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Reliability of large three-dimensional nanosystems

Keywords

reliability, nanosystem, asymptotic approach, limit reliability function

Abstract

Basic notions and agreements on reliability of three-dimensional nanosystems are introduced. The asymptotic approach to the three-dimensional nanosystem reliability investigation is presented and the nanosystem limit reliability function is defined. Auxiliary theorems on limit reliability functions of three-dimensional nanosystems composed of large number of independent nanocomponents are formulated and the classes of limit reliability functions for a homogeneous series and series-parallel nanosystems are fixed. A model of a three-dimensional series and series-parallel nanosystem with dependent nanocomponents is created and the class of limit reliability functions identical with the class in the previous case is fixed as well. The asymptotic approach to reliability evaluation of exemplary three-dimensonal series and series-parallel nanosystem with dependent nanocomponents is presented and its accuracy is discussed.

1. Introduction

A nanosystem is a device which is a system engineered in a nanoscale, in other words, at least one of its dimensions is in size range of 1 to 100 nanometers (10^{-9} to 10^{-7} meters) and which is made up of individual nanocomponents. We could ponder nanosystems as large systems because they could be built of a large number of nanocomponents. In that case, the determination of an exact reliability of the nanosystem could lead us to very complicated formula. This happens mostly when survival functions of nanocomponents are dependent on each other. It makes that obtain results are often useless for practical purpose. Asymptotic approach to reliability evaluation of nanosystems is a solution to this problem. If we assume that the number of nanocomponents tends to infinity and find the limit reliability of the nanosystem, we can receive a simply function which approximate the reliability function. Main results concerned with the asymptotic approach to the reliability of large nanosystems which nanocomponents are dependent of each other and the dependence between nanocomponents is decreasing when the distance between them tends to infinity are presented. There are also considered models of series and series-parallel nanosystems with dependent nanocomponents which asymptotic reliability functions are determined using modified lemmas which are used in the investigation of limit reliability functions of nanosystems with independent nanocomponents.

2. Reliability of three-dimensional nanosystems

We consider a three-dimensional nanosystem composed of

$$n = l_1 m_1 + l_2 m_2 + \ldots + l_{k_n} m_{k_n},\ n \in N_+,$$

nanocomponents E_{111}, E_{112}, …, E_{11m_1}, E_{121}, E_{122}, …, E_{12m_1}, …, $E_{1l_1m_1}$, E_{211}, E_{212},…, $E_{2l_2m_2}$, ..., $E_{k_n l_{k_n} m_{k_n}}$, where l_i, $m_i \in N_+$, $i = 1, 2, \ldots, k_n \in N_+$. They are arranged in order shown in *Figure 1*.
We denote the sets of indexes by

$$W((l_i, m_i) : i = 1,\ldots, k_n) = \{(i, j, \upsilon) : j = 1,\ldots, l_i,\ \upsilon = 1,\ldots, m_i,\ i = 1,\ldots, k_n\},$$

$$D_{k_n l_n m_n} = W(\underbrace{(l_n, m_n),\ldots,(l_n, m_n)}_{k_n \text{ pairs of } (l_n, m_n)}),$$

$$Z_{k_n l_n m_n} = \{((i, j, \upsilon),(i', j', m')) \in D_{k_n l_n m_n} \times D_{k_n l_n m_n} : (i < i') \vee (i = i' \wedge j < j') \vee (i = i' \wedge j = j' \wedge \upsilon < \upsilon')\}.$$

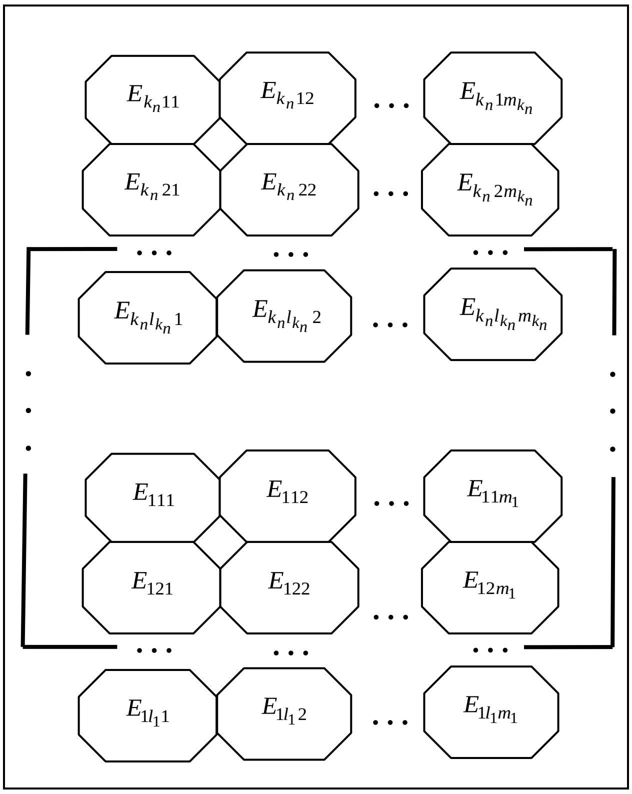

Figure 1. A graph of the three-dimensional nanosystem composed of nanocomponents $E_{111}, E_{112}, ..., E_{11m_1}, E_{121}, E_{122}, ..., E_{1l_1m_1}, ..., E_{k_nl_{k_n}m_{k_n}}$, l_i, $m_i \in N_+$, $i = 1, 2, \ldots, k_n$, $k_n \in N_+$

Moreover, we mark by $s_{ij\upsilon}(t)$, $t \in <0, +\infty)$, $(i, j, \upsilon) \in W((l_i, m_i)$: $i = 1,\ldots, k_n)$, a nanocomponent $E_{ij\upsilon}$ displacement stochastic process which is equal to 0 when a nanocomponent $E_{ij\upsilon}$ is displaced from its initial position at the moment t, $t \in <0, +\infty)$, and it is equal to 1 when a nanocomponent $E_{ij\upsilon}$ is not displaced from its initial position at this moment.
We assume that a nanocomponent $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i)$: $i = 1,\ldots, k_n)$, is not displaced at the moment $t = 0$ and we mark by $T_{ij\upsilon}$ a non-negative continuous random variable that represents the time at which a nanocomponent $E_{ij\upsilon}$ becomes displaced from its initial position. Further, the random variable $T_{ij\upsilon}$ will also be called the time up to displacement of a nanocomponent $E_{ij\upsilon}$ from its initial position.
From this fact we receive that

$$T_{ij\upsilon} = t_{ij\upsilon} \text{ if and only if } s_{ij\upsilon}(t_{ij\upsilon}^-) = 1 \text{ and } s_{ij\upsilon}(t_{ij\upsilon}) = 0$$

for $t_{ij\upsilon} > 0$, $(i, j, \upsilon) \in W((l_i, m_i)$: $i = 1,\ldots, k_n)$.
We denote by

$$F_{ij\upsilon}(t) = P(T_{ij\upsilon} \le t), \quad (1)$$
$$t \in (-\infty, +\infty), (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n),$$

the distribution function of the time up to displacement $T_{ij\upsilon}$ of the nanocomponent $E_{ij\upsilon}$.
Definition 2.1. A nanocomponent $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, is failed if it is displaced from its initial position.
Definition 2.2. A function

$$R_{ij\upsilon}(t) = P(T_{ij\upsilon} > t), \quad (2)$$
$$t \in (-\infty, +\infty), (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n),$$

is called a reliability function of a nanocomponent $E_{ij\upsilon}$.

Corollary 2.1. Between the distribution function $F_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i)$: $i = 1,\ldots, k_n)$, of a nanocomponent $E_{ij\upsilon}$ and its reliability function $R_{ij\upsilon}$ the following relationship

$$F_{ij\upsilon}(t) + R_{ij\upsilon}(t) = 1, \quad (3)$$

holds for $t \in (-\infty, +\infty)$.

