Journal of Polish Safety and Reliability Association
Summer Safety and Reliability Seminars, Volume 6, Number 2, 2015

Kołowrocki Krzysztof
Maritime University, Gdynia, Poland

Tchórzewska-Cieślak Barbara

Piegdoń Izabela
Rzeszow University of Technology, Rzeszow, Poland

Modelling reliability of water supply network

Keywords

modelling, water supply, reliability, operation process

Abstract

The operation process of the complex technical system is considered and its operation states are introduced. The semi-Markov process is used to construct a general probabilistic model of the considered complex technical system operation processes. Further, a general reliability analytical model of complex two-state technical systems related to their operation processes is constructed. An exemplary application of this model to a simplified fragment of the main water supply network operating at variable conditions is presented as well.

1. Introduction

Water supply network, whose main aim is to deliver water for inhabitants of urban and rural areas, consists of functionally interrelated subsystems, which create an integral whole. The primary and basic subject to which the concept of water safety concerns is the water consumer. The secondary subject is the supplier – the manufacturer of water. The water supply technical reliability considered here means providing stable contribution conditions allowing to satisfy a current demand for water in sufficient quantity in convenient time for water consumers. Whereas, the technical unreliability of main water network causes both water loss, interruptions in water supply or delivery the water to customers with quality incompatible with the standards. The most commonly reported adverse events in water network are water pipes failures and failures on water fittings. This also refers to the leaks on water pipes, and there is no need to disconnect the section of pipe from operation. Due to the specificity of work of water pipes the failures removal process is inherent with maintaining the reliability of water network, and the priority is to provide consumers with water of appropriate quality, at a suitable pressure at any time. Water pipes failures, its causes, effects, methods of detecting and counteracting were and are the subject of many research and implementation work [12]-[13], [16]-[17], [20]-[22].

The result of water pipes failure may be a break in the water supply for a number of consumers or may be revealed in lowering the pressure in the water network below the required value, which results in a lack or limitation of water supply to consumers, especially those who live in the upper floors of buildings. Hydraulic conditions in the main network co-decide about the changes of the physico-chemical and bacteriological water composition (so-called biological and chemical stability of tap water) [6], [12], [17]. Another aspect of this is also a significantly oversized water pipes, their age (technical condition), resulting in secondary pollution of water in the network [6], [12].

The paper does not consider the quality of water intended for human consumption, only quantitative analysis dependent on the basic hydraulic parameters of water mains which are the flow rate Q and the pressure P. In the reliability engineering theory to estimate the indicators of the reliability of main network and thus to ensure water supply, distributions of the random variable T of suitability of objects, such as exponential, Weibull and normal, are used.

2. Operation of water main

The water mains are water pipes forming the skeleton of the water supply system, whose main operating task is to distribute water throughout the supply area with the required operating pressure. The water mains constitute a system of pipes of given diameter, connected in a geometrical sense in ring systems (closed, circular) (favorable from the reliability point of view) or branched systems.

The direction of flow is regulated by means of suitably located valves. The basic hydraulic parameters of the water main operation are the flow rate Q and the pressure P. The pressure losses in water network are due to the hydraulic resistance of pipes and flow rate and their value can be calculated from the relation:

$$\Delta h = K \cdot Q^2 \tag{1}$$

where

Δh– drop in pressure in water pipe for l m length,
K–resistance of 1 m segment, s^2/m^5,
Q – flow rate,m^3/s.

The changes in flow rate in the water pipe are due to the changing conditions of water uptake by water consumers per day. These changes are characterized by the indicators of hourly inequality and network hydraulic calculations are primarily referred to such values of flow rate Q as: hourly average Q_{ah} [m^3/h], hourly maximum Q_{maxh} [m^3/h] and hourly minimum Q_{minh}[m^3/h].

Flow rate values are regulated automatically in the water pumping stations which often cooperate with water tanks and the priority is to ensure stable conditions of water uptake by consumers in each operating condition of water supply network. The studies of water supply network failure rate [6], [13], [16]-[17] showed that the failure rate depends not only on the technical condition of the pipe but also on the variable operating conditions.

It has been shown in the literature that the failure rate of water pipes is influenced by many factors, e.g. unstable ground, the age of the pipe, function of pipe (transit, main, distribution and water supply connections), water mains pressure, flow rate, ground corrosivity, pipe material, the method of joining pipes. The studies show also the impact of the seasons and related to it air, ground and water temperature, on water pipes failure rate [6], [12]-[13], [20].

