories of information. Similarly, remaining at classical sets, we can apply various non-additive monotone measures (e.g. Sugeno's $\lambda$ measure, Dempster-Shafer measure, Choquet measure of n-th order or the general lower and upper probability function), creating new theories.

There are, therefore, a lot of formal theories of uncertainty. Each of them is more or less general, and any two theories at the same level of generality may not be mutually comparable. Each particular problem requires the use of such a theory, which would make it possible for a decision-maker to express his or ignorance and to protect against ignoring any information, relevant in a given situation. Currently, within the framework of the so-called imperfect knowledge trend, efforts are made to create GTU – the Generalized Theory of Uncertainty, which would go beyond the classical theory of probability and classical set theory, would characterize each type of uncertainty and work at four levels: formalization, computational tools, measurement and methodology [21].

In each of the above theories, uncertainty is represented by the so-called uncertainty function, assigning each possible realization from the set a number from the interval [0,1], which determines the degree of certainty that a specific opportunity arises. Examples of uncertainty functions include: the probability function, the possibility function, the function of faith and credibility or function of the lower and upper probability. In each theory, the uncertainty function meets certain requirements that differentiate the various theories. The measure of uncertainty for a specific type of theory is the functional that assigns a non-negative real number to each function. Typical examples of uncertainty measures are the Shannon entropy [11] and Hartley measure. A functional representing the uncertainty measure must meet a number of requirements. Admittedly, mathematical formalization of each of the requirements depends on the theory used, however, these requirements can be represented in the general form as:

- Additivity - the uncertainty in the total data representations is equal to the sum of uncertainty of individual representations of data,
- Subadditivity - the uncertainty in the total representation of data cannot be greater than the total uncertainty of the sum of individual data representation,
- Range- the uncertainty is contained in the interval (0, M), where 0 is related to the function that describes the complete certainty and M depends on the size of the set used and the selected unit of measure,
- Continuity – the functional must be continuous,
- Expansibility - developing a set of alternatives by adding alternatives cannot change the level of uncertainty
- Consistency - if uncertainty can be calculated in different ways (allowed in the method), the result must be the same,
- Monotonocity - if the data form an increasing series, their measure of uncertainty also increases and vice versa, if the data in a series are decreasing, uncertainty also decreases,
- Coordinate invariance - a measure of uncertainty cannot change with the isometric coordinate transformations.

These requirements must be fulfilled by all types of uncertainty that exist in the theory. Uncertainty of information can be expressed through measures of probability, possibility and necessity, faith and credibility. Measures of possibility, necessity, faith (conceivability) and credibility are dual, i.e., if the event is necessary, the counter-event is impossible, if the event is credible, the counter- event is inconceivable.

There are three main principles of uncertainty management:

- The principle of minimum uncertainty – we accept only those solutions for which the loss of information (resulting from simplifications, transformations conflict solutions) is minimal, i.e. we choose solutions with a minimum of uncertainty,
- The principle of maximum uncertainty - we accept all solutions, after making sure that the information that raises doubt is reliable,
- The principle of uncertainty invariance - the level of uncertainty should be kept at each transition from one mathematical approach to another.

## 2.2. Formalized languages

Currently, three formalized languages are used to describe sets: CST – the Classical Sets Theory, SFST – the Standard Fuzzy Sets Theory and NFST – the Nonstandard Fuzzy Sets Theory. The first two theories are thoroughly described in the literature, are well-developed and widely used, while the last one is a relatively new theory and not yet fully developed.

### 2.2.1. The classical set theory

In the classical set theory, it is assumed that each element of the considered space X belongs to either set A $(x \in A)$ defined on the space X ( $A \in P(X)$ ), or it complements the set A ( $x \in \overline{A}$), that is, no element can belong simultaneously to both sets. The characteristic function (membership function) of the set A is

$$m_A : X \to \{0,1\} \text{ and } m_A = \begin{cases} 1 \text{ for } & x \in A \\ 0 \text{ for } & x \notin A \end{cases} \qquad (1)$$

for each $x \in X$ .