Definition 2.3. A three-dimensional nanosystem is called homogeneous if all its nanocomponents have the same reliability function $R(t)$, $t \in (-\infty, +\infty)$ i.e.

$$R_{ij\upsilon}(t) = P(T_{ij\upsilon} > t) = R(t), \quad (4)$$
$$t \in (-\infty, +\infty), (i, j, \upsilon) \in W((l_i, m_i): i = 1,\ldots, k_n).$$

Further, we mark by $s(t)$, $t \in <0, +\infty)$ a nanosystem failure stochastic process which is equal to 0 when a nanosystem is failed at the moment t, $t \in <0, +\infty)$ and it is equal to 1 when a nanosystem is not failed at this moment.
We assume that a nanosystem is not failed at the moment $t = 0$ and we mark by T a non-negative continuous random variable that represents the time at which a nanosystem becomes failed. Next, the random variable T will also be called the time up to nanosystem failure or the nanosystem lifetime.
From the above assumptions we conclude that

$$T = t, \text{ if and only if } s(t^-) = 1 \text{ and } s(t) = 0, \text{ for } t > 0.$$

We denote by

$$\boldsymbol{F}(t) = P(T \le t), t \in (-\infty, +\infty), \quad (5)$$

the distribution function of the nanosystem lifetime.

Definition 2.4. We call a function

$$\boldsymbol{R}(t) = P(T > t), t \in (-\infty, +\infty), \quad (6)$$

the reliability function of the three-dimensional nanosystem.

Definition 2.5. A function

$$\boldsymbol{R}_{k_n,l_1,\ldots,l_{k_n},m_1,\ldots,m_{k_n}}(t) = P(T > t), t \in (-\infty,+\infty),$$

where

$$T = \varphi(T_{ij\upsilon}: (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)),$$

and φ is the three-dimensional nanosystem reliability structure function dependent on the nanosystem model and expressing the relationship between the nanosystem lifetime and its nanocomponents times up to their displacements from their initial positions, is called the reliability function of the three-dimensional nanosystem composed of $n \in N_+$ nanocomponents $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$.

Definition 2.6. A three-dimensional nanosystem is called series if it is not failed if and only if all its nanocomponents $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, are not displaced.

Corollary 2.2. The lifetime of a three-dimensional series nanosystem composed of n nanocomponents $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, is equal to

$$T = \min_{1 \le i \le k_n} \{ \min_{\substack{1 \le j \le l_i \\ 1 \le \upsilon \le m_i}} \{T_{ij\upsilon}\}\}, \quad (7)$$

where $T_{ij\upsilon}$ are the nanocomponents $E_{ij\upsilon}$ displacement times.

Definition 2.7. We call a three-dimensional nanosystem series-parallel if its lifetime T is given by

$$T = \max_{1 \le i \le k_n} \{ \min_{\substack{1 \le j \le l_i, \\ 1 \le \upsilon \le m_i}} \{T_{ij\upsilon}\}\}. \quad (8)$$

Definition 2.8 The nanocomponents $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, displacement times $T_{ij\upsilon}$ are independent random variables if and only if

$$\boldsymbol{R}(t_{ij\upsilon} : (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n))$$
$$= \prod_{i=1}^{k_n} [\prod_{\substack{j=1,\ldots,l_i, \\ \upsilon=1,\ldots,m_i}} R_{ij\upsilon}(t_{ij\upsilon})],$$

where

$$\boldsymbol{R}(t_{ij\upsilon} : (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n))$$
$$= P(T_{ij\upsilon} > t_{ij\upsilon} : (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n))$$

for $t_{ij\upsilon} \in (-\infty, +\infty)$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, is a joint reliability function of a random vector

$$(T_{ij\upsilon}, (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n))$$

and

$$R_{ij\upsilon}(t_{ij\upsilon}) = \boldsymbol{R}(-\infty,\ldots,-\infty, t_{ij\upsilon}, -\infty,\ldots,-\infty),$$
$$t_{ij\upsilon} \in (-\infty, +\infty), (i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n),$$

are the reliability functions of the nanocomponents $E_{ij\upsilon}$ defined by (2).

Corollary 2.3. If nanocomponents $E_{ij\upsilon}$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, displacement times $T_{ij\upsilon}$ of the three-dimensional nanosystem are independent random variables, then the reliability function of the three-dimensional

a) series nanosystem is given by

$$\boldsymbol{R}_{k_n,l_1,\ldots,l_{k_n},m_1,\ldots,m_{k_n}}(t) = \prod_{i=1}^{k_n} [\prod_{\substack{j=1,\ldots,l_i, \\ \upsilon=1,\ldots,m_i}} R_{ij\upsilon}(t)], \quad (9)$$

b) series-parallel nanosystem is given by

$$\mathbf{R}_{k_n,l_1,\ldots,l_{k_n},m_1,\ldots,m_{k_n}}(t)$$
$$= 1 - \prod_{i=1}^{k_n} [1 - \prod_{\substack{j=1,\ldots,l_i, \\ \upsilon=1,\ldots,m_i}} R_{ij\upsilon}(t)], \quad (10)$$

where $R_{ij\upsilon}(t)$, $t \in (-\infty, +\infty)$, $(i, j, \upsilon) \in W((l_i, m_i) : i = 1,\ldots, k_n)$, are the reliability functions of its nanocomponents defined by (2).

Definition 2.9. We call three-dimensional nanosystem regular if

$$l_1 = l_2 = \ldots = l_{k_n} = l_n,\ l_n \in N_+,$$
$$m_1 = m_2 = \ldots = m_{k_n} = m_n,\ m_n \in N_+.$$

We mark by

$$\boldsymbol{R}_{k_n,l_n,m_n}(t) = \boldsymbol{R}_{k_n,l_n,\ldots,l_n,m_n,\ldots,m_n}(t),\ t \in (-\infty,+\infty),$$
$$k_n,\ l_n,\ m_n \in N_+,$$

the reliability function of the three-dimensional regular nanosystem.

Corollary 2.4. Under assumptions from *Corollary 2.5* and assuming that the pondered nanosystem is homogeneous the reliability function of the three-dimensional

a) regular series nanosystem is given by

$$\boldsymbol{R}_{k_n,l_n,m_n}(t) = [R(t)]^{k_n l_n m_n} = [R(t)]^n, \quad (11)$$

b) regular series-parallel nanosystem is given by

$$\boldsymbol{R}_{k_n,l_n,m_n}(t) = 1 - [1 - [R(t)]^{l_n \cdot m_n}]^{k_n}, \quad (12)$$

where k_n, l_n, m_n, $n \in N_+$ and $R(t)$, $t \in (-\infty, +\infty)$, is the reliability function of its nanocomponents defined by (4).

Further, we will also mull over more general case when the nanocomponents $E_{ij\upsilon}$, $(i, j, \upsilon) \in D_{k_n l_n m_n}$, displacement times $T_{ij\upsilon}$ of the regular three-dimensional nanosystem are dependent random variables, formulated in the following assumption.

Assumption 2.1. The dependence between $T_{ij\upsilon}$ and $T_{i'j'\upsilon'}$, $((i, j, \upsilon), (i', j', \upsilon')) \in Z_{k_n l_n m_n}$, decreases with the increasing distance $d((i, j, \upsilon), (i', j', \upsilon'))$ between them in that way they are independent when this distance tends to infinity.