3. Modelling complex technical system operation process

The operation processes of real technical systems which include collective water supply system are very complex and it is difficult to analyse these systems reliability with respect to changing in time their operation conditions that often are essential in this analysis. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics is often very difficult to fix. Usually, the system environment and infrastructure have either an explicit or an implicit strong influence on the system operation process. As a rule, some of the initiating environment events and infrastructure conditions define a set of different operation states of the technical systems in which the systems change their reliability structure and their components reliability parameters. A convenient tool for analysing this problem is semi-Markov modelling [1]-[3], [5], [7], [11], [15] of the systems operation processes proposed in this paper.

3.1. Semi-Markov model of system operation process

We assume that the system during its operation process is taking $v, v \in N$, different operation states $z_1, z_2, \ldots, z_v$. Further, we define the system operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the system operation process Z(t) is a semi-Markov process [3], [5], [7], [11], [15] with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,...,v,$ $b \neq l.$ Under these assumptions, the system operation process may be described by:

- the vector of the initial probabilities $p_b(0) = P(Z(0) = z_b),$ $b = 1,2,...,v,$ of the system operation process $Z(t)$ staying at particular operation states at the moment $t = 0$.

$$[p_b(0)]_{1\times v} = [p_1(0), p_2(0),..., p_v(0)] \tag{2}$$

- the matrix of probabilities p_{bl}, $b, l = 1,2,...,v,$ $b \neq l,$ of the system operation process $Z(t)$ transitions between the operation states z_b and z_l

$$[p_{bl}]_{v\times v} = \begin{bmatrix} p_{11} \ p_{12} \cdots p_{1v} \\ p_{21} \ p_{22} \cdots p_{2v} \\ \cdots \\ p_{v1} \ p_{v2} \cdots p_{vv} \end{bmatrix} \tag{3}$$

where by formal agreement $p_{bb} = 0$ for $b = 1,2,..., v;$

- the matrix of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $b, l = 1,2,...,v,\ b \neq l,$ of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states

$$[H_{bl}(t)]_{v\times v} = \begin{bmatrix} H_{11}(t)\ H_{12}(t) \ldots H_{1v}(t) \\ H_{21}(t)\ H_{22}(t) \ldots H_{2v}(t) \\ \ldots \\ H_{v1}(t)\ H_{v2}(t) \ldots H_{vv}(t) \end{bmatrix} \quad (4)$$

where by formal agreement $H_{bb}(t) = 0$ for $b = 1,2,..., v.$

It is possible to show the matrix of the conditional density functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states corresponding to the conditional distribution functions $H_{bl}(t)$

$$[h_{bl}(t)]_{v\times v} = \begin{bmatrix} h_{11}(t)\ h_{12}(t) \ldots h_{1v}(t) \\ h_{21}(t)\ h_{22}(t) \ldots h_{2v}(t) \\ \ldots \\ h_{v1}(t)\ h_{v2}(t) \ldots h_{vv}(t) \end{bmatrix} \quad (5)$$

where

$$h_{bl}(t) = \frac{d}{dt}[H_{bl}(t)] \text{ for } b, l = 1,2,..., v,\ b \neq l,$$

and by formal agreement $h_{bb}(t) = 0$ for $b = 1,2,...,v.$

The mean values of the conditional sojourn times θ_{bl} are given by [11]

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t) = \int_0^\infty t h_{bl}(t)dt,\ b,l = 1,2,...,v,\ b \neq l. \quad (6)$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,...,v,$ of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,...,v,$ are given by [3], [5], [7], [11], [15]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t),\ b = 1,2,...,v. \quad (7)$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,...,v,$ at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl},\ b = 1,2,...,v, \quad (8)$$

where M_{bl} are defined by the formula (6) in a case of any distribution of sojourn times θ_{bl}.
The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in \langle 0,+\infty),\ b = 1,2,...,v,$$

are given by [3], [5], [6], [10]-[12]

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l},\ b = 1,2,...,v, \quad (9)$$

where M_b, $b = 1,2,...,v,$ are given by (8), while the steady probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \quad (10)$$

In the case of a periodic system operation process, the limit transient probabilities p_b, $b = 1,2,...,v,$ at the operation states defined by (9), are the long term proportions of the system operation process $Z(t)$ sojourn times at the particular operation states z_b, $b = 1,2,...,v.$

Other interesting characteristics of the system operation process $Z(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, $b = 1,2,...,v,$ during the fixed system operation time. It is well known [3], [5]-[6], [10]-[12] that the system operation process total sojourn times $\hat{\theta}_b$ at the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta,\ b = 1,2,...,v, \quad (11)$$

where p_b are given by (9).