Two sets are equal only when every element of one of them is the element of the other and vice versa. Two sets of the same number of elements are called equinumerous.
The interference based on binary logic and classic sets is simple and unambiguous, but in many cases insufficient to describe the complex reality.

### 2.2.2. The standard fuzzy sets theory

The concept of fuzzy sets was introduced by L.A. Zadeh [19] as a generalization of the classical set theory. In the case of fuzzy sets, each element of space X can belong partially to a set A, and partly to its complement $\overline{A}$. Fuzzy sets are defined by the membership function corresponding to the function characteristics of classical sets. Each element of the set X has the assigned value that defines the degree of membership to the fuzzy set. Membership of standard fuzzy sets is in the range [0,1] and if the maximum value equals 1, we deal with normal fuzzy sets. Thus, the membership function of the set X is :

$$\mu_A : X \to [0,1] \qquad (2)$$

We can distinguish three cases here:

a) $\mu_A(x) = 1$- means full membership in the fuzzy set *A;*

b) $\mu_A(x) = 0$- means the lack of membership in the fuzzy set *A;*

c) $0 < \mu_A(x) < 1$- means a partial membership in the fuzzy set *A.*

A fuzzy set *A* is contained in the fuzzy set *B* only when $\mu_A(x) < \mu_B(x)$ for each $x \in X$, and the fuzzy set *A* equals the fuzzy set *B* only when. $\mu_A(x) = \mu_B(x)$. The complement of the set *A* is a fuzzy set $\overline{A}$ with a membership function. $\mu_{\overline{A}} = 1 - \mu_A$.
Although the inference based on the fuzzy set theory and multi-valued logic is more complex and less intuitive, however thanks to widely available computer tools supporting the process of the fuzzy inference it is becoming more common [7].

## 2.3. Monotonic measures

### 2.3.1. Additive measures - the "numeric" probability

A classic tool in studying uncertainty is the probability theory. Applying it leads to assumptions, such as that the rate of uncertainty should be measurable (using simple methods) and have a numerical rating. Another assumption used in the studies of uncertainty is the independence of the system components. Uncertainty and uncertain information are modelled in this case also by a probability distribution. For a finite set *X* of mutually exclusive alternatives, a probability distribution function for each $x \in X$ is

$$p(x) \in [0,1] \quad \sum_{x \in X} p(x) = 1 \qquad (3)$$

### 2.3.2. Non-additive measures

A characteristic of non-additive measures is that the probability of the sum of mutually exclusive (independent) events does need not to equal probability of these events. This reflects a subjective assessment of probability by a decision-maker, which is not necessarily an objective probability. This condition is fulfilled by measures based on the possibility theory and the imprecise theory of probabilities.

A) The possibility theory
The concept of possibility and necessity is the oldest and most fundamental concept of measures, derived from the ideas of Aristotle. However, the theoretical basis for this concept was developed by L.A. Zadeh in the 1970s [20]. It is assumed that there is a set of mutually exclusive alternatives *X*. The basic information that we can obtain on the set *X* (based on different types of tests) is the information that certain alternatives from a set *X* are impossible. After rejecting impossible alternatives, we obtain a set of possible alternatives *E*, which is a subset of X. The characteristic function of the set *E* (also known as the basic function of possibility) is:

$$r_E(x) = \begin{cases} 1 & \text{for } x \in E \\ 0 & \text{for } x \notin E \end{cases} \tag{4}$$

A function of possibility defined on a power set *P*(*X*) is given by the formula:

$$Pos_E(A) = \max_{x \in A} r_E(x) \ \text{ for each } \ A \in P(X) \tag{5}$$

A real alternative may belong to a set *A* if *A* contains at least one element of the set *E*. Functions of necessity can be formulated as follows:

$$Nec_E(A) = 1 - Pos_E(\bar{A}) \ \text{ for each } \ A \in P(X) \tag{6}$$

The real alternative is necessary in A only when it is not possible that it is in a completion of A. An uncertainty measure of a finite set of possible alternatives *E* is Hartley's measure expressed by the formula:

$$H(Pos_E) = \log_2 |E| \tag{7}$$

B) The theory of imprecise probabilities
The theory of imprecise probabilities is used for experimental data burdened with uncertainty, for which it is difficult or even impossible to calculate the probability characteristics. A characteristic of all imprecise probabilities inaccuracy is that the data can be described by means of the low or high probability functions ($\underline{g}$ i $\overline{g}$). Functions $\underline{g}$ i $\overline{g}$ are regular monotonic measures and they fulfil the following conditions [15]:

$$\sum_{x \in X} \underline{g}(\{x\}) \le 1, \qquad \sum_{x \in X} \overline{g}(\{x\}) \ge 1 \tag{8}$$