3. Asymptotic approach to reliability of three-dimensional nanosystems

Considering the reliability of three-dimensional nanosystems we assume that the distributions of the nanocomponents displacement times and the nanosystem lifetime T do not necessarily have to be concentrated in the interval $< 0, +\infty)$. It means that a reliability function $\boldsymbol{R}(t)$, $t \in (-\infty, +\infty)$, does not have to satisfy the usually demanded condition

$$\boldsymbol{R}(t) = 1 \text{ for } t < 0,$$

At the same time, from the achieved results on the generalized reliability functions, for particular cases, the same properties of the normally used reliability functions appear.

From that assumption it follows that between a reliability function $\boldsymbol{R}(t)$, $t \in (-\infty, +\infty)$, and a distribution function $\boldsymbol{F}(t)$ there exists a relationship given by

$$\boldsymbol{R}(t) = 1 - \boldsymbol{F}(t) \text{ for } t \in (-\infty, +\infty).$$

Thus, the following corollary is obvious.

Corollary 3.1. A reliability function $\boldsymbol{R}(t)$ is nonincreasing, right-continuous and $\boldsymbol{R}(-\infty) = 1$, $\boldsymbol{R}(+\infty) = 0$.

Definition 3.1. A reliability function $\boldsymbol{R}(t)$ is called degenerate if there exists $t_0 \in (-\infty, +\infty)$, such that

$$\boldsymbol{R}(t) = 1 \text{ for } t < t_0 \text{ and } \boldsymbol{R}(t) = 0 \text{ for } t \geq t_0.$$

Corollary 3.2. A function

a) $\boldsymbol{R}(t) = 1 - \exp(-V(t))$, $t \in (-\infty, +\infty)$,

b) $\overline{\boldsymbol{R}}(t) = \exp(-\overline{V}(t))$, $t \in (-\infty, +\infty)$,

is a reliability function if and only if

a) a function $V(t)$ is non-negative, non-increasing, right continuous, $V(-\infty) = +\infty$, $V(+\infty) = 0$,

b) a function $\overline{V}(t)$ is non-negative, non-decreasing, right continuous, $\overline{V}(-\infty) = 0$, $\overline{V}(+\infty) = +\infty$,

and moreover, $V(t)$ and $\overline{V}(t)$ can be identically equal to $+\infty$ in an interval.

Agreement 3.1. In further considerations if we use symbols $V(t)$ and $\overline{V}(t)$ we always mean functions of properties given in *Corollary 3.2*.

If $V(t)$ and $\overline{V}(t)$ are identically equal to $+\infty$ we assume that

$$\exp(-\overline{V}(t)) = 0 \text{ and } \exp(-V(t)) = 0.$$

If we say that $V(t)$ and $\overline{V}(t)$ are a non-negative, non-decreasing or non-increasing and right-continuous we mean the intervals where $V(t)$, $\overline{V}(t) \neq +\infty$.

Moreover, we denote the set of continuity points of a reliability function $\boldsymbol{R}(t)$ by $C_{\boldsymbol{R}}$, $\overline{\boldsymbol{R}}(t)$ by $C_{\overline{\boldsymbol{R}}}$, the set of continuity points of a function $V(t)$ and points such that $V(t) = +\infty$ by C_V and similarly, the set of continuity points of a function $\overline{V}(t)$ and points such that $\overline{V}(t) = +\infty$ by $C_{\overline{V}}$.

Definition 3.2. A function $V(t)$ is called degenerate if there exists $t_0 \in (-\infty, +\infty)$, such that

$$V(t) = +\infty \text{ for } t < t_0 \text{ and } V(t) = 0 \text{ for } t \geq t_0$$

and similarly, a function $\overline{V}(t)$ is called degenerate if there exists $t_0 \in (-\infty, +\infty)$, such that

$$\overline{V}(t) = 0 \text{ for } t < t_0 \text{ and } \overline{V}(t) = +\infty \text{ for } t \geq t_0.$$

Under this definition the following corollary is clear.

Corollary 3.3. A reliability function

a) $\boldsymbol{R}(t) = 1 - \exp(-V(t))$, $t \in (-\infty, +\infty)$,

b) $\overline{\boldsymbol{R}}(t) = \exp(-\overline{V}(t))$, $t \in (-\infty, +\infty)$,

is degenerate if and only if

a) a function $V(t)$ is degenerate,

b) a function $\overline{V}(t)$ is degenerate.

The asymptotic approach to the reliability of nanosystems depends on the investigation of limit distributions of a standardised random variable $(T - a_n)/b_n$ where T is the lifetime of a nanosystem and $a_n > 0$, $b_n \in (-\infty, +\infty)$, are suitably chosen numbers called normalising constants. Since

$$P((T - b_n)/a_n > t) = P(T > a_n t + b_n)$$
$$= \boldsymbol{R}_{k_n,l_n,m_n}(a_n t + b_n),$$

where $\boldsymbol{R}_{k_n,l_n,m_n}(t)$ is a reliability function of a regular nanosystem composed of $n \in N_+$ nano-components, then the following definition becomes natural.

Definition 3.4. A reliability function $\mathfrak{R}(t)$ is called a limit reliability function or an asymptotic reliability function of a regular nanosystem having a reliability function $\boldsymbol{R}_{k_n,l_n,m_n}(t)$ if there exist normalising constants $a_n > 0$, $b_n \in (-\infty, +\infty)$, such that

$$\lim_{n\to+\infty} \boldsymbol{R}_{k_n,l_n,m_n}(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}}.$$

Thus, if the asymptotic reliability function $\mathfrak{R}(t)$ of a system is known, then for sufficiently large $n \in N_+$, the approximate formula

$$\boldsymbol{R}_{k_n,l_n,m_n}(t) \cong \mathfrak{R}((t - b_n)/a_n),\ t \in (-\infty, +\infty), \qquad (13)$$

may be used instead of the system exact reliability function $\boldsymbol{R}_{k_n,l_n,m_n}(t)$. From the condition

$$\lim_{n\to+\infty} \boldsymbol{R}_{k_n,l_n,m_n}(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}},$$

it follows that setting

$$\alpha_n = a a_n,\ \beta_n = b a_n + b_n,$$

where $a > 0$ and $b \in (-\infty, +\infty)$, for $t \in C_{\mathfrak{R}}$ we receive

$$\lim_{n\to+\infty} \boldsymbol{R}_{k_n,l_n,m_n}(\alpha_n t + \beta_n) =$$
$$= \lim_{n\to+\infty} \boldsymbol{R}_{k_n,l_n,m_n}(a_n(at + b) + b_n) = \mathfrak{R}(at + b)$$

Hence, if $\mathfrak{R}(t)$ is the limit reliability function of a system, then $\mathfrak{R}(at + b)$ with arbitrary $a > 0$ and $b \in (-\infty, +\infty)$, is also its limit reliability function. That fact, in a natural way, yields the concept of a type of limit reliability function.

Definition 3.5. The limit reliability functions $\mathfrak{R}_0(t)$ and $\mathfrak{R}(t)$ are said to be of the same type if there exist numbers $a > 0$ and $b \in (-\infty, +\infty)$, such that

$$\mathfrak{R}_0(t) = \mathfrak{R}(at + b) \text{ for } t \in (-\infty, +\infty),$$

Agreement 3.2. In further considerations we assume the following notation:
$x(n) << y(n)$ or $x(n) = o(y(n))$, where $x(n)$ and $y(n)$ are positive functions, means that $x(n)$ is of order much less than $y(n)$ in a sense

$$\lim_{n\to+\infty} x(n)/y(n) = 0.$$

4. Limit reliability of the three-dimensional nanosystem with independent nanocomponents

The investigations of limit reliability functions of homogeneous regular nanosystems with independent nanocomponents are based on following auxiliary lemmas.