3.2. Operation process of main network

The considered water main network supplies in water town to a town with a total capacity 76 896 m^3/d. Average daily production of treated water is 40000m^3/d.
The total length of the main water supply network is 50*km*.
The city *X* also exploited:

- emergency water intake with a capacity of 240 m^3/d,
- 34 water pumping stations,
- 11 clean water tanks with a total capacity of 34,400 m^3,
- 186 public wells which are located on city property *X*,
- Q_{ah} average hourly water demand characterized by the irregularity of the partition of water during 24 hours,
- Q_h efficiency of second stage pumping station,
- The network operates under varying conditions of water demand:
  - $Q_h<Q_{ah}$,
  - $Q_h>Q_{ah}$.

The system is composed of three subsystems (main pipes) S_1, S_2 and S_3. The general functional structure of the system is showed in *Figure 1*.

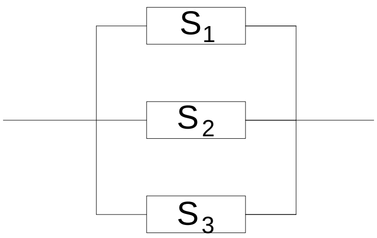

Figure 1. The scheme of the system functional structure

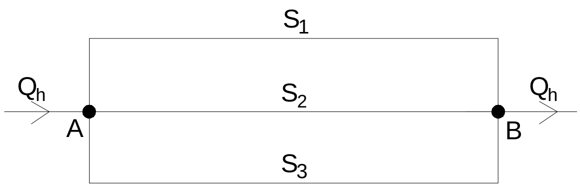

Figure 2. The analyzed main water network

However, the pipeline system reliability structure and its subsystems and components reliability depend on its changing in time operation states [8], [10], [12]. The scheme of analysed system - the annular main network –consists of three main water pipes (each pipe 1200*m* long), composed of*l*= 300 components(each element 4*m* long) with the length *d* and the numbers *l* of the particular pipes segments respectively:

- S_1: *d*=400*mm*, *l*=300,
- S_2: *d*=600*mm*, *l*=300,
- S_3: *d*=500*mm*, *l*=300.

Since the particular components of the subsystem S_i, $i=1,2,3$, have an exponential reliability function

$$R_i(t)=\exp[\lambda_i t], \qquad i=1,2,3, \tag{12}$$

then the reliability function of the subsystem S_i, $i=1,2,3$, is given by

$$R_i(t)=\exp[\lambda_i lt],\ i=1,2,3, \tag{13}$$

where
λ_i- the failure rate of a subsystem S_i, $i=1,2,3$, components,
$l-$ the number of a subsystem S_i, $i=1,2,3$, components.
Taking into account expert opinions on the varying in time operation process of the considered piping system, we distinguish the following operation states [8], [10], [12]:

- an operation state z_1- transport of water from part *A* to *B* using three out of three pipeline subsystemsS_1, S_2 and S_3(a series reliability structure) with condition of water demand $Q_h<Q_{ah}$.The subsystemsS_1, S_2 and $S_3$are each composed of*l*=300 components with failure rates λ_i =0,00028, *i*= 1,2,3;
- an operation state z_2- transport of water from part A to B using three out of three pipeline subsystems S_1, S_2 and S_3(a series reliability structure) with condition of water demand $Q_h>Q_{ah}$. The subsystems S_1, S_2 and S_3 are each composed of l = 300 components with failure rates λ_i = 0,00032, *i*= 1,2,3;
- an operation state z_3- transport of water from part A to B using two out of three pipeline subsystems S_1, S_2 and S_3 (a ”2 out of 3” reliability structure) with condition of water demand $Q_h<Q_{ah}$. The subsystems S_1, S_2 and S_3 are each composed of l = 300 components with failure rates λ_i = 0,00028, *i*= 1,2,3;
- an operation state z_4- transport of water from part A to B using two out of three pipeline subsystems S_1, S_2 and S_3 (a ”2 out of 3” reliability structure)with condition of water demand $Q_h>Q_{ah}$. The subsystems S_1, S_2 and S_3 are each composed of l = 300 components with failure rates λ_i = 0,00032, *i*= 1,2,3.

The above assumptions mean that according to (13), the subsystems the subsystem S_i, $i = 1,2,3,$ depending on the operate states, have the following reliability functions:

$$R_i(t) = \exp[-0.00028 \cdot 300t] = \exp[-0.0084t],$$
$$i = 1,2,3, (14)$$

at the operation state z_1;

$$R_i(t) = \exp[-0.00032 \cdot 300t] = \exp[-0.0096t],$$
$$i = 1,2,3, \qquad (15)$$

at the operation state z_2;

$$R_i(t) = \exp[-0.00028 \cdot 300t] = \exp[-0.0084t],$$
$$i = 1,2,3, \qquad (16)$$

at the operation state z_3;

$$R_i(t) = \exp[-0.00032 \cdot 300t] = \exp[-0.0096t],$$
$$i = 1,2,3, \qquad (17)$$

at the operation state z_4.

