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Reliability of the exemplary multistate series system with dependent components

Keywords

reliability, dependent components, local load sharing, ageing components, multistate series system

Abstract

In the paper a multistate approach to reliability analysis of series systems with dependent components according to the local load sharing rule is proposed. As a particular case, the reliability function of a multistate series system composed of dependent components having exponential reliability functions is determined. The mean values and standard deviations of the multistate system lifetimes in the reliability state subsets and the mean values of its lifetimes in the particular reliability states are determined. Application of the proposed model of components' dependency to the reliability analysis of the exemplary system is presented. The exemplary system risk function and the moment of exceeding by the system the critical reliability state are given.

1. Introduction

In reliability analysis the independence of system components is often assumed. This assumption means that failure of a component has no influence on the remaining surviving components. However, in many real technical systems after failure of any system components its load is transmitted to the remaining surviving components. Then, dependencies among system's components have significant influence on the reliability of a system. Thus, modeling systems with interdependent components is an important issue.

A change of a stress in a system caused by changing reliability state by one or several of system components may have a significant effect on the reliability states of remaining system components. Depending on the analyzed system structure and behaviour of the system components we can consider different types of inside systems dependencies. We can consider equal load sharing models [4]-[5], [8], [14], [17]-[20], which consider equal sharing of a stress on remaining components, and local load sharing models [10]-[11], [16], in which a stress has the strongest impact on the nearest neighbours of a component that has changed the reliability state.

In a multistate system with dependent components we may consider the dependency of changes of their ageing reliability states and assume that after changing the reliability state subset by one of system components to the worse reliability state subset, lifetimes of remaining system components in this reliability state subset decrease [2]-[3].

2. Local load sharing model of components dependency

We suppose as in [12] that all components and a system under consideration have the reliability state set $\{0,1,...,z\}$, $z \geq 1$, where the state 0 is the worst and the state z is the best. The state of a system and components degrades with time without repair. Further, we consider a multistate series system, defined in [12], composed of n ageing and independent components with the reliability functions of its components

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1), ..., R_i(t,z)],\ i = 1,2,...,n, \quad (1)$$

where

$$R_i(t,u) = P(T_i(u) > t),\ i = 1,\ldots,n, \quad (2)$$

and $T_i(u)$, i = 1,2,...,n, are independent random variables representing lifetimes of components E_i in the reliability state subset $\{u,u+1,...,z\}$. Similarly, as in [11], we define the multistate reliability function of a system as a vector

$$\boldsymbol{R}(t,\cdot) = [\boldsymbol{R}(t,0), \boldsymbol{R}(t,1), \ldots, \boldsymbol{R}(t,z)],\ t \geq 0, \quad (3)$$

where its coordinates

$$\boldsymbol{R}(t,u) = P(T(u) > t),\ u = 0,1,...,z,$$

and $T(u)$ is a random variable representing the lifetime of the system in the reliability state subset $\{u,u+1,...,z\}$.

Taking into account the dependence of components, we assume that after changing the reliability state subset by one of system components to the worse reliability state subset, lifetimes of remaining system components in the reliability state subsets decrease mostly for neighbour components in first line, then less for neighbour components in second line and so on. Further, we call this rule of components dependency a local load sharing rule. More exactly, in this rule if the system component E_j, j = 1,…,n, gets out of the reliability state subset $\{u,u+1,\ldots,z\}$, u = 1,2,…,z, the reliability parameters of remaining system components are changing dependently of the distance from the component that has got out of the reliability state subset $\{u,u+1,\ldots,z\}$, u = 1,2,…,z, expressed by index d. The meaning of d is illustrated in *Figure 1*.

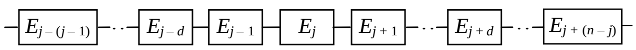

Figure 1. The meaning of the distance d

We denote by $E[T_i(u)]$ and $E[T_{i/j}(u)]$, i = 1,2,...,n, j = 1,2,...,n, u = 1,2,…,z, the mean values of system components lifetimes $T_i(u)$ and $T_{i/j}(u)$, respectively, before and after departure of one fixed component E_j, j = 1,…,n, from the reliability state subset $\{u,u+1,\ldots,z\}$, u = 1,2,…,z. With this notation, in considered local load sharing rule, the mean values of components lifetimes in the reliability state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, υ = u,u-1,…,1, u = 1,2,…,z, are decreasing according to the following formula:

$$E[T_{i/j}(\upsilon)] = q(\upsilon, d_{ij}) \cdot E[T_i(\upsilon)],$$

$$i = 1,\ldots,n,\ j = 1,\ldots,n,\ \upsilon = u, u-1, \ldots, 1, \quad (4)$$

where $q(\upsilon, d_{ij})$, $0 < q(\upsilon, d_{ij}) \leq 1$, i = 1,…,n, j = 1,…,n, and $q(\upsilon,0)$ = 1, for υ = u,u–1,…,1, u = 1,2,…,z-1, are non-increasing coefficients of components' distance $d_{ij} = |i - j|$ from the component that has got out of the reliability state subset $\{u,u+1,\ldots,z\}$, u = 1,2,…,z.

We denote by

$$R_{i/j}(t,\upsilon) = P(T_{i/j}(\upsilon) > t),\ i = 1,\ldots,n,\ j = 1,\ldots,n, \quad (5)$$

for υ = u,u–1,…,1, u = 1,2,…,z–1, the coordinates of the reliability function

$$R_{i/j}(t,\cdot) = [1, R_{i/j}(t,1), \ldots, R_{i/j}(t,u)], \quad (6)$$

of a system component E_i, i = 1,…,n, after departure of the jth component E_j, j = 1,…,n, from the reliability state subset $\{u,u+1,\ldots,z\}$, u = 1,2,…,z.

3. Reliability of multistate series system with dependent components

In [6] the below formulated theorem is proofed.

Proposition 1. If in a multistate series system components are dependent according to the local load sharing rule and have reliability functions given by (1)-(2), then its reliability function is given by the vector

$$\boldsymbol{R}_{LLS}(t,\cdot) = [1, \boldsymbol{R}_{LLS}(t,1), \ldots, \boldsymbol{R}_{LLS}(t,z)],\ t \geq 0, \quad (7)$$

with the coordinates

$$\boldsymbol{R}_{LLS}(t,u) = \prod_{i=1}^{n} R_i(t,u+1) \cdot \prod_{i=1}^{n} R_i(t,u) + \int_0^t \sum_{j=1}^{n} [f_j(a,u+1) \cdot \prod_{\substack{i=1 \\ i \neq j}}^{n} R_i(a,u+1) \cdot \prod_{i=1}^{n} R_{i/j}(t-a,u)$$

$$\cdot \prod_{i=1}^{n} R_i(a,u)]da,\ u = 1,2,\ldots,z-1, \quad (8)$$

$$\boldsymbol{R}_{LLS}(t,z) = \prod_{i=1}^{n} R_i(t,z), \quad (9)$$

where:

$R_i(t,u+1)$ – the reliability function coordinate of a component E_i, i = 1,…,n, i.e. the probability that its

lifetime in the reliability state subset $\{u+1,\ldots,z\}$, $u = 1,2,\ldots,z-1$, is greater than t,
$R_i(t,u)-$ the reliability function coordinate of a component E_i, $i = 1,\ldots,n$, i.e. the probability that its lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, is greater than t,
$f_j(t,u+1)-$ the density function coordinate of a component E_j, $j = 1,\ldots,n$, lifetime in the reliability state subset $\{u+1,\ldots,z\}$, $u = 1,2,\ldots,z-1$,
$R_{i/j}(t,u)-$ the reliability function coordinate of a component E_i, $i = 1,\ldots,n$, i.e. the probability that its lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z-1$, after departure from the reliability state subset $\{u+1,\ldots,z\}$, $u = 1,2,\ldots,z-1$, by the component E_j, $j = 1,\ldots,n$, is greater than t.

Further, we consider a homogeneous multistate series system with components dependent according to the local load sharing rule having reliability functions

$$R(t,\cdot)=[1, R(t,1),\ldots,R(t,z)],\ t\geq 0, \tag{10}$$

and for that system we get a particular case of *Proposition 1* formulated below.