Lemma 4.1. If

(i) $\overline{\mathfrak{R}}(t) = \exp(-\overline{V}(t))$, is a non-degenerate reliability function,

(ii) $\overline{\boldsymbol{R}}_{k_n,l_n,m_n}(t)$ is the reliability function of a homogeneous regular series nanosystem with independent nanocomponents defined by (12),

(iii) $a_n > 0$, $b_n \in (-\infty, +\infty)$

then

$$\lim_{n\to+\infty} \overline{\mathbf{R}}_{k_n,l_n,m_n}(a_n t + b_n) = \overline{\mathfrak{R}}(t) \text{ for } t \in C_{\overline{\mathfrak{R}}} \qquad (14)$$

if and only if

$$\lim_{n\to+\infty} nF(a_n t + b_n) = \overline{V}(t) \text{ for } t \in C_{\overline{V}}. \qquad (15)$$

Lemma 4.2. If

(i) $k_n \to k > 0$, $l_n \cdot m_n \to +\infty$,

(ii) $\mathfrak{R}(t)$ is a non-degenerate reliability function,

(iii) $\boldsymbol{R}_{k_n,l_n,m_n}(t)$ is the reliability function of a homogeneous regular series-parallel nanosystem with independent nanocomponents defined by (13),

(iv) $a_n > 0$, $b_n \in (-\infty, +\infty)$

then

$$\lim_{n\to+\infty} \boldsymbol{R}_{k_n,l_n,m_n}(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}},$$

if and only if

$$\lim_{n\to\infty} [R(a_n t + b_n)]^{l_n \cdot m_n} = \mathfrak{R}_0(t) \text{ for } t \in C_{\mathfrak{R}_0},$$

where $\mathfrak{R}_0(t)$ is a non-degenerate reliability function and

$$\mathfrak{R}(t) = 1 - [1 - \mathfrak{R}_0(t)]^k \text{ for } t \in (-\infty, +\infty).$$

Lemma 4.1 - 4.2 are an essential tool in finding limit reliability functions of homogeneous regular series and series-parallel nanosystems with independent nanocomponents. Their various proofs may be found in [1], [4] and [5]. They also are the basis for fixing the class of all possible limit reliability functions of these systems. These classes are determined by the following theorems proved in [1], [4] and [5].

Theorem 4.1. The only non-degenerate limit reliability functions of the homogeneous regular three-dimensional series nanosystem with independent nanocomponents are:

$$\overline{\mathfrak{R}}_1(t) = \begin{cases} \exp[-(-t)^{-\alpha}], & t < 0 \\ 0, & t \ge 0, \end{cases} \quad \text{for } \alpha > 0,$$

$$\overline{\mathfrak{R}}_2(t) = \begin{cases} 1, & t < 0 \\ \exp[-t^{\alpha}], & t \ge 0, \end{cases} \quad \text{for } \alpha > 0,$$

$$\overline{\mathfrak{R}}_3(t) = \exp[-\exp[t]] \text{ for } t \in (-\infty, +\infty).$$

The classes of limit reliability functions of a homogeneous regular three-dimensional series-parallel nanosystem with independent nanocomponents depend on the relationships between numbers k_n and $l_n \cdot m_n$ [5].

Theorem 4.2. If $k_n \to k$, $k > 0$, and $l_n \cdot m_n \to +\infty$, then the only non-degenerate limit reliability functions of the homogeneous regular series-parallel nanosystem with independent nanocomponents are:

$$\mathfrak{R}_1(t) = \begin{cases} 1 - (1 - \exp[-(-t)^{-\alpha}])^k, & t < 0 \\ 0, & t \ge 0, \end{cases} \quad \text{for } \alpha > 0,$$

$$\mathfrak{R}_2(t) = \begin{cases} 1, & t < 0 \\ 1 - (1 - \exp[-t^{\alpha}])^k, & t \ge 0, \end{cases} \quad \text{for } \alpha > 0,$$

$$\mathfrak{R}_3(t) = 1 - (1 - \exp[-\exp(t)])^k \text{ for } t \in (-\infty, +\infty).$$

5. Limit reliability of the three-dimensional nanosystem with dependent nanocomponents

To investigate the limit reliability functions of some three-dimensional homogeneous regular series and series-parallel nanosystems with dependent nanocomponents which satisfy *Assumption 2.1* we can use modified *Lemma 4.1* and *Lemma 4.2*.

Theorem 5.1. If the joint reliability function of the homogeneous regular series nanosystem is given by

$$\boldsymbol{R}_{k_n,l_n,m_n}(t_{ij\upsilon} : (i,j,\upsilon) \in D_{k_n l_n m_n}) = [\prod_{(i,j,\upsilon)\in D_{k_n l_n m_n}} R(t_{ij\upsilon})] \cdot H(t_{ij\upsilon} : (i,j,\upsilon) \in D_{k_n l_n m_n}), \quad (16)$$

where

$$H(t_{ij\upsilon} : (i,j,\upsilon) \in D_{k_n l_n m_n}) = \prod_{((i,j,\upsilon),(i',j',\upsilon'))\in Z_{k_n l_n m_n}} h[d((i,j,\upsilon),(i',j',\upsilon')), R(t_{ij\upsilon}), R(t_{i'j'\upsilon'})],$$

$$d((i,j,\upsilon),(i',j',\upsilon')) = \sqrt{(i-i')^2 + (j-j')^2 + (\upsilon-\upsilon')^2},$$

$t_{ij\upsilon}$, $t_{i'j'\upsilon'} \in (-\infty, +\infty)$, $((i, j, \upsilon), (i', j', \upsilon')) \in Z_{k_n l_n m_n}$,
$R(t)$ is a reliability function of the nanocomponent,

$$h : N_+ \times <0, 1>^2 \to <0, 1>, \quad (17)$$

$$\lim_{k\to+\infty} h(k, x, y) = 1, \; x, y \in <0, 1>, \quad (18)$$

$$h(k,x,y) = h(k,y,x), \; x, y \in <0, 1>, \; k \in N_+, \quad (19)$$

$$h(k,1,y) = 1, \; y \in <0, 1>, \; k \in N_+, \quad (20)$$

$h(k,x,y)$ is increasing for fixed $x, y \in <0, 1>$

and for fixed $k \in N_+$, $x \in <0, 1>$, (21)

$\overline{\mathfrak{R}}(t) = \exp(-\overline{V}(t))$, $t \in (-\infty, +\infty)$ is a non-degenerate reliability function, $a_n > 0$, $b_n \in (-\infty, +\infty)$,

$$h(1, R(a_n t + b_n), R(a_n t + b_n)) = 1 - o(1/n^2), \quad (22)$$

then

$$\lim_{n\to+\infty} \overline{\boldsymbol{R}}_{k_n,l_n,m_n}(a_n t + b_n) = \overline{\mathfrak{R}}(t) \text{ for } t \in C_{\overline{\mathfrak{R}}} \tag{23}$$

if and only if

$$\lim_{n\to+\infty} nF(a_n t + b_n) = \overline{V}(t) \text{ for } t \in C_{\overline{V}}. \tag{24}$$