The upper probability is a subadditive measure, and the lower probability is a superadditive measure. The classic probability measure is a special case of imprecise probabilities, for which lower and upper probabilities are equal.
On the basis of mentioned above theories, it seems to be logical to describe the uncertainty of prevalence of the rare, difficult to predict events, by a use of the standard fuzzy sets theory as the formalized language, and the basic function of possibility as the non-additive monotone measure. This method of the uncertainty description was adopted in the next part of this work, as the basis for the design of an expert model to assess the supply chains resilience.

## 3. Modelling supply chain resilience

Supply chain resilience is a multi-dimensional phenomenon. Supply networks are becoming more complex, dynamically changing nets. A supply chain could be very lean and efficient; if it is unable to find an alternative route of delivery quickly, it will be susceptible to system shocks and disturbances. Many of the processes of supply chain management may unwittingly contribute to the creation of a system that, while responsive and efficient in the steady state, is so tightly coupled that it cannot prevent the escalation of threats and also has insufficient slack to cope with the demands of the event once it occurs [9].
MIT research group [12], [13] defines supply chain resilience as, *"the ability to react to unexpected disruption and restore normal supply network operations."* Sheffi examined the ways in which companies can recover from high-impact disruptions and focused on actions to lower vulnerability and increase resilience. These include:

- Reducing likelihood of disruptions through monitoring and detecting weakest signals, demand-responsive supply chains, supply-chain wide collaboration, redundancy;
- Operational flexibility through standardization of parts facilitating interchangeability, postponement or mass customization strategy to respond to unpredictable demand changes, customer and supplier relation management and multiple sourcing.

Christopher and Peck [4] defined supply chain resilience as *"the ability of the supply chain to return to its original state or move to a new, more desirable state after being disturbed"*. So most of the definitions assume resilience as the ability to deal with unexpected events successfully after they have actually happened. M. Christopher and H. Peck notified the five broad elements of supply chain resilience:

- Supply chain understanding (pitch points, bottlenecks, critical path);
- Supply base strategy (risk awareness, audited monitoring);
- Design principles (keep several option open! – efficiency vs. redundancy, decoupling point, critical nodes);
- Collaboration (knowledge shared by partners, Supply Chain Event Management - SCEM);
- Supply chain agility (visibility, communication, velocity, acceleration);
- Supply chain risk management culture (nothing is possible without leadership from the top of the organization!).

Supply chain resilience is the ability and capacity to withstand systemic discontinuities and adapt to new risk environments. So supply chain resilience can be defined as not only the ability to maintain control over performance variability in the face of disturbance but also a property of being adaptive and capable of sustained response to sudden and significant shifts in the environment.

On the basis of these definitions, we propose a model of supply chains resilience in the form of a multidimensional vector R, which consists of three basic attributes constituting the resilience of the whole system: security (SE), survivability (SU) and recovery (RE) in the presence of the disruptive events.

$$R = \{SE, SU, RE\} \tag{9}$$

***Security (SE)*** – protection of a system from malicious intent, is displayed by:

- Confidentiality (CON) of information to unauthorized users,
- Integrity (INT) - impossibility of introducing changes into the system by unauthorized users, and
- Accessibility (ACC) for authorized users only.

***Survivability (SU)*** - the capability of a system to fulfil its mission, in a timely manner, in the presence of attacks, failures, or accidents, is described by:

- Detectability (DET) - early threats recognition, supervision and monitoring,
- Robustness (ROB) - resistance and redundancy,
- Adaptability (ADA) - flexibility, agility, fault tolerance.