Proposition 2. If in a homogeneous multistate series system components are dependent according to the local load sharing rule and have reliability functions given by (10), then its reliability function is given by the vector

$$\boldsymbol{R}_{LLS}(t,\cdot)=[1,\boldsymbol{R}_{LLS}(t,1),\ldots,\boldsymbol{R}_{LLS}(t,z)],\ t\geq 0, \tag{11}$$

with the coordinates

$$\boldsymbol{R}_{LLS}(t,u)=[R(t,u+1)]^n\cdot[R(t,u)]^n + \int_0^t \sum_{j=1}^n [f(a,u+1)\cdot[R(a,u+1)]^{n-1}\cdot\prod_{i=1}^n R_{i/j}(t-a,u)$$

$$\cdot[R(a,u)]^n]da,\ u=1,2,\ldots,z-1, \tag{12}$$

$$\boldsymbol{R}_{LLS}(t,z)=[R(t,z)]^n, \tag{13}$$

where:
$R(t,u+1)-$ the reliability function coordinate of a component i.e. the probability that its lifetime in the reliability state subset $\{u+1,\ldots,z\}$, $u=1,2,\ldots,z-1$, is greater than t,
$R(t,u)-$ the reliability function coordinate of a component i.e. the probability that its lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$, $u=1,2,\ldots,z$, is greater than t,
$f(t,u+1)-$ the density function coordinate of a component lifetime in the reliability state subset $\{u+1,\ldots,z\}$, $u=1,2,\ldots,z-1$,
$R_{i/j}(t,u)-$ the reliability function coordinate of a component E_i, $i = 1,\ldots,n$, i.e. the probability that its lifetime in the reliability state subset $\{u,u+1,\ldots,z\}$, $u=1,2,\ldots,z-1$, after departure from the reliability state subset $\{u+1,\ldots,z\}$, $u=1,2,\ldots,z-1$, by the component E_j, $j = 1,\ldots,n$, is greater than t.

Next, we assume that the reliability functions of system components (10) have exponential coordinates

$$R(t,u)=\exp[-\lambda(u)t],\ t\geq 0,\ \lambda(u)\geq 0. \tag{14}$$

where $\lambda(u)$, $u = 1,2,\ldots,z$, are components intensities of departure from the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. Then, according to the relationship between the lifetime mean value in this reliability state subset and the intensity of departure from this reliability state subset, we get the formula for the intensities $\lambda_{i/j}(\upsilon)$, $i = 1,\ldots,n$, $j = 1,\ldots,n$, $\upsilon = u,u-1,\ldots,1$, of components departure from this reliability state subset after departure of the jth component E_j, $j = 1,\ldots,n$. Namely, from formula (4), we obtain

$$\lambda_{i/j}(\upsilon)=\frac{\lambda(\upsilon)}{q(\upsilon,d_{ij})},\ \upsilon=u,u-1,\ldots,1, \tag{15}$$

where $d_{ij}=|i-j|$ and $0<q(\upsilon,d_{ij})\leq 1$, for $i = 1,\ldots,n$, $j = 1,\ldots,n$, and $q(\upsilon,0) = 1$ for $\upsilon = u, u-1,\ldots,1$, $u = 1,2,\ldots,z-1$, are non-increasing coefficients of components distance from the component that has got out of the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$.
In this case *Proposition 2* takes the form presented below.

Proposition 3. If in a homogeneous multistate series system components are dependent according to the local load sharing rule and have reliability functions (10) with exponential coordinates given by (14), then its reliability function is given by the vector

$$\boldsymbol{R}_{LLS}(t,\cdot)=[1,\boldsymbol{R}_{LLS}(t,1),\ldots,\boldsymbol{R}_{LLS}(t,z)],\ t\geq 0, \tag{16}$$

with the coordinates

$$\boldsymbol{R}_{LLS}(t,u) = \exp[-n(\lambda(u+1)+\lambda(u))t]$$
$$+\sum_{j=1}^{n}\frac{\lambda(u+1)}{\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}-n(\lambda(u+1)+\lambda(u))}$$
$$\cdot[\exp[-n(\lambda(u+1)+\lambda(u))t]$$
$$-\exp[-\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}t]],\ u=1,2,\ldots,z-1, \quad (17)$$

$$\boldsymbol{R}_{LLS}(t,z) = \exp[-n\lambda(z)t]. \quad (18)$$

From *Proposition 3*, we immediately obtain two corollaries concerned with basic reliability characteristics of a homogeneous multistate series system.

