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Complex system operation cost optimization

Keywords

safety, cost, operation process, optimization, exemplary application

Abstract

The general model of a complex system changing its safety structure, its components safety parameters and its operation cost during the variable operation process and linear programming are applied to optimize the system operation process in order to get the system operation cost optimal values. The optimization problem allowing to find the optimal values of the transient probabilities of the complex system operation process at the particular operation states that minimize the system unconditional operation cost mean value in the safety states subset not worse than a critical system safety state under the assumption that the system conditional operation cost mean values in this safety state subset at the particular operation states are fixed is presented. Further, the procedure of finding the optimal values operation cost is presented and applied to the exemplary complex technical system.

1. Introduction

To tie the investigations of the complex technical system safety together with the investigations of its operation the semi-Markov processes models can be used to describe this system operation processes [1]-[6]. These models, under the assumption on the system structure multistate model [11]-[14] can be used to construct the general safety model of the complex multistate system changing its safety structure and its components safety parameters during variable operation process [3]-[6]. Further, using this general model, it is possible to find the complex system main safety characteristics such as the system safety function, the system mean lifetimes in system safety subsets and risk function [4]-[6] and its operation process cost. Having these characteristics it is possible to optimize the system operation process to get their optimal values [7]-[10]. To this end the linear programming [1] can be applied for minimizing the system operation cost.

2. Paper preparation

Considering the equation (23) in [6] for the system unconditional safety function by the analogous way we may introduce the instantaneous system operation cost on the form of vector

$$\boldsymbol{C}(t,\cdot) = [1,\ \boldsymbol{C}(t,1), ...,\ \boldsymbol{C}(t,z)\ ], \quad (1)$$

with the coordinates given by

$$\boldsymbol{C}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{C}(t,u)]^{(b)} \text{ for } t \geq 0, \quad (2)$$

$$u = 1,2,...,z,$$

where $[\boldsymbol{C}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the coordinates of the system conditional instantaneous operation costs in the safety state subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z$, at the system operation states z_b, $b = 1,2,...,\nu$, and p_b, $b = 1,2,...,\nu$, are the system operation process limit transient probabilities determined in [6]. Thus, it is naturally assumed that the system instantaneous operation cost depends significantly on the system safety state and the system operation state as well. This dependency is also clearly expressed in the linear equation

$$c(u) \cong \sum_{b=1}^{\nu} p_b c_b(u) \quad (3)$$

for the mean value of the system total unconditional operation costs in the safety state subsets

$\{u, u+1, ..., z\}$, $u = 1,2,...,z$, where $c_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, are the mean values of the system total conditional operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the particular system operation states z_b, $b = 1,2,...,\nu$, determined by

$$c_b(u) = \int_0^{\mu_b(u)} [\boldsymbol{C}(t,u)]^{(b)} dt, \quad (4)$$

$u = 1,2,...,z$, $b = 1,2,...,\nu$, where

$$\mu_b(u) = E[T^{(b)}(u)]$$

are the mean values of the system conditional lifetimes $T^{(b)}(u)$ in the safety state subset $\{u, u+1, ..., z\}$ at the operation state z_b, $b = 1,2,...,\nu$, given by

$$\mu_b(u) = \int_0^{\infty} [\boldsymbol{S}(t,u)]^{(b)} dt, \ u = 1,2,...,z, \quad (5)$$

and $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the system safety function defined by (22) in [6] and p_b are given by (20) in [6].

3. System operation cost minimization

From the linear equations (3), we can see that the mean value of the system total unconditional operation cost $c(u)$, $u = 1,2,...,z$, is determined by the limit values of transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation process at the operation states z_b, $b = 1,2,...,\nu$, and by the mean values $c_b(u)$ of the system total conditional operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the particular system operation states z_b, $b = 1,2,...,\nu$, that by (4) are dependent on the mean values $\mu_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, of the system conditional lifetimes and by the system conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$ in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the system operation states z_b, $b = 1,2,...,\nu$.