***Recovery (RE)*** – failure removal (restoration) in acceptable time and costs, is divided into:

- Susceptibility to repair (SUS) and
- Availability of Repair Resources (ARR).

Methods of parameters description with use of linguistic variables allow using fuzzy sets as a tool for building expert system, in which linguistic variables are used as inputs variables of the system. The application of fuzzy sets theory in this case is reasonable because expert's knowledge can be used to build a suitable rule base [2], [3]. Expert knowledge on the impact of the various parameters on result is expressed in the form of "if … then" rules. The knowledge encoded in a rule base is derived from human experience and intuition as well as on the basis of theoretical and practical understanding of the properties of the studied object. The main task of this deduction system is to calculate the approximate value of the output variable based on the share of each rule from the rule base with an appropriate factor determining the "validity" of the rule. Fuzzy logic based systems are a kind of expert system built on a knowledge base that contains inference algorithms in the form of a rule base. What distinguishes fuzzy inference in terms of concept from conventional inference is the lack of an analytical description. The approximate inference mechanism transforms knowledge from the rule base into a non-fuzzy form. The non-fuzzy form of the result is obtained in the process of defuzzification. Defuzzification is interpreting the membership degrees of fuzzy sets into a real value. The correctness of selection of rules as well as the shape and ranges of the membership function is verified with a rules viewer and simulation. The rules viewer displays a roadmap of the whole fuzzy inference process. It also shows how the shape of certain membership functions influences the overall result.

The software WinFACT [18] was used for building a system for evaluating resilience level. WinFACT provides FLOP (Fuzzy Logic Operating Program) tools for creating and editing fuzzy inference systems or integrating our fuzzy systems into simulations with BORIS (Block ORIented Simulation). The fuzzy shell FLOP allows the design and the analysis of rule based systems on the basis of fuzzy logic. The program offers the following options: definition of linguistic variables and corresponding terms, creation of rule bases, realization of inference processes, evaluation of transfer characteristic curves and maps, simulation based on recorded data and creation of fuzzy controller files for the block oriented simulation system BORIS [18].

The evaluation system of supply chain resilience is a hierarchical structure. In the first sequence the *Security (SE), Survivability (SU) and Recovery (RE)*, were evaluated, and the information about them can be represented by the membership function of fuzzy system. Trapezium membership functions are associated with the numerical value RE, SU, SE. Each of these parameters was divided in to three categories- *Low*, *Moderate* and *High* from range 0 to 1. Level of resilience (R) is an output of this system and it is depends on the value of inputs and on the "knowledge" which is implemented in the rules base in a fuzzy system. The structure of the resilience category evaluation system is show in *Figure 1*. Each of the input parameters of the system, namely, SE, SU and RE is determined by another independent fuzzy system.

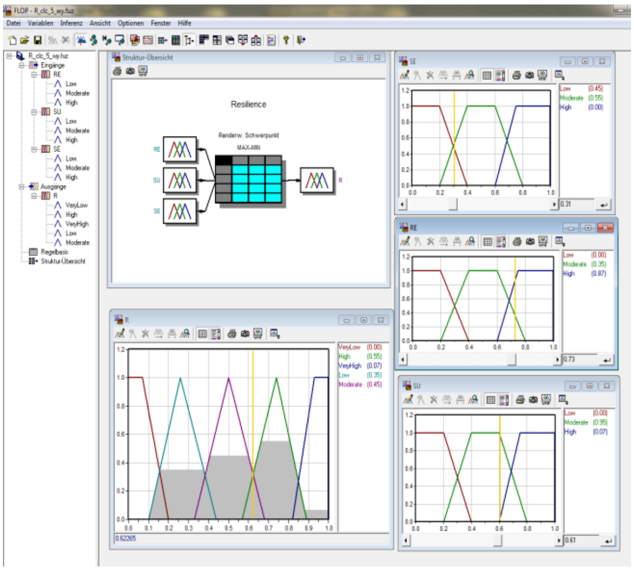


*Figure 1.* Structure of the resilience evaluation system (own work)

An example of a system for evaluation survivability level shown in *Figure 2*. The trapezium membership functions type for inputs and output variables were also applied. The parameter ROB is divided in three categories, and parameters DET and ADA are divided only in to two categories. In the case of using trapezoidal membership function models for linguistic variables, one may assume that that the measure of uncertainty in quantitative estimates is the angle of inclination of the sides of the trapezoids (a right angle corresponds to a lack of uncertainty in the estimate, and the smaller the angle, the larger the uncertainty). So slope of each input variables could be interpreted as an uncertainty of input parameter estimation.