Corollary 1. If in a homogeneous multistate series system components are dependent according to the local load sharing rule and have reliability functions (10) with exponential coordinates given by (14), then its mean lifetimes in the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, are given by

$$\mu_{LLS}(u) = \frac{1}{n(\lambda(u+1)+\lambda(u))}$$
$$+\sum_{j=1}^{n}\frac{\lambda(u+1)}{\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}-n(\lambda(u+1)+\lambda(u))}$$
$$\cdot[\frac{1}{n(\lambda(u+1)+\lambda(u))}-\frac{1}{\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}}],$$
$$u=1,2,\ldots,z-1, \quad (19)$$

$$\mu_{LLS}(z) = \frac{1}{n\lambda(z)} \quad (20)$$

and the standard deviations of the system sojourn times in the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, are given by

$$\sigma_{LLS}(u) = \sqrt{n_{LLS}(u)-[\mu_{LLS}(u)]^2},$$
$$u=1,2,\ldots,z-1, \quad (21)$$

where

$$n_{LLS}(u) = \frac{2}{[n(\lambda(u+1)+\lambda(u))]^2}$$
$$+2\sum_{j=1}^{n}\frac{\lambda(u+1)}{\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}-n(\lambda(u+1)+\lambda(u))}$$
$$\cdot[\frac{1}{[n(\lambda(u+1)+\lambda(u))]^2}-\frac{1}{[\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}]^2}],$$
$$u=1,2,\ldots,z-1, \quad (22)$$

and

$$\sigma_{LLS}(z) = \frac{1}{n\lambda(z)}. \quad (23)$$

Corollary 2. If in a homogeneous multistate series system components are dependent according to the local load sharing rule and have reliability functions (10) with exponential coordinates given by (14), then the intensity of its departure from the reliability state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, is given by

$$\boldsymbol{\lambda}_{LLS}(t,u) = \{n(\lambda(u+1)+\lambda(u))$$
$$+\sum_{j=1}^{n}\frac{\lambda(u+1)}{\lambda(u)(\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}-n)-n\lambda(u+1)}$$
$$\cdot[n(\lambda(u+1)+\lambda(u))-\lambda(u)\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}]$$
$$\cdot\exp[n\lambda(u+1)t+\lambda(u)(n-\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})})t]]\}$$
$$/\{1+\sum_{j=1}^{n}\frac{\lambda(u+1)}{\lambda(u)(\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})}-n)-n\lambda(u+1)}$$
$$\cdot[1-\exp[n\lambda(u+1)t+\lambda(u)(n-\sum_{i=1}^{n}\frac{1}{q(u,d_{ij})})t]]\},$$
$$u=1,2,\ldots,z-1, \quad (24)$$

$$\boldsymbol{\lambda}_{LLS}(t,z) = n\lambda(z). \quad (25)$$

4. Application

The increasing complexity of today's infrastructure and technical systems causes the need of development of new models that incorporate the dependencies among system's components. Potential applications of load-sharing models can be found in many areas, including textile engineering and materials testing [7]-[8], [10]-[11], [16]-[17], technical systems reliability analysis [4]-[5], software reliability, civil and structural engineering [1], [14], safety assessment (for example of power plant [1]) and others. In this paper we present reliability analysis of the exemplary system with interdependent components, however future studies on the presented model of dependency with practical applications are planned.

4.1. Reliability of exemplary system

We consider an exemplary system *S* as a homogeneous series system composed of 3 components E_i, i = 1,2,3, with reliability structure presented in *Figure 2*.

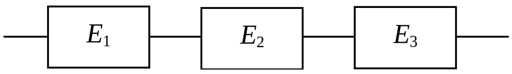

Figure 2. The scheme of the exemplary system *S* reliability structure

We assume that the system *S* is a 5-state system and we arbitrarily distinguish the following five reliability states of the system and its components:

- a reliability state 4 – the system operation is fully effective,
- a reliability state 3 – the system operation is less effective because of ageing,
- a reliability state 2 – the system operation is less effective because of ageing and dangerous for the environment,
- a reliability state 1 – the system operation is less effective because of ageing and more dangerous for the environment,
- a reliability state 0 – the system is destroyed.