Proof: Obviously function

$\boldsymbol{R}_{k_n,l_n,m_n}(t_{ij\upsilon} : (i,j,\upsilon) \in D_{k_n l_n m_n})$, $t_{ij\upsilon} \in (-\infty, +\infty)$, $(i,j,\upsilon) \in D_{k_n l_n m_n}$,

given by (16), is a joint reliability function.
Moreover,

$$\lim_{d((i,j,\upsilon),(i',j',\upsilon'))\to+\infty} P(T_{ij\upsilon} > t_{ij\upsilon}, T_{i'j'\upsilon'} > t_{i'j'\upsilon'},) =$$
$$= \lim_{d((i,j,\upsilon),(i',j',\upsilon'))\to+\infty} \boldsymbol{R}_{k_n,l_n,m_n}(-\infty,\ldots,-\infty, t_{ij\upsilon}, -\infty,\ldots,-\infty,$$
$$-\infty,\ldots,-\infty, t_{i'j'\upsilon'}, -\infty,\ldots,-\infty) =$$
$$= \lim_{d((i,j,\upsilon),(i',j',\upsilon'))\to+\infty} R(t_{ij\upsilon}) R(t_{i'j'\upsilon'}) \cdot$$
$$\cdot\, h[d((i,j,\upsilon),(i',j',\upsilon')), R(t_{ij\upsilon}), R(t_{i'j'\upsilon'})] =$$
$$= R(t_{ij\upsilon}) \cdot R(t_{i'j'\upsilon'}),$$

for $t_{ij\upsilon}$, $t_{i'j'\upsilon'} \in (-\infty, +\infty)$, $((i,j,\upsilon), (i',j',\upsilon')) \in Z_{k_n l_n m_n}$, so this model of the nanosystem fulfills *Assumption 2.1.*
Further, for simplify our notation we mark by

$$g_1(a,b,c,t) =$$
$$= \prod_{\substack{i=1,\ldots,a,\\ j=1,\ldots,b,\\ \upsilon=1,\ldots,c}} h(\sqrt{i^2+j^2+k^2}, R(t), R(t))^{4(a-i)(b-j)(c-\upsilon)},$$

$$g_1(a,b,c,t) =$$
$$= \prod_{\substack{i=1,\ldots,a,\\ j=1,\ldots,b}} h(\sqrt{i^2+j^2}, R(t), R(t))^{2(a-i)(b-j)c},$$

$$g_3(a,b,t) = \prod_{i=1,\ldots,a} h(i, R(t), R(t))^{(a-i)b},$$

where $t \in (-\infty, +\infty)$ and $a,b,c \in N_+$.
We can clearly see that

$$H(t,t,\ldots,t) = g_1(k_n, l_n, m_n, t) \cdot$$
$$\cdot\, g_2(k_n, l_n, m_n, t) \cdot g_2(l_n, m_n, k_n, t) \cdot g_2(m_n, k_n, l_n, t) \cdot$$
$$\cdot\, g_3(k_n, l_n m_n, t) \cdot g_3(l_n, m_n k_n, t) \cdot g_3(m_n, k_n l_n, t)$$

for $t \in (-\infty, +\infty)$.
According to conditions (17) and (21) we obtain for $t \in (-\infty, +\infty)$

$$[h(1, R(t), R(t))]^{13n^2} \le H(t,t,\ldots,t) \le 1.$$

Introduce constants $a_n > 0$, $b_n \in (-\infty, +\infty)$, which satisfy the condition (22). Thus, from (22) we get

$$\lim_{n\to+\infty} [h(1, R(a_n t + b_n), R(a_n t + b_n))]^{13n^2} =$$
$$= \lim_{n\to+\infty} [1 - o(1/n^2)]^{13n^2} =$$
$$= \lim_{n\to+\infty} \exp[-o(1/n^2) \cdot 13n^2] = \exp[0] = 1,$$

for all $t \in (-\infty, +\infty)$. It follows that, according to the squeeze theorem

$$\lim_{n\to+\infty} H(a_n t + b_n, \ldots, a_n t + b_n) = 1, t \in (-\infty, +\infty). \tag{25}$$

Next, assume that

$$\lim_{n\to+\infty} nF(a_n t + b_n) = \overline{V}(t), t \in C_{\overline{V}},$$

for $a_n > 0$, $b_n \in (-\infty, +\infty)$. Of course, using (25) for $t \in C_{\overline{V}}$ we receive

$$\lim_{n\to+\infty} \overline{\boldsymbol{R}}_{k_n,l_n,m_n}(a_n t + b_n) = \lim_{n\to+\infty} [R(a_n t + b_n)]^n =$$
$$= \lim_{n\to+\infty} \exp[-nF(a_n t + b_n)] = \exp[-\overline{V}(t)] = \overline{\mathfrak{R}}(t).$$

Further, assume that

$$\lim_{n\to+\infty} \overline{\boldsymbol{R}}_{k_n,l_n,m_n}(a_n t + b_n) = \overline{\mathfrak{R}}(t) = \exp[-\overline{V}(t)],$$

for $t \in C_{\overline{\mathfrak{R}}}$, $a_n > 0$, $b_n \in (-\infty, +\infty)$. Hence, according to (25) we can see that

$$\lim_{n\to+\infty} \overline{\boldsymbol{R}}_{k_n,l_n,m_n}(a_n t + b_n) = \lim_{n\to+\infty} [R(a_n t + b_n)]^n =$$
$$= \exp[-\overline{V}(t)],$$

And

$$\lim_{n\to+\infty} [R(a_n t + b_n)]^n = \lim_{n\to+\infty} \exp[-nF(a_n t + b_n)] =$$

$$= \exp[-\overline{V}(t)], t \in C_{\overline{V}},$$

so consequently

$$\lim_{n\to+\infty} nF(a_n t + b_n) = \overline{V}(t) \text{ for } t \in C_{\overline{V}},$$

what completes the proof. □

Theorem 5.2. The only non-degenerate limit reliability functions of the homogeneous regular three-dimensional series system with dependent nanocomponents, which fulfill assumptions from *Theorem 5.1*, are same as functions from *Theorem 4.1*.

Example 5.1. Consider function h given by

$$h(k, x, y) = 1- c\cdot[(1 - x)(1 - y)]^q / k \quad (26)$$

for $c \in (0, 1>$, $q > 1$, $x, y \in <0, 1>$, $k \in N_+$.
It is easy to prove that function h satisfies conditions (17)-(21). Moreover, if we assume that

$$\lim_{n\to+\infty} nF(a_n t + b_n) = \overline{V}(t) \text{ for } t \in C_{\overline{V}},$$

for $a_n > 0$, $b_n \in (-\infty, +\infty)$, we will obtain

$$\lim_{n\to+\infty} \frac{h(1, R(a_n t + b_n), R(a_n t + b_n)) - 1}{1/n^2} =$$

$$= \lim_{n\to+\infty} \frac{-c\cdot([1 - R(a_n t + b_n)]\cdot[1 - R(a_n t + b_n)])^q}{1/n^2} =$$

$$= \lim_{n\to+\infty} -c\cdot\left(F(a_n t + b_n)\right)^{2q}\cdot n^2 =$$

$$= \lim_{n\to+\infty} -c\cdot\left(nF(a_n t + b_n)\right)^{2q}\cdot n^{2-2q} =$$

$$= -c[\overline{V}(t)]^{2q}\cdot 0 = 0, \; c \in (0, 1>, \; q > 1, \; t \in C_{\overline{V}}.$$

Thus,

$$h(1, R(a_n t + b_n), R(a_n t + b_n)) = 1 - o(1/n^2)$$

for $n \in N_+$, $t \in C_{\overline{V}}$.