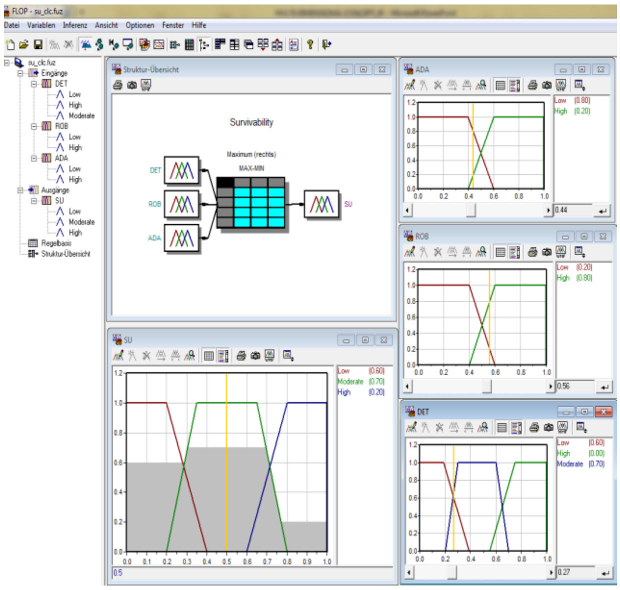


*Figure 2.* Structure of the system for evaluation survivability level (own work)

Structure of the resilience level simulation model is shown in *Figure 3*.

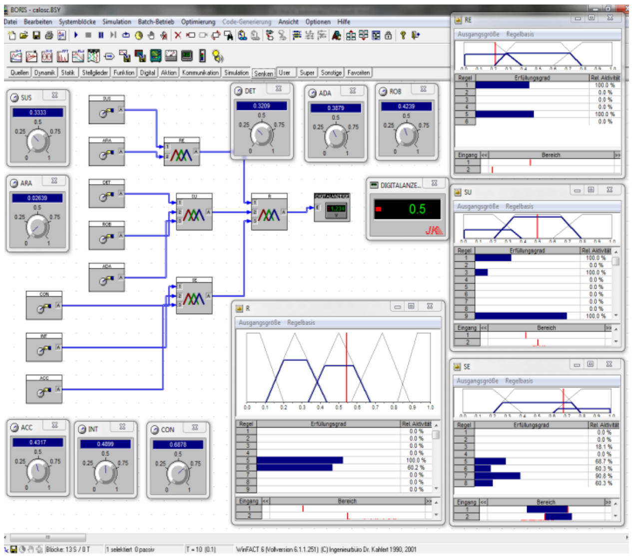

*Figure 3.* Structure of the resilience level simulation model (own work)

Simulations were carried out in the BORIS software to observe the impact of changes in inputs parameters on the output. Each input parameter can be set at a level ranging from 0 to 1. The implemented simulation system allows for continuous observation of changes in output depending on the value of input signals. Simulation of the system can run continuously. In addition, during the simulation, it is possible to observe the degree to which input variables belong to given membership functions and it is also possible to identify the rules involved in generating system output as well as changes in resilience level that are functions of changes in inputs values.

## 4. Conclusion

The framework for evaluation of supply chain resilience proposed in this paper is a universal, "shell" type model, that can be applied to verifying and validating the vulnerability of logistics systems, especially at the design stage. Adapting this tool to the needs of a particular type of system or a specific practical case requires the estimation of numerical values (or ranges) corresponding to each parameter class.
In the case of using triangular or trapezoidal membership function models for linguistic variables, the measure of uncertainty in quantitative estimates is the angle of inclination of the sides of the triangles or trapezoids (the smaller the angle, the larger the uncertainty).

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