To have the assumption on ageing satisfied, we assume that there are possible the transitions between the components reliability states only from better to worse ones. Moreover, we assume that the system and its components critical reliability state is r = 2.
Further, we assume that the system components lifetimes in the reliability states are expressed in years and they have the identical reliability functions

$$R(t,\cdot) = [1, R(t,1), R(t,2), R(t,3), R(t,4)],\ t \ge 0, \quad (26)$$

with the coordinates that by the assumption are exponential of the forms

$$R(t,1) = \exp[-0.25t],\ R(t,2) = \exp[-0.5t],$$
$$R(t,3) = \exp[-0.75t],\ R(t,4) = \exp[-t]. \quad (27)$$

From *Proposition 3* according to (16)-(18), the system reliability function is given by

$$\boldsymbol{R}_{LLS}(t,\cdot) = [1, \boldsymbol{R}_{LLS}(t,1), \ldots, \boldsymbol{R}_{LLS}(t,z)],\ t \ge 0, \quad (28)$$

with the coordinates

$$\boldsymbol{R}_{LLS}(t,1) = \exp[-3(\lambda(2)+\lambda(1))t] + \sum_{j=1}^{3} \frac{\lambda(2)}{\lambda(1)\sum_{i=1}^{3}\frac{1}{q(1,d_{ij})} - 3(\lambda(2)+\lambda(1))} \cdot [\exp[-3(\lambda(2)+\lambda(1))t] - \exp[-\lambda(1)\sum_{i=1}^{3}\frac{1}{q(1,d_{ij})}\cdot t]], \quad (29)$$

$$\boldsymbol{R}_{LLS}(t,2) = \exp[-3(\lambda(3)+\lambda(2))t] + \sum_{j=1}^{3} \frac{\lambda(3)}{\lambda(2)\sum_{i=1}^{3}\frac{1}{q(2,d_{ij})} - 3(\lambda(3)+\lambda(2))} \cdot [\exp[-3(\lambda(3)+\lambda(2))t], - \exp[-\lambda(2)\sum_{i=1}^{3}\frac{1}{q(2,d_{ij})}\cdot t]] \quad (30)$$

$$\boldsymbol{R}_{LLS}(t,3) = \exp[-3(\lambda(4)+\lambda(3))t] + \sum_{j=1}^{3} \frac{\lambda(4)}{\lambda(3)\sum_{i=1}^{3}\frac{1}{q(3,d_{ij})} - 3(\lambda(4)+\lambda(3))} \cdot [\exp[-3(\lambda(4)+\lambda(3))t] - \exp[-\lambda(3)\sum_{i=1}^{3}\frac{1}{q(3,d_{ij})}\cdot t]], \quad (31)$$

$$\boldsymbol{R}_{LLS}(t,4) = \exp[-3\lambda(4)t], \quad (32)$$

where $q(\upsilon, d_{ij})$, $0 < q(\upsilon, d_{ij}) \le 1$, i = 1,2,3, j = 1,2,3, and $q(\upsilon,0)$ = 1, $\upsilon = u, u-1, \ldots, 1$, u = 1,2,3, are non-increasing coefficients of components' distance $d_{ij} = |i-j|$ from the component that has got out of the reliability state subset $\{u, u+1, \ldots, 4\}$, u = 1,2,3,4.
For a particular system, the non-increasing coefficients $q(\upsilon, d_{ij})$, i = 1,2,3, j = 1,2,3, $\upsilon = u, u-1, \ldots, 1$, u = 1,2,3, can be defined differently depending on the specifics of the system and its components. In presented example of a multistate series system, we assume that this coefficient is given by the formula

$$q(\upsilon, d_{ij}) = 1 - [q(\upsilon)]^{d_{ij}},\ i, j = 1,2,3,\ i \ne j, \quad (33)$$

where

$$q(\upsilon) = \frac{1}{2^{\upsilon}},\ \upsilon = u, u-1, \ldots, 1,\ u = 1,2,3, \quad (34)$$

and distance between components $d_{ij} = |i-j|$, i = 1,2,3, j = 1,2,3.

Further for the non-increasing coefficients of components' distance, defined by (33)-(34), we get

$$q(1,0)=1,\ q(1,1)=0.5,\ q(1,2)=0.75, \qquad (35)$$

and for the coordinates of component reliability functions, given by (26)-(27), i.e. for the component intensities

$$\lambda(1)=0.25,\ \lambda(2)=0.5,\ \lambda(3)=0.75,\ \lambda(4)=1, \quad (36)$$

the system reliability function coordinate $\boldsymbol{R}_{LLS}(t,1)$, given by (29), takes the form

$$\boldsymbol{R}_{LLS}(t,1)=\exp(-2.25t) - 0.86\cdot[\exp(-2.25t)-\exp(-1.08t)] - 0.5\cdot[\exp(-2.25t)-\exp(-1.25t)],\ t\geq 0. \qquad (37)$$