Theorem 5.3. If the joint reliability function of the homogeneous regular series-parallel nanosystem is the same as the joint reliability function given by (16), fulfils conditions (17)-(22) and

(i) $k_n \equiv k > 0$, $l_n \cdot m_n \to +\infty$,
(ii) $\mathfrak{R}(t)$ is a non-degenerate reliability function,
(iii) $a_n > 0$, $b_n \in (-\infty, +\infty)$,

then

$$\lim_{n\to+\infty} \boldsymbol{R}_{k,l_n,m_n}(a_n t + b_n) = \mathfrak{R}(t) \text{ for } t \in C_{\mathfrak{R}}$$

if and only if

$$\lim_{n\to+\infty} [R(a_n t + b_n)]^{l_n\cdot m_n} = \mathfrak{R}_0(t) \text{ for } t \in C_{\mathfrak{R}_0},$$

where $\mathfrak{R}_0(t)$ is a non-degenerate reliability function and

$$\mathfrak{R}(t) = 1 - [1 - \mathfrak{R}_0(t)]^k \text{ for } t \in (-\infty, +\infty).$$

Proof: First we must answer the question how the reliability function of the homogeneous regular series-parallel nanosystem with dependent nanocomponents which satisfies *Assumption 2.1* looks like. From (8) we get

$$\boldsymbol{R}_{k,l_n,m_n}(t) = P(\max_{i=1,\dots,k}\{\min_{\substack{j=1,\dots,l_n,\\ \upsilon=1,\dots,m_n}}\{T_{ij\upsilon}\}\} > t) =$$

$$= P(\bigcup_{i=1,\dots,k}\bigcap_{\substack{j=1,\dots,l_n,\\ \upsilon=1,\dots,m_n}}\{T_{ij\upsilon} > t\}).$$

Further, for simplify our notation, we denote by

$$A_i(t) = \bigcap_{\substack{j=1,\dots,l_n,\\ \upsilon=1,\dots,m_n}}\{T_{ij\upsilon} > t\}, \; i = 1,\dots, k, \; t \in (-\infty, +\infty).$$

Hence,

$$\boldsymbol{R}_{k,l_n,m_n}(t) = \sum_{i=1,\dots,k} P(A_i(t)) + \quad (27)$$

$$- \sum_{\substack{i_1,i_2=1,\dots,k,\\ i_1<i_2}} P(A_{i_1}(t)\cap A_{i_2}(t)) + \dots +$$

$$+ (-1)^{k+1}\cdot P(A_1(t)\cap\dots\cap A_k(t)), \; t \in (-\infty, +\infty).$$

Moreover for $i = 1,\dots, k$, $t \in (-\infty, +\infty)$,

$$P(A_i(t)) = \boldsymbol{R}_{k,l_n,m_n}(-\infty,\dots,-\infty, \underbrace{t,\dots,t,}_{\text{positions } (i-1)\cdot l_n\cdot m_n+1,\dots,i\cdot l_n\cdot m_n} -\infty,\dots,-\infty) =$$

$$= R^{l_n m_n}(t)\cdot\overline{H}(t),$$

where

$$\overline{H}(t) = \prod_{j=1,\dots,l_n} h(j, R(t), R(t))^{(l_n-j)m_n}\cdot \quad (28)$$

$$\cdot \prod_{\substack{j=1,\dots,l_n,\\ \upsilon=1,\dots,m_n}} h(\sqrt{j^2+\upsilon^2}, R(t), R(t))^{2(l_n-j)(m_n-\upsilon)}\cdot$$

$$\cdot \prod_{\upsilon=1,\dots,m_n} h(\upsilon, R(t), R(t))^{l_n(m_n-\upsilon)}, \; t \in (-\infty, +\infty),$$

because $P(A_i(t))$ could be pondered as the reliability function of homogeneous regular two-dimensional series nanosystem composed of $l_n \cdot m_n \in N_+$ nanocomponents. Similarly

$$P(A_{i_1}(t) \cap \ldots \cap A_{i_\varepsilon}(t)) = (R^{l_n m_n}(t) \cdot \overline{H}(t))^{\varepsilon} \cdot \cdot \prod_{\substack{\eta_1, \eta_2 = i_1, \ldots, i_\varepsilon, \\ \eta_1 < \eta_2}} \overline{\overline{H}}(t, \eta_2 - \eta_1), \qquad (29)$$

where

$$\overline{\overline{H}}(t,i) = h(i, R(t), R(t))^{l_n m_n} \cdot \qquad (30)$$
$$\cdot \prod_{\substack{j=1,\ldots,l_n, \\ \upsilon=1,\ldots,m_n}} h[\sqrt{i^2 + j^2 + \upsilon^2}, R(t), R(t)]^{4(l_n - j)(m_n - \upsilon)} \cdot$$
$$\cdot \prod_{j=1,\ldots,l_n} h[\sqrt{i^2 + j^2}, R(t), R(t)]^{2(l_n - j)m_n} \cdot$$
$$\cdot \prod_{\upsilon=1,\ldots,m_n} h[\sqrt{i^2 + \upsilon^2}, R(t), R(t)]^{2 l_n (m_n - \upsilon)},$$

$i \in N_+$, $t \in (-\infty, +\infty)$ and $i, i_1, \ldots, i_\varepsilon$, $\varepsilon = 1, \ldots, k$. Since according to (17), (22) we have

$$1 \geq \overline{\overline{H}}(t,i) \geq h(1, R(t), R(t))^{9(l_n \cdot m_n)^2},$$
$$1 \geq \overline{H}(t) \geq h(1, R(t), R(t))^{4(l_n \cdot m_n)^2},$$

for $i = 1, \ldots, k$, $t \in (-\infty, +\infty)$, then

$$R^{\varepsilon \cdot l_n m_n}(t) \geq P(A_{i_1}(t) \cap \ldots \cap A_{i_\varepsilon}(t)) \geq$$
$$\geq R^{\varepsilon \cdot l_n m_n}(t) \cdot h(1, R(t), R(t))^{13\varepsilon^2 \cdot (l_n \cdot m_n)^2} \geq$$
$$\geq R^{\varepsilon \cdot l_n m_n}(t) \cdot h(1, R(t), R(t))^{13 n^2}.$$

Moreover, using (22) we receive

$$\lim_{n \to \infty} [h(1, R(a_n t + b_n), R(a_n t + b_n))]^{n^2} =$$
$$= \lim_{n \to \infty} [1 - o(1/n^2)]^{n^2} = 1,$$

for $t \in (-\infty, +\infty)$, $a_n > 0$, $b_n \in (-\infty, +\infty)$. It follows that, according to the squeeze theorem

$$\lim_{n \to \infty} P(A_{i_1}(a_n t + b_n) \cap \ldots \cap A_{i_\varepsilon}(a_n t + b_n)) =$$
$$= \lim_{n \to \infty} R^{\varepsilon \cdot l_n m_n}(a_n t + b_n),$$

where $t \in (-\infty, +\infty)$ and $i, i_1, \ldots, i_\varepsilon$, $\varepsilon = 1, \ldots, k$. Consequently for $t \in (-\infty, +\infty)$

$$\lim_{n \to \infty} \boldsymbol{R}_{k, l_n, m_n}(a_n t + b_n) =$$
$$= \lim_{n \to \infty} \sum_{\varepsilon=1,\ldots,k} [(-1)^{\varepsilon+1} \cdot \sum_{i_1 < i_2 < \ldots < i_\varepsilon = 1,\ldots,k} P(A_{i_1}(t) \cap \ldots \cap A_{i_\varepsilon}(t)) =$$
$$= \sum_{\varepsilon=1,\ldots,k} [(-1)^{\varepsilon+1} \cdot$$
$$\cdot (\sum_{i_1 < i_2 < \ldots < i_\varepsilon = 1,\ldots,k} [\lim_{n \to \infty} P(A_{i_1}(t) \cap \ldots \cap A_{i_\varepsilon}(t))])] =$$
$$= \sum_{\varepsilon=1,\ldots,k} [(-1)^{\varepsilon+1} \cdot \sum_{i_1 < i_2 < \ldots < i_\varepsilon = 1,\ldots,k} \lim_{n \to \infty} R^{\varepsilon \cdot l_n \cdot m_n}(a_n t + b_n)] =$$
$$= \sum_{\varepsilon=1,\ldots,k} [(-1)^{\varepsilon+1} \binom{k}{\varepsilon} \cdot \lim_{n \to \infty} R^{\varepsilon \cdot l_n \cdot m_n}(a_n t + b_n)] =$$
$$= 1 - [1 - \sum_{\varepsilon=1,\ldots,k} [(-1)^{\varepsilon} \binom{k}{\varepsilon} \cdot \lim_{n \to \infty} R^{\varepsilon \cdot l_n \cdot m_n}(a_n t + b_n)]] =$$
$$= 1 - [1 - \lim_{n \to \infty} R^{l_n \cdot m_n}(a_n t + b_n)]^k. \qquad (31)$$