Therefore, the system operations cost optimization based on the linear programming [1], [4], can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation process at the operation states to minimize the mean value $c(u)$ of the system total unconditional operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, under the assumption that the mean values $c_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,...,z$, of the system total conditional operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the particular system operation states z_b, $b = 1,2,...,\nu$, are fixed. As a special and practically important case of the above formulated system operations cost optimization problem, if r, $r = 1,2,...,z$, is a system critical safety state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation process at the system operation states to minimize the mean value $c(r)$ of the system total unconditional operation costs in the safety state subset $\{r, r+1, ..., z\}$, $r = 1,2,...,z$, under the assumption that the mean values $c_b(r)$, $b = 1,2,...,\nu$, $r = 1,2,...,z$, of the system total conditional operation costs in this safety state subsets are fixed. More exactly, we may formulate the optimization problem as a linear programming model with the objective function of the following form

$$c(r) = \sum_{b=1}^{\nu} p_b c_b(r) \quad (6)$$

for a fixed $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b, \ b = 1,2,...,\nu, \ \sum_{b=1}^{\nu} p_b = 1, \quad (7)$$

where $c_b(r)$, $c_b(r) \ge 0$, $b = 1,2,...,\nu$, are fixed mean values of the system conditional operation costs in the safety state subset $\{r, r+1, ..., z\}$ and

$$\breve{p}_b, \ 0 \le \breve{p}_b \le 1 \text{ and}$$

$$\hat{p}_b, \ 0 \le \hat{p}_b \le 1, \ \breve{p}_b \le \hat{p}_b, \ b = 1,2,...,\nu, \quad (8)$$

are lower and upper bounds of the unknown transient probabilities p_b, $b = 1,2,...,\nu$, respectively.

Now, we can obtain the optimal solution of the formulated by (6)-(8) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,...,\nu$, that minimize the objective function given by (6).

First, we arrange the mean values of the system total conditional operation costs $c_b(r)$, $b = 1,2,...,\nu$, in non-decreasing order

$$c_{b_1}(r) \le c_{b_2}(r) \le \ldots \le c_{b_\nu}(r),$$

where $b_i \in \{1,2,\ldots,\nu\}$ for $i = 1,2,\ldots,\nu.$
Next, we substitute

$$x_i = p_{b_i}, \; \breve{x}_i = \breve{p}_{b_i}, \; \hat{x}_i = \hat{p}_{b_i} \text{ for } i = 1,2,\ldots,\nu \quad (9)$$

and we minimize with respect to x_i, $i = 1,2,\ldots,\nu$, the linear form (6) that after this transformation takes the form

$$c(r) = \sum_{i=1}^{\nu} x_i c_{b_i}(r) \quad (10)$$

for a fixed $r \in \{1,2,\ldots,z\}$ with the following bound constraints

$$\breve{x}_i \le x_i \le \hat{x}_i, \; i = 1,2,\ldots,\nu, \; \sum_{i=1}^{\nu} x_i = 1, \quad (11)$$

where $c_{b_i}(r)$, $c_{b_i}(r) \ge 0$, $i = 1,2,\ldots,\nu$, are fixed mean values of the system conditional operation costs in the safety state subset $\{r, r+1,\ldots,z\}$ arranged in non-decreasing order and

$$\breve{x}_i, \; 0 \le \breve{x}_i \le 1 \text{ and } \hat{x}_i, \; 0 \le \hat{x}_i \le 1, \; \breve{x}_i \le \hat{x}_i, \quad (12)$$

$$i = 1,2,\ldots,\nu,$$

are lower and upper bounds of the unknown probabilities x_i, $i = 1,2,\ldots,\nu$, respectively.
To find the optimal values of x_i, $i = 1,2,\ldots,\nu$, we define

$$\breve{x} = \sum_{i=1}^{\nu} \breve{x}_i, \; \hat{y} = 1 - \breve{x} \quad (13)$$

and

$$\breve{x}^0 = 0, \; \hat{x}^0 = 0 \text{ and } \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i, \; \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \quad (14)$$

for $I = 1,2,\ldots,\nu.$
Next, we find the largest value $I \in \{0,1,\ldots,\nu\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \quad (15)$$

and we fix the optimal solution that minimize (10) in the following way:

i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \text{ and } \dot{x}_i = \breve{x}_i \text{ for } i = 2,3,\ldots,\nu; \quad (16)$$

ii) if $0 < I < \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,I, \; \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}$$

$$\text{and } \dot{x}_i = \breve{x}_i \text{ for } i = I+2, I+3,\ldots,\nu; \quad (17)$$

iii) if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \text{ for } i = 1,2,\ldots,\nu. \quad (18)$$