The influence of the above system operation states changing on the changes of the pipeline system reliability structure. At the system operation states z_1 and z_2, the system is composed of the subsystems S_1, S_2 and $S3$showed in *Figure 3*.

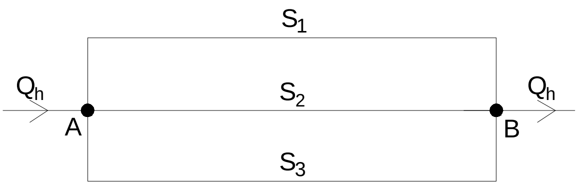

Figure 3. The functional scheme of water transportation system at the operation state $z_1$and z_2

At these operation states, the piping network is a series system in the reliability sense.
At the system operation states$z_3$and z_4, the system is composed of either two out of three subsystems or three subsystems. The functional combination of working main pipes were showed in *Figure 4*.
At these operation states, the piping network is a "2 out of 3" system in the reliability sense.
To identify the unknown parameters of the water main network operation process the suitable statistical data coming from its real realizations should be collected. The lack of sufficient statistical data about the system operation process causes that it is not possible to estimate exactly its operation parameters. However, even on the basis of the fragmentary statistical data coming from experts, the water main network system operation process probabilities p_{bl} of transitions from the operation state z_b into the operation state z_l, $b, l = 1, 2, ..., 4,$ $b \neq l,$ can be evaluated approximately. Their approximate evaluations are given in the matrix below:

$$[p_{bl}] = \begin{bmatrix} 0 & 0,75 & 0,2 & 0,05 \\ 0,65 & 0 & 0,2 & 0,15 \\ 0,6 & 0,2 & 0 & 0,2 \\ 0,5 & 0,4 & 0,1 & 0 \end{bmatrix} \qquad (14)$$

Unfortunately, it is not possible to identify completely the matrix of the conditional distribution functions $[H_{bl}(t)]_{7x7}$ of the sojourn times θ_{bl} for $b, l = 1, 2, ..., 4,$ $b \neq l,$ and consequently, it is also not possible to determine the vector $[H_b(t)]_{1x7}$ of the unconditional distribution functions of the sojourn times θ_b of this system operation process at the operation states z_b, $b, l = 1, 2, ..., 4,$ given by (7).

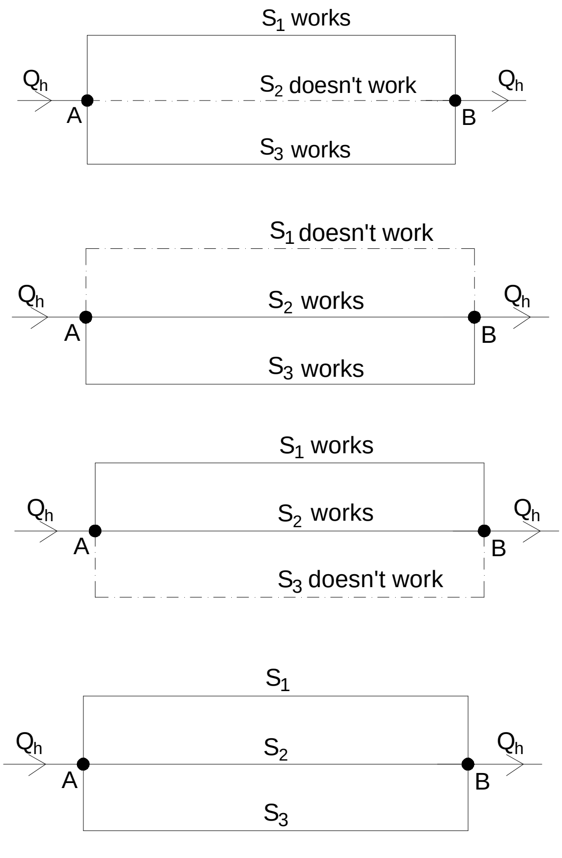

Figure 4. The schemes of water transportation system at the operation state $z_3$and z_4