Further, assume that

$$\lim_{n \to \infty} [R(a_n t + b_n)]^{l_n \cdot m_n} = \boldsymbol{\Re}_0(t), t \in C_{\boldsymbol{\Re}_0}, \qquad (32)$$

where $\boldsymbol{\Re}_0(t)$ is a non-degenerate reliability function, $a_n > 0$, $b_n \in (-\infty, +\infty)$, $t \in (-\infty, +\infty)$. Hence,

$$\lim_{n \to \infty} \boldsymbol{R}_{k, l_n, m_n}(a_n t + b_n) =$$
$$= 1 - [1 - \lim_{n \to \infty} R^{l_n \cdot m_n}(a_n t + b_n)]^k =$$
$$= 1 - [1 - \boldsymbol{\Re}_0(t)]^k = \boldsymbol{\Re}(t), t \in (-\infty, +\infty).$$

Next, we assume that

$$\lim_{n \to +\infty} \boldsymbol{R}_{k, l_n, m_n}(a_n t + b_n) = \boldsymbol{\Re}(t) \text{ for } t \in C_{\boldsymbol{\Re}}. \qquad (33)$$

Thus

$$\lim_{n \to \infty} \boldsymbol{R}_{k, l_n, m_n}(a_n t + b_n) =$$
$$= 1 - [1 - \lim_{n \to \infty} R^{l_n \cdot m_n}(a_n t + b_n)]^k =$$
$$= 1 - [1 - \boldsymbol{\Re}_0(t)]^k = \boldsymbol{\Re}(t), t \in (-\infty, +\infty).$$

so finally

$$\lim_{n \to \infty} [R(a_n t + b_n)]^{l_n \cdot m_n} = \boldsymbol{\Re}_0(t), t \in C_{\boldsymbol{\Re}_0}. \qquad \square$$

Theorem 5.4. The only non-degenerate limit reliability functions of the homogeneous regular three-dimensional series-parallel system with dependent nanocomponents, which satisfy assumptions from *Theorem 5.3*, are same as functions from *Theorem 4.2*.

6. An example

Example 6.1. Mull over the three-dimensional series nanosystem which reliability function is given by (16) where function

$$h(k, x, y) = 1 - [(1 - x)(1 - y)]^{1.1} / k, \quad (34)$$

where $x, y \in <0, 1>$, $k \in N_+$.

Obviously $h(k, x, y)$ given by (34) is an example of the function (26), so fulfills conditions (17)-(22). Moreover, assume that the survival function of nanocomponents

$$R(t) = \begin{cases} 1, & t < 0, \\ \exp[-\lambda t], & t \geq 0, \end{cases} \quad \text{for } \lambda > 0. \quad (35)$$

Introduce constants

$$a_n = 1/(\lambda n),\ b_n = 0,\ n \in N_+, \quad (36)$$

then we receive

$$\lim_{n \to +\infty} nF(a_n t + b_n) = \lim_{n \to +\infty} n \cdot (1 - \exp[-t/n]) =$$
$$= \lim_{n \to +\infty} n \cdot (t/n - o(1/n)) = t, t > 0, \quad (37)$$

and

$$\lim_{n \to +\infty} nF(a_n t + b_n) = 0, t \leq 0. \quad (38)$$

Using this fact and *Theorem 5.1* we obtain that the asymptotic reliability function of considered nanosystem is

$$\overline{\mathfrak{R}}_2(t) = \lim_{n \to +\infty} \overline{\boldsymbol{R}}_{k_n, l_n, m_n}(t/(\lambda n)) = \begin{cases} 1, & t < 0 \\ \exp[-t], & t \geq 0. \end{cases}$$

Assume that $k_{900} = 4$, $l_{900} = 15$, $m_{900} = 15$, $\lambda = 1/90$. Then the exact reliability function of the nanosystem

$$\overline{\boldsymbol{R}}_{4,15,15}(t) = \exp(-10t) \cdot$$
$$\cdot \prod_{\substack{i=1,\dots,4,\\ j=1,\dots,15,\\ \upsilon=1,\dots,15}} [1 - \frac{(1-e^{-t/90})^{2.2}}{\sqrt{i^2 + j^2 + \upsilon^2}}]^{4(4-i)(15-j)(15-\upsilon)} \cdot$$
$$\cdot [\prod_{\substack{i=1,\dots,4,\\ j=1,\dots,15}} [1 - \frac{(1-e^{-t/90})^{2.2}}{\sqrt{i^2 + j^2}}]^{2(4-i)(15-j)\cdot 15}]^2 \cdot$$
$$\cdot [\prod_{\substack{i=1,\dots,15,\\ j=1,\dots,15}} [1 - \frac{(1-e^{-t/90})^{2.2}}{\sqrt{i^2 + j^2}}]^{2(15-i)(15-j)\cdot 4}] \cdot$$
$$\cdot [\prod_{i=1,\dots,15} [1 - \frac{(1-e^{-t/90})^{2.2}}{i}]^{(15-i)\cdot 60}]^2 \cdot$$
$$\cdot [\prod_{i=1,\dots,4} [1 - \frac{(1-e^{-t/90})^{2.2}}{i}]^{(4-i)\cdot 225}], t > 0,$$

and

$$\overline{\boldsymbol{R}}_{4,15,15}(t) = 1, t \leq 0,$$

could be approximated by

$$\overline{\boldsymbol{R}}_{4,15,15}(t) \cong \overline{\mathfrak{R}}_2(10t) = \begin{cases} 1, & t < 0 \\ \exp[-10t], & t \geq 0. \end{cases} \quad (39)$$

This follows from (13).

Similar consideration we can perform for the three-dimensional series-parallel nanosystem in which reliability function is given by (16), function h by (34), $R(t)$ by (35), and $k_n \equiv k > 0$ is a positive constant.

It this case, since for constants (36) according to (37) and (38)

$$\lim_{n \to +\infty} nF(a_n t + b_n) = \begin{cases} 0, & t < 0, \\ t, & t \geq 0, \end{cases}$$

then

$$\lim_{n \to +\infty} [R(a_n t + b_n)]^{l_n \cdot m_n}$$
$$= (\lim_{n \to +\infty} [R(a_n t + b_n)]^n)^{1/k} = (\lim_{n \to +\infty} [1 - F(a_n t + b_n)]^n)^{1/k}$$
$$= (\lim_{n \to +\infty} \exp[-nF(a_n t + b_n)])^{1/k} = \begin{cases} 1, & t < 0 \\ \exp[-t/k], & t \geq 0. \end{cases}$$

From this fact and *Theorem 5.3* follow that the asymptotic reliability function of this series-parallel nanosystem is

$$\mathfrak{R}_2(t) = \lim_{n \to +\infty} \mathbf{R}_{k, l_n, m_n}(t/(\lambda n))$$
$$= \begin{cases} 1, & t < 0 \\ 1 - [1 - \exp[-t/k]]^{1/k}, & t \geq 0. \end{cases}$$

Next, assume that l_{900}, $m_{900} = 15$, k = 4 and $\lambda = 1/90$.