Finally, after making the inverse to (9) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1,2,\ldots,\nu, \quad (19)$$

that minimize the mean value of the system total unconditional operation costs in the safety state subset $\{r, r+1,\ldots,z\}$, defined by the linear form (6), giving its minimum value in the following form

$$\dot{c}(r) = \sum_{b=1}^{\nu} \dot{p}_b c_b(r) \quad (20)$$

for a fixed $r \in \{1,2,\ldots,z\}$.
From the expression (20) for the minimum mean value $\dot{c}(r)$ of the system unconditional operation cost in the safety state subset $\{r, r+1,\ldots,z\}$, replacing in it the critical safety state r by the safety state u, $u = 1,2,\ldots,z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional operation costs in the safety state subsets $\{u, u+1,\ldots,z\}$ of the form

$$\dot{c}(u) = \sum_{b=1}^{\nu} \dot{p}_b c_b(u) \text{ for } u = 1,2,\ldots,z. \quad (21)$$

Further, according to (1)-(2), the corresponding system optimal unconditional instantaneous operation cost is given by the vector

$$\dot{\boldsymbol{C}}(t,\cdot) = [1, \; \dot{\boldsymbol{C}}(t,1), \ldots, \; \dot{\boldsymbol{C}}(t,z)], \quad (22)$$

with the coordinates given by

$$\dot{\boldsymbol{C}}(t,u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{C}(t,u)]^{(b)} \quad (23)$$

for $t \ge 0$, $u = 1,2,\ldots,z.$

And, the optimal solutions for the mean values of the system unconditional operation costs in the particular safety states are

$$\dot{\bar{c}}(u) = \dot{c}(u) - \dot{c}(u+1), \; u = 1,\ldots,z-1,$$
$$\dot{\bar{c}}(z) = \dot{c}(z). \quad (24)$$

Moreover, considering (27) and (28) given in [6], the corresponding optimal system critical operation cost function

$$\dot{\boldsymbol{c}}(t) = 1 - \dot{\boldsymbol{C}}(t,r), \; t \geq 0, \quad (25)$$

and the optimal moment when the system operation cost exceeds a permitted level δ, respectively are given by

$$\dot{\tau} = \dot{\boldsymbol{c}}^{-1}(\delta), \quad (26)$$

where $\dot{\boldsymbol{C}}(t,r)$ is given by (23) for $u = r$ and $\dot{\boldsymbol{c}}^{-1}(t)$, if it exists, is the inverse function of the optimal critical operation cost function $\dot{\boldsymbol{c}}(t)$.

4. Application

We consider the exemplary critical infrastructure, considered in [6]. This system safety structure and its components safety parameters depend on its changing in time operation states with arbitrarily fixed the number of the system operation process states $\nu = 3$.
The considered in [6] system is a “4 out of 6”-series system composed of dependent components E_{ij}, $i = 1,2,3,$ $j = 1,2,\ldots,6,$ operating at three operation states z_1, z_2 and z_3.
At the operation state z_1 the system is composed of three “4 out of 6” subsystems linked in series and composed of components E_{ij}, $i = 1,2,3,$ $j = 1,2,\ldots,6.$
At the operation state z_2 the system is composed of two “4 out of 6” subsystems linked in series and composed of components E_{ij}, $i = 1,2,$ $j = 1,2,\ldots,6.$
At the operation state z_3, the system is composed of one “4 out of 6” subsystem composed of components E_{ij}, $i = 1,$ $j = 1,2,\ldots,6.$
We arbitrarily assume that the transient probabilities of the system at particular operation states z_1, z_2 and z_3 respectively are [6]

$$p_1 = 0.4, \; p_2 = 0.4, \; p_3 = 0.2. \quad (27)$$

We distinguished four safety states of the system components 0, 1, 2, 3, i.e. $z = 3,$ and we fix that the critical safety state is $r = 2$ [6].
We arbitrarily assume that the system conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$, in the safety state subsets $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z,$ $b = 1,2,\ldots,\nu,$ are constant at the system operation states z_b, $b = 1,2,\ldots,\nu,$ and in the safety subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, are given in the *Table1* bellow
Table 1. The values of the conditional instantaneous operation costs $[\boldsymbol{C}(t,u)]^{(b)}$, $b = 1,2,3,$ $u = 1,2,3,$ of the system total conditional operation costs in the safety state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$.