Presented methods are based on the basis of data coming from practice and collected by experts operating this piping system, some hypotheses on the forms of the distributions describing the system operation process.
In this case, having these distributions identified, it is possible to evaluate the mean values $M_{bl} = E[\theta_{bl}]$ of the conditional sojourn times θ_{bl} at the particular operation states, using the general formula (6). Otherwise, if the collected statistical data is not sufficient to test and to accept the forms of the distributions of the piping system operation process conditional sojourn times θ_{bl}, their mean values $M_{bl} = E[\theta_{bl}]$ may be estimated by applying the formula for the empirical man values of the conditional sojourn times at the particular operation states. As the results of using the last of these two possibilities, the approximate evaluations of these mean values, are as follows:

$$\begin{array}{llll} M_{12} = 993 & M_{21} = 674 & M_{31} = 410 & M_{41} = 386 \\ M_{13} = 2207 & M_{23} = 2207 & M_{32} = 2080 & M_{42} = 1009 \\ M_{14} = 2650 & M_{24} = 2608 & M_{34} = 1999 & M_{43} = 2616 \end{array} \quad (15)$$

In this way, the water main network operation process is approximately defined and we may predict its main characteristics. Applying (8), (12) and (13), the unconditional mean sojourn times of the piping system operation process at the particular operation states are:

$$M_1 = E[\theta_1] = p_{12}M_{12} + p_{13}M_{13} + p_{14}M_{14}$$
$$M_1 = E[\theta_1] = 0,75 \cdot 993 + 0,2 \cdot 2207 + 0,05 \cdot 2650 \cong 1318,65$$

$$M_2 = E[\theta_2] = p_{21}M_{21} + p_{23}M_{23} + p_{24}M_{24}$$
$$M_2 = E[\theta_2] = 0,65 \cdot 674 + 0,2 \cdot 2207 + 0,15 \cdot 2608 \cong 1270,7$$

(16)

$$M_3 = E[\theta_3] = p_{31}M_{31} + p_{32}M_{32} + p_{34}M_{34}$$
$$M_3 = E[\theta_3] = 0,6 \cdot 410 + 0,2 \cdot 2080 + 0,2 \cdot 1999 \cong 1061,8$$

$$M_4 = E[\theta_4] = p_{41}M_{41} + p_{42}M_{42} + p_{43}M_{43}$$
$$M_4 = E[\theta_4] = 0,5 \cdot 386 + 0,4 \cdot 1009 + 0,1 \cdot 2616 \cong 858,2$$

Considering (12) in the system of equations (10) that takes the form

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4] = [\pi_1, \pi_2, \pi_3, \pi_4][p_{bl}]_{4x4} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \end{cases} \quad (17)$$

we get the solution:

$$\pi_1 = 0.317, \ \pi_2 = 0.284, \ \pi_3 = 0.194,$$
$$\pi_4 = 0.205. \quad (18)$$

Applying the formulas (14) and (9) we get the limit values of the piping system operation process transient probabilities $p_b(t)$ at the operation states z_b, $b = 1, 2, ..., 4$:

$$p_1 = 0.3601, \ p_2 = 0.3109, \ p_3 = 0.1775,$$
$$p_4 = 0.1516. \quad (19)$$

Considering the above result, after applying (11), the expected values of the total sojourn times $\hat{\theta}_b$, $b = 1, 2, ..., 4$ of the system operation process at the particular operation states z_b, $b = 1, 2, ..., 4$ during the fixed operation time $\theta = 1$ year = 365 days, amounts to:

$$\begin{array}{l} E[\hat{\theta}_1] = 0,3601 \text{ year} = 131,4 \text{ days} \\ E[\hat{\theta}_2] = 0,3109 \text{ year} = 113,5 \text{ days} \\ E[\hat{\theta}_3] = 0,1775 \text{ year} = 64,8 \text{ days} \\ E[\hat{\theta}_4] = 0,1515 \text{ year} = 55,3 \text{ days} \end{array} \quad (20)$$

4. Reliability of complex technical system at variable operation conditions

Most real technical systems are structurally very complex and they often have complicated operation processes. Large numbers of components and subsystems and their operating complexity cause that the evaluation and prediction of their reliability is difficult. The time dependent interactions between the systems' operation processes operation states changing and the systems' structures and their components reliability states changing processes are evident features of most real technical systems. The common reliability and operation analysis of these complex technical systems is of great value in the industrial practice. The convenient tools for analysing this problem are the two-state system's reliability modelling [7], [11] commonly used with the semi-Markov modelling [3], [7], [11].

4.1. General approach to reliability analysis of complex systems

We assume that every operation state of the object operation process $Z(t)$, $t \in \langle 0, +\infty)$, described in section 2, have an influence on the object reliability [7]. Therefore, the object reliability at the particular operation state z_b, $b=1,2,...,v$, can be described using the conditional reliability function

$$\boldsymbol{R}^{(b)}(t) = P(T^{(b)} > t \mid Z(t) = z_b),\ t \in \langle 0, +\infty),$$
$$b = 1,2,...,\nu, \tag{21}$$

that is the conditional probability that the object conditional lifetime $T^{(b)}$ is greater than t, while the object operation process $Z(t)$ is at the operation state z_b, $b=1,2,...,\nu$ [7].