Using (27)-(30) we obtain the exact reliability function of this nanosystem

$$\mathbf{R}_{4,15,15}(t) = 4P(A(t)) - [P(A(t))]^2$$
$$\cdot (3\overline{\overline{H}}(t,1) + 2\overline{\overline{H}}(t,2) + \overline{\overline{H}}(t,3)) + [P(A(t))]^3$$
$$\cdot (2\overline{\overline{H}}(t,1)^2\,\overline{\overline{H}}(t,2) + 2\overline{\overline{H}}(t,1)\overline{\overline{H}}(t,2)\overline{\overline{H}}(t,3))$$
$$- [P(A(t))]^4 \cdot \overline{\overline{H}}(t,1)^3\,\overline{\overline{H}}(t,2)^2\,\overline{\overline{H}}(t,3),\ t > 0,$$

where

$$P(A(t)) = \overline{H}(t)\cdot \exp[-5t/2], t > 0,$$

$$\overline{H}(t) = \prod_{\substack{j=1,\ldots,15,\\ \upsilon=1,\ldots,15}} [1 - \frac{(1-e^{-t/90})^{2.2}}{\sqrt{j^2+\upsilon^2}}]^{2(15-i)(15-\upsilon)}$$

$$\cdot [\prod_{j=1,\ldots,15} [1 - \frac{(1-e^{-t/90})^{2.2}}{j}]^{(15-j)\cdot 15}]^2, t > 0,$$

$$\overline{\overline{H}}(t,i) = \prod_{\substack{j=1,\ldots,15,\\ \upsilon=1,\ldots,15}} [1 - \frac{(1-e^{-t/90})^{2.2}}{\sqrt{i^2+j^2+\upsilon^2}}]^{4(15-i)(15-\upsilon)}$$

$$\cdot [\prod_{j=1,\ldots,15} [1 - \frac{(1-e^{-t/90})^{2.2}}{\sqrt{i^2+j^2}}]^{2(15-j)\cdot 15}]^2$$

$$\cdot [1 - \frac{(1-e^{-t/90})^{2.2}}{i}]^{225}, i \in \{1,2,3\}, t > 0,$$

and

$$\boldsymbol{R}_{4,15,15}(t) = 1, t \le 0,$$

which could be approximated by

$$\boldsymbol{R}_{4,15,15}(t) \cong \mathfrak{R}_2(10t) = \begin{cases} 1, & t < 0 \\ 1 - [1 - \exp[-5t/2]]^4, & t \ge 0. \end{cases}$$

The expected values of considered series nanosystem lifetime T_1, series-parallel nanosystem lifetime T_2 and their standard deviations, in seconds, calculated on the basis of the above approximate result, respectively are:

$E[T_1] \cong 0.10$ sec, $\sigma_1 \cong 0.10$ sec,

and

$E[T_2] \cong 0.83$ sec, $\sigma_2 \cong 3.11$ s.

Tables 1-2 and *Figures 2-3* are presenting the differences between the values of nanosystem's exact reliability functions and the values of the nanosystem's approximate reliability functions. We can see that they are very small, what justifies the correctness of the approximation.

Table 1. The differences between the values of the series nanosystem exact and approximate reliability function

$t\,[s]$	$\overline{\boldsymbol{R}}_{4,15,15}(t)$	$\overline{\mathfrak{R}}_2[\frac{t-b_n}{a_n}]$	$\overline{\boldsymbol{R}}_{4,15,15}(t) - \overline{\mathfrak{R}}_2(10t)$
0	1	1.	0
0.025	0.778009	0.778801	-0.000792
0.050	0.603703	0.606531	-0.002827
0.075	0.467014	0.472367	-0.005353
0.100	0.360071	0.367879	-0.007809
0.125	0.276639	0.286505	-0.009866
0.150	0.211756	0.223130	-0.011374
0.175	0.161473	0.173774	-0.012301
0.200	0.122645	0.135335	**-0.012690**
0.225	0.092778	0.105399	-0.012621
0.250	0.069895	0.082085	-0.012190
0.275	0.052434	0.063928	-0.011494
0.300	0.039166	0.049787	-0.010621
0.325	0.029128	0.038774	-0.009647
0.350	0.021566	0.030197	-0.008631
0.375	0.015896	0.023518	-0.007622
0.400	0.011664	0.018316	-0.006652

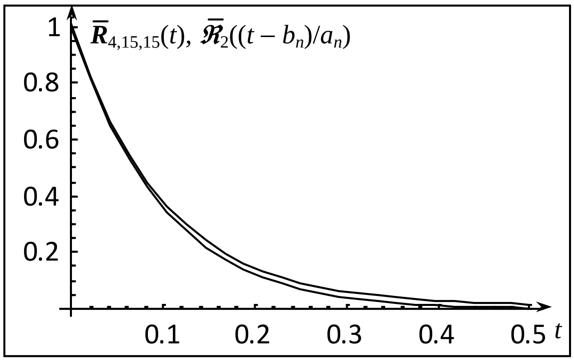

Figure 2. The graphs of the exact and approximate reliability functions of the exemplary homogeneous regular series three-dimensional nanosystem

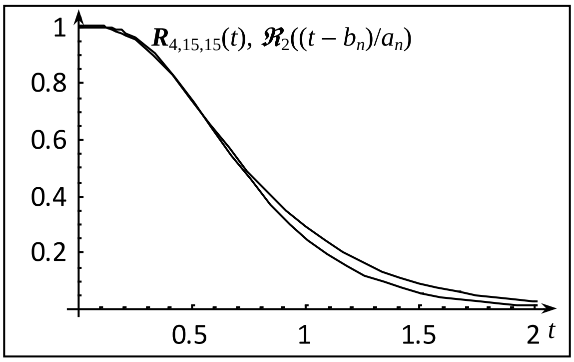

Figure 3. The graphs of the exact and approximate reliability functions of the exemplary homogeneous regular series-parallel three-dimensional nanosystem

Table 2. The differences between the values of the series-parallel nanosystem exact and approximate reliability function

t [s]	$\boldsymbol{R}_{4,15,15}(t)$	$\Re_2[\frac{t-b_n}{a_n}]$	$\boldsymbol{R}_{4,15,15}(t)-\Re_2(10t)$
0	1	1	0
0.1	0.998025	0.997606	0.000419
0.2	0.979219	0.976031	0.003187
0.3	0.929011	0.922495	0.006516
0.4	0.846572	0.840339	0.006234
0.5	0.741330	0.740842	0.000487
0.6	0.626213	0.635755	-0.009543
0.7	0.512777	0.533990	-0.021213
0.8	0.409031	0.441027	-0.031996
0.9	0.319236	0.359503	-0.040267
1.0	0.244686	0.290079	-0.045394
1.1	0.184737	0.232219	**-0.047482**
1.2	0.137713	0.184763	-0.047050
1.3	0.101546	0.146307	-0.044761
1.4	0.074165	0.115428	-0.041262
1.5	0.053705	0.090804	-0.037099
1.6	0.038583	0.071274	-0.032691
1.7	0.027515	0.055848	-0.028333
1.8	0.019483	0.043701	-0.024218
1.9	0.013700	0.034160	-0.020460

7. Conclusion

The asymptotic reliability function of the three-dimensional homogeneous regular series nanosystem in which times up to displacement of nanocomponents, which make up this nanosystem, from their initial positions are dependent non-negative continuous random variables and the dependence between two nanocomponents decreasing with increasing the distance between them, was investigated in [3] with using copula functions to create the joint reliability function of this nanosystem.

In this paper was showed one example of the reliability function of the three-dimensional homogeneous regular series and series-parallel nanosystem which takes into account dependencies between times up to displacement of nanocomponents and its asymptotic reliability function is the same as a asymptotic reliability function of this nanosystem when times up to displacement of nanocomponents are independent. To investigate of the limit reliability function of these reliability function we used modified theorems which investigate limit reliability function of the three-dimensional homogeneous regular series and series-parallel nanosystem with independent times up to displacement of nanocomponents. This allowed us to determinate the classes of limit reliability functions and approximate the nanosystem exact reliability function which is given by very complicated formula when times up to displacement of nanocomponents are dependent on each other.

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