$[\boldsymbol{C}(t,u)]^{(b)}$	$b = 1$	$b = 2,$	$b = 3,$
$u = 1$	89	286	389
$u = 2$	29	103	130
$u = 3$	10	40	50

Considering the mean values $\mu_b(u) = E[T^{(b)}(u)]$ of the system conditional lifetimes $T^{(b)}(u)$ in the safety state subset $\{u, u+1,\ldots,z\}$ at the operation state z_b, $b = 1,2,\ldots,\nu,$

$$\mu_1(1) = 0.28, \; \mu_1(2) = 0.14, \; \mu_1(3) = 0.09,$$

$$\mu_2(1) = 0.69, \; \mu_2(2) = 0.34, \; \mu_2(3) = 0.23,$$

$$\mu_3(1) = 1.5, \; \mu_3(2) = 0.75, \; \mu_3(3) = 0.50,$$

calculated in [6] and applying the formula (4) we get the approximate mean values $c_b(u)$, $b = 1,2,3,$ $u = 1,2,3,$ of the system total conditional operation costs in the safety state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, given in the *Table2* bellow.
Table 2. The mean values $c_b(u)$, $b = 1,2,3,$ $u = 1,2,3,$ of the system total conditional operation costs in the safety state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$.

$c_b(u)$	$b = 1$	$b = 2,$	$b = 3,$
$u = 1$	25	40	35
$u = 2$	20	35	30
$u = 3$	15	30	25

Taking $c_b(u)$ $b = 1,2,3,$ $u = 1,2,3,$ from *Table 2* and the transient probabilities p_b, $b = 1,2,3,$ (27) the mean values of the system total unconditional operation costs in the safety state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, according to (3) are

$$c(1) \cong p_1 c_1(1) + p_2 c_2(1) + p_3 c_3(1)$$
$$\cong 0.4 \cdot 25 + 0.4 \cdot 40 + 0.2 \cdot 35 = 33, \quad (28)$$

$$c(2) \cong p_1 c_1(2) + p_2 c_2(2) + p_3 c_3(2)$$
$$\cong 0.4 \cdot 20 + 0.4 \cdot 35 + 0.2 \cdot 30 = 28, \quad (29)$$

$$c(3) \cong p_1 c_1(3) + p_2 c_2(3) + p_3 c_3(3)$$

$$\cong 0.4 \cdot 15 + 0.4 \cdot 30 + 0.2 \cdot 25 = 23. \qquad (30)$$

To find the optimal values of those system cost characteristics, we conclude that the objective function defined by (6), in this case, as the system critical state is $\boldsymbol{r} = 2$, according to (29), takes the form

$$c(2) = p_1 \cdot 20 + p_2 \cdot 35 + p_3 \cdot 30. \qquad (31)$$

Arbitrarily assumed, the lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown optimal values of transient probabilities p_b, $b = 1,2,3,$ respectively are:

$$\breve{p}_1 = 0.3, \ \breve{p}_2 = 0.3, \ \breve{p}_3 = 0.1$$

$$\hat{p}_1 = 0.5, \ \hat{p}_2 = 0.5, \ \hat{p}_3 = 0.3.$$

Therefore, according to (7)-(8), we assume the following bound constraints

$$0.3 \le p_1 \le 0.5, \ 0.3 \le p_2 \le 0.5, \ 0.1 \le p_3 \le 0.3. \qquad (32)$$

$$\sum_{b=1}^{3} p_b = 1. \qquad (33)$$

Now, before we find optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,3,$ that minimize the objective function (31), we arrange the mean values of the system conditional operation cost $c_b(2)$, $b = 1,2,3,$ in non-decreasing order

$$c_1(2) \le c_3(2) \le c_2(2). \qquad (34)$$

Further, according to (9), we substitute

$$x_1 = p_1, \ x_2 = p_3, \ x_3 = p_2,$$

and

$$\breve{x}_1 = \breve{p}_1 = 0.3, \ \breve{x}_2 = \breve{p}_3 = 0.1, \ \breve{x}_3 = \breve{p}_2 = 0.3;$$

$$\hat{x}_1 = \hat{p}_1 = 0.5, \ \hat{x}_2 = \hat{p}_3 = 0.3, \ \hat{x}_3 = \hat{p}_2 = 0.5, \qquad (36)$$

and we minimize with respect to x_i, $i = 1,2,3,$ the linear form (31) that according to (10)-(12) takes the form