Further, we denote the object unconditional lifetime by T and the unconditional reliability function of the object by

$$\boldsymbol{R}(t) = P(T > t),\ \ t \in \langle 0, +\infty). \tag{22}$$

In the case when the object operation time θ is large enough, the unconditional reliability function of the object is approximated by [7]

$$\boldsymbol{R}(t) \cong \sum_{b=1}^{\nu} p_b \boldsymbol{R}^{(b)}(t),\ t \in \langle 0, +\infty), \tag{23}$$

where p_b, $b=1,2,...,\nu$, are the object operation process limit transient. The mean value of the object unconditional lifetime T is given by

$$\mu \cong \sum_{b=1}^{\nu} p_b \mu_b, \tag{24}$$

where μ_b are the mean values of the object conditional lifetimes $T^{(b)}$ at the operation state z_b, $b=1,2,...,\nu$, given by

$$\mu_b = \int_0^{+\infty} \boldsymbol{R}^{(b)}(t)dt,\ \ b = 1,2,...,\nu, \tag{25}$$

$\boldsymbol{R}^{(b)}(t)$, $b=1,2,...,\nu$, are defined by (19) and p_b are given by (9).

4.2. Reliability of main water network at variable operation conditions

It is assume for main water network that its subsystems S_υ, $\upsilon = 1,2,3,$ are composed of two-state, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,3,$ with the conditional three-state reliability functions

$$[\boldsymbol{R}_{ij}^{(\upsilon)}(t)]^{(b)},\ \ b = 1,2,...,4, \tag{26}$$

with the exponential coordinates

$$[\boldsymbol{R}_{ij}^{(\upsilon)}(t)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}]^{(b)} t], \tag{27}$$

different at various operation states z_b, $b=1,2,...,4$, and with the failure rates respectively

$$[\lambda_{ij}^{(\upsilon)}]^{(b)},\ b = 1,2,...,4. \tag{28}$$

The influence of the system operation states changing on the changes of the system reliability structure and its components reliability functions is as follows.

- At the system operation state z_1, the piping network has the reliability series structure composed of the subsystems S_1, S_2 and $S_3$with functional structure illustrated in Figure 3 and the subsystems reliability functions are given by (14), i.e.

$$\boldsymbol{R}_i^{(1)}(t) = R^{(1)}(t) = \exp[-0.084t],\ i = 1,2,3. \tag{29}$$

Thus, at the operation state z_1, the pipeline system conditional reliability function is given by

$$\boldsymbol{R}^{(1)}(\mathrm{t}) = \prod_{i-1}^{3} R_i^{(1)}(t) = [R^{(1)}(t)]^3 = [\exp[-0.084t]]^3$$
$$= \exp[-0.252t]$$

- At the system operation state z_2, the piping network hasthe reliability series structurecomposed of the subsystems S_1, S_2 and S_3 illustrated in Figure 3,and the subsystems reliability functions are given by (15), i.e.

$$\boldsymbol{R}_i^{(2)}(t) = R^{(2)}(t) = \exp[-0.096t],\ i = 1,2,3. \tag{30}$$

Thus, at the operation state z_2, the pipeline system conditional reliability function is given by

$$\boldsymbol{R}^{(2)}(\mathrm{t}) = \prod_{i-1}^{3} R_i^{(2)}(t) = [R^{(2)}(t)]^3 = [\exp[-0.096t]]^3$$
$$= \exp[-0.288t]$$

- At the system operation state z_3, the piping network has a "2 out of 3" reliability structure composed of the with subsystems S_1, S_2 and S_3 illustrated in Figure 4 and the subsystems reliability functions are given by (16), i.e.

$$\boldsymbol{R}_i^{(3)}(t) = R^{(3)} \exp[-0.084t],\ i = 1,2,3. \qquad (31)$$

Thus, the network conditional reliability function is given by

$$\boldsymbol{R}^{(3)}(\mathrm{t}) = 1 - [\ [\tbinom{3}{0} R^{(3)}(t)]^0 [1 - R^{(3)}(t)]^3 + \tbinom{3}{1} [R^{(3)}(t)]^1 [1 - R^{(3)}(t)]^2\ ]$$
$$= 1 - [[1 - \exp[-0.084t]^3 + 3\exp[-0.084t][1 - \exp[-0.084t]]^2]$$
$$= 1 - [1 - 3\exp[-0.084t] + 3\exp[-0.168t] - \exp[-0.252t]]$$
$$+ 3\exp[-0.084t] - 6\exp[-0.168t] + 3\exp[-0.252t]]$$
$$= 3\exp[-0.168t] - 2\exp[-0.252t].$$

- At the system operation state z_4, the piping network has a "2 out of 3" reliability structure composed of the subsystems S_1, S_2 and S_3 illustrated in Figure 4 and the subsystems reliability functions are given by (17), i.e.

$$\boldsymbol{R}_i^{(4)}(t) = R^{(4)} \exp[-0.096t],\ i = 1,2,3. \qquad (32)$$

Thus, the network conditional reliability function is given by

$$\boldsymbol{R}^{(4)}(\mathrm{t}) = 1 - [\ [\tbinom{3}{0} R^{(4)}(t)]^0 [1 - R^{(4)}(t)]^3 + \tbinom{3}{1} [R^{(4)}(t)]^1 [1 - R^{(4)}(t)]^2\ ]$$
$$= 1 - [[1 - \exp[-0.096t]^3 + 3\exp[-0.096t][1 - \exp[-0.096]]^2]$$
$$= 1 - [1 - 3\exp[-0.096t] + 3\exp[-0.192t] - \exp[-0.288t]]$$
$$+ 3\exp[-0.096t] - 6\exp[-0.192t] + 3\exp[-0.288t]]$$
$$= 3\exp[-0.192t] - 2\exp[-0.288t].$$

For the system of the water main network unconditional reliability function is given as follows:

$$\boldsymbol{R}(\mathrm{t}) \cong p_1\ \boldsymbol{R}^{(1)}(\mathrm{t}) + p_2\ \boldsymbol{R}^{(2)}(\mathrm{t}) + p_3\ \boldsymbol{R}^{(3)}(\mathrm{t}) + p_4\ \boldsymbol{R}^{(4)}(\mathrm{t}),$$

i.e.

$$\boldsymbol{R}(\mathrm{t}) = 0,3601 \cdot [\boldsymbol{R}(\mathrm{t})]^{(1)} + 0,3109 \cdot [\boldsymbol{R}(\mathrm{t})]^{(2)} + 0.1775 \cdot [\boldsymbol{R}(\mathrm{t})]^{(3)} + 0.1515 \cdot [\boldsymbol{R}(\mathrm{t})]^{(4)}$$

for $t \geq 0$,

and finally, we get

$$\boldsymbol{R}(\mathrm{t}) \cong 0.3601\exp[-0.252t]$$
$$+0.3109\exp[-0.288t]$$
$$+0.1775[3\exp[-0.168t] - 2\exp[-0.252t]]$$
$$+0.1515[3\exp[-0.192t] - 2\exp[-0.288t]]. \qquad (33)$$

Hence, the mean value of the of the piping network unconditional lifetime is given by

$$\mu = \int_0^{+\infty} \boldsymbol{R}(\mathrm{t})dt \cong 0.3601\frac{1}{0.252} + 0.3109\frac{1}{0.288} + 0.1775[3\frac{1}{0.168} - 2\frac{1}{0.252}] + 0.1515[3\frac{1}{0.192} - 2\frac{1}{0.288}] \cong 5.5845 \text{ years.} \qquad (34)$$

The variance of the of the piping network unconditional lifetime can be calculated as follows [11]

$$\sigma^2 = 2\int_0^{+\infty} t\boldsymbol{R}(\mathrm{t})dt - \mu^2 = 2[0.3601\frac{1}{0.252^2} + 0.3109\frac{1}{0.288^2} + 0.1775[3\frac{1}{0.168^2} - 2\frac{1}{0.252^2}] + 0.1515[3\frac{1}{0.192^2} - 2\frac{1}{0.288^2}]] - 5.5845^2$$
$$\cong 62.7432 - 31.1866 = 31.5566.$$

Hence, the standard deviation of the piping network unconditional lifetime amounts

$$\sigma = \sqrt{31.5566} = 5.6175 \text{ years.} \qquad (35)$$

The determined values given by (34) and (35) are convergent to the real values for the considered water main piping system.

5. Conclusions

Collective water supply system is one of the technical systems, which reliability is strictly related with its operation. Water supply network, which is the main element of water distribution in supply areas, works at variable operation conditions. These conditions are associated primarily with changes in flow and pressure, which affects its reliability. Therefore, in the paper a modelling of the main network reliability under varying operating conditions was proposed. The considered is a simplified model of a fragment of the main annular network. The results justifies that this type of modelling allows to determine the reliability characteristics of the whole water supply network operating at the variable conditions. In the future, the authors plan to consider operation of water supply network including changes of the diameter, length and utilities of the considered partly water network.