$$c(2) = x_1 \cdot 20 + x_2 \cdot 30 + x_3 \cdot 35, \qquad (37)$$

with the following bound constraints

$$0.3 \le x_1 \le 0.5, \ 0.1 \le x_2 \le 0.3, \ 0.3 \le x_3 \le 0.5. \qquad (38)$$

$$\sum_{i=1}^{3} x_i = 1. \qquad (39)$$

According to (13), we calculate

$$\breve{x} = \sum_{i=1}^{3} \breve{x}_i = 0.7, \ \hat{y} = 1 - \breve{x} = 1 - 0.7 = 0.3 \qquad (40)$$

and according to (14), we determine

$$\breve{x}^0 = 0, \ \hat{x}^0 = 0, \ \hat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.3, \ \hat{x}^1 = 0.5, \ \hat{x}^1 - \breve{x}^1 = 0.2,$$

$$\breve{x}^2 = 0.4 \ \hat{x}^2 = 0.8, \ \hat{x}^2 - \breve{x}^2 = 0.4,$$

$$\breve{x}^3 = 0.7, \ \hat{x}^3 = 1.3, \ \hat{x}^3 - \breve{x}^3 = 0.6. \qquad (41)$$

From the above, as according to (40), the inequality (15) takes the form

$$\hat{x}^I - \breve{x}^I < 0.3, \qquad (42)$$

it follows that the largest value $I \in \{0,1,2,3\}$ such that this inequality holds is $I = 1$.
Therefore, we fix the optimal solution that minimize linear function (37) according to the rule (17). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.5,$$

$$\dot{x}_2 = \hat{y} - \hat{x}^1 + \breve{x}^1 + \breve{x}_2 = 0.3 - 0.5 + 0.3 + 0.1 = 0.2,$$

$$\dot{x}_3 = \breve{x}_3 = 0.3. \qquad (43)$$

Finally, after making the inverse to (19) substitution, we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_1 = 0.5, \ \dot{p}_2 = \dot{x}_3 = 0.3, \ \dot{p}_3 = \dot{x}_2 = 0.2, \qquad (44)$$

(35)

that minimize the mean value of the exemplary system total unconditional operation costs $c(2)$ in the safety state subset $\{2,3\}$ expressed by the linear form (31) giving, according to (20) and (44), its optimal value

$$\dot{c}(2) = \dot{p}_1 \cdot 20 + \dot{p}_2 \cdot 35 + \dot{p}_3 \cdot 30$$

$$= 0.5 \cdot 20 + 0.3 \cdot 35 + 0.2 \cdot 30 \cong 26.5 \qquad (45)$$

Substituting the optimal solution (44) into the formula (21), we obtain the optimal solution for the mean values of the exemplary system total unconditional operation costs in the safety state subsets $\{1,2,3\}$ and $\{3\}$, that are as follows

$$\dot{c}(1) = \dot{p}_1 \cdot 25 + \dot{p}_2 \cdot 40 + \dot{p}_3 \cdot 35$$

$$= 0.5 \cdot 25 + 0.3 \cdot 40 + 0.2 \cdot 35 \cong 31.5, \quad (46)$$

$$\dot{c}(3) = \dot{p}_1 \cdot 15 + \dot{p}_2 \cdot 30 + \dot{p}_3 \cdot 25$$

$$= 0.5 \cdot 15 + 0.3 \cdot 30 + 0.2 \cdot 25 \cong 21.5, \quad (47)$$

and according to (24), the optimal values of the mean values of the considered exemplary system unconditional operation costs in the particular safety states 1, 2 and 3, respectively are

$$\dot{\bar{c}}(1) = \dot{c}(1) - \dot{c}(2) \cong 5,$$

$$\dot{\bar{c}}(2) = \dot{c}(2) - \dot{c}(3) \cong 5$$

$$\dot{\bar{c}}(3) = \dot{c}(3) = 21.5. \quad (48)$$

5. Conclusion

The procedure of using the general safety analytical model of complex multistate technical systems related to their operation processes prezented in [4] and the liner programing [1] was presented to the optimization of the operation processes and cost of complex system. Next the procedure was applied to the optimization of an exemplary "*m* out of *l*"-series critical infrastructure operation cost. The mean values of the considered system total unconditional operation costs were evaluated and minimize after its operation process optimization.

Presented in this paper tools can be useful in operation cost optimization of a very wide class of real technical systems operating at the varying conditions that have an influence on changing their safety structures and their components safety parameters. The results can be interesting for safety practitioners from various industrial sectors.

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