References

[1] Ferreira, F. & Pacheco, A. (2007). *Comparison of level-crossing times for Markov and semi-Markov processes.*Statistics & Probability Letters, 77, 2, 151-157.

[2] Glynn, P. W.& Haas, P. J. (2006). *Laws of large numbers and functional central limit theorems for generalized semi-Markov processes*. Stochastic Model, 22, 2, 201-231.

[3] Grabski, F. (2015). *Semi-Markov Processes: Applications in Systems Reliability and Maintenance*. Elsevier.

[4] Guze, S., Kołowrocki, K. & Soszyńska, J. (2008). *Modelling environment and infrastructure influence on reliability and operation processes of port transportation systems.* Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, 2, 1, 179-188.

[5] Habibullah, M. S., Lumanpauw, E., Kołowrocki, K., Soszyńska, J. & Ming, N. G. (2009). *A computational tool for general model of industrial systems. operation processes.* Electronic Journal of Reliability & Risk Analysis: Theory & Application, 2, 4, 181-191.

[6] Hotloś, J. (2007). *Ilościowa ocean wpływu wybranych czynników na parametry i koszty eksploatacyjne sieci wodociągowych.* Monografia, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2007,

[7] Kołowrocki, K. (2004). *Reliability of Large and Complex Systems.* Elsevier, Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney – Tokyo.

[8] Kołowrocki, K. & Soszyńska, J. (2009). Modelling environment and infrastructure influence on reliability and operation process of port oil transportation system. *Electronic Journal of Reliability & Risk Analysis: Theory & Application*, 2, 3, 131-142.

[9] Kołowrocki, K. & Soszynska, J. (2010). Reliability modeling of a port oil transportation system's operation processes. *International Journal of Performance Engineering*, 6, 1, 77-87.

[10] Kołowrocki, K. & Soszynska, J. (2010). Reliability, availability and safety of complex technical systems: modelling – identification – prediction – optimization. Summer Safety & Reliability Seminars. *Journal of Polish Safety and Reliability Association*, 4(1):133-158.

[11] Kołowrocki, K. & Soszyńska-Budny, J. (2011). *Reliability and Safety of Complex Technical Systems and Processes: Modeling-Identification-Prediction-Optimization*, London, Dordrecht, Heildeberg, New York, Springer.

[12] Kwietniewski, M., Kłos-Trębaczkiewicz, H., & Roman, M. (1993). *Niezawodność wodociągów i kanalizacji.* Arkady, Warszawa.

[13] Kwietniewski, M. & Rak, J. (2010). *Niezawodność infrastruktury wodociągowej i kanalizacyjnej w Polsce.* Studia z zakresu inżynierii, Polska Akademia Nauk, Warszawa.

[14] Limnios, N. & Oprisan, G. (2005). *Semi-Markov Processes and Reliability.*Birkhauser, Boston.

[15] Mercier, S. (2008). Numerical bounds for semi-Markovian quantities and application to reliability.*Methodol and Comput in ApplProbab* 10, 2, 179-198.

[16] Rak, J. (2009). Selected problems of water supply safety. *Environment Protection Engineering*, 35, 2, 23-28.

[17] Rak, J. & Tchórzewska-Cieślak, B. (2013). *Ryzyko w eksploatacji systemów zbiorowego zaopatrzenia w wodę.* Wydawnictwo Seidel-Przywecki Sp. z o. o.

[18] Soszyńska, J. (2007). *Systems reliability analysis in variable operation conditions.* PhD Thesis, Gdynia Maritime University-System Research Institute Warsaw, (in Polish).

[19] Soszyńska, J., Kołowrocki, K., Blokus-Roszkowska, A. & Guze, S. (2010). Prediction of complex technical systems operation processes.Summer Safety & Reliability Seminars. *Journal of Polish Safety and Reliability Association*, 4, 2, 379-510.

[20] Tchórzewska-Cieślak, B. (2009). Water supply system reliability management. *Environmental Protection Engineering*, 35, 2, 29-35.

[21] Tchórzewska-Cieślak, B., Boryczko, K. & Piegdoń, I. (2015). Possibilistic risk analysis of failure in water supply network. *Safetyand Reliability: Methodology and Applications* – Nowakowski – et al. (Eds), Taylor &Francis Group, London. 1473 – 1480.

[22] Tchórzewska-Cieślak, B., Piegdoń, I. & Szpak, D. (2014). Water supply of urban agglomeration in crisis situation.Summer Safety & Reliability Seminars. *Journal of Polish Safety and Reliability Association* 5, 1, 143-155.