Similarly, for the non-increasing coefficients of components' distance

$$q(2,0)=1,\ q(2,1)=0.75,\ q(2,2)=0.94, \qquad (38)$$

and for the component intensities given by (36), the system reliability function coordinate $\boldsymbol{R}_{LLS}(t,2)$ given by (30), takes following form

$$\boldsymbol{R}_{LLS}(t,2)=\exp(-3.75t) - 0.73\cdot[\exp(-3.75t)-\exp(-1.7t)] - 0.39\cdot[\exp(-3.75t)-\exp(-1.83t)],\ t\geq 0. \qquad (39)$$

And further, for the non-increasing coefficients of components' distance

$$q(3,0)=1,\ q(3,1)=0.88,\ q(3,2)=0.98, \qquad (40)$$

and for the component intensities given by (36), we obtain the system reliability function coordinate $\boldsymbol{R}_{LLS}(t,3)$ given by (31), of the form

$$\boldsymbol{R}_{LLS}(t,3)=\exp(-5.25t) - 0.69\cdot[\exp(-5.25t)-\exp(-2.37t)] - 0.36\cdot[\exp(-5.25t)-\exp(-2.46t)],\ t\geq 0. \qquad (41)$$

Finally, the system reliability function coordinate $\boldsymbol{R}_{LLS}(t,4)$ given by (32), for the component intensities (33), takes following form

$$\boldsymbol{R}_{LLS}(t,4)=\exp[-3t],\ t\geq 0. \qquad (42)$$

The reliability function coordinates of the exemplary series system are illustrated in *Figure 3*.

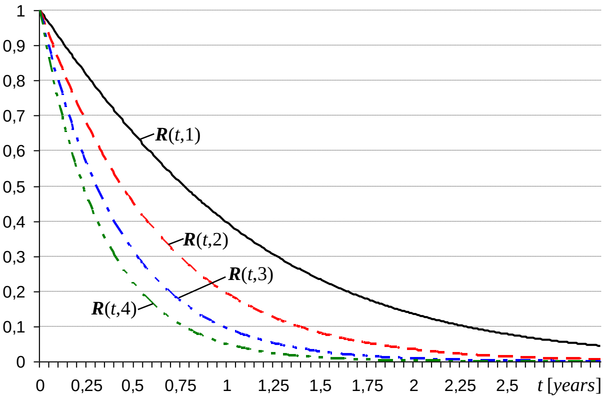

Figure 3. The graph of the exemplary system reliability function coordinates

From *Corollary 1*, according to (19)-(20), the mean lifetimes of the exemplary system in the reliability state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, are respectively given by

$$\mu_{LLS}(1)=\frac{1}{3(\lambda(2)+\lambda(1))} + \frac{2\cdot\lambda(2)}{\lambda(1)[\frac{1}{q(1,0)}+\frac{1}{q(1,1)}+\frac{1}{q(1,2)}]-3(\lambda(2)+\lambda(1))} \cdot[\frac{1}{3(\lambda(2)+\lambda(1))}-\frac{1}{\lambda(1)[\frac{1}{q(1,0)}+\frac{1}{q(1,1)}+\frac{1}{q(1,2)}]}] + \frac{\lambda(2)}{\lambda(1)[\frac{1}{q(1,0)}+\frac{2}{q(1,1)}]-3(\lambda(2)+\lambda(1))} \cdot[\frac{1}{3(\lambda(2)+\lambda(1))}-\frac{1}{\lambda(1)[\frac{1}{q(1,0)}+\frac{2}{q(1,1)}]}], \qquad (43)$$

$$\mu_{LLS}(2)=\frac{1}{3(\lambda(3)+\lambda(2))} + \frac{2\cdot\lambda(3)}{\lambda(2)[\frac{1}{q(2,0)}+\frac{1}{q(2,1)}+\frac{1}{q(2,2)}]-3(\lambda(3)+\lambda(2))} \cdot[\frac{1}{3(\lambda(3)+\lambda(2))}-\frac{1}{\lambda(2)[\frac{1}{q(2,0)}+\frac{1}{q(2,1)}+\frac{1}{q(2,2)}]}] + \frac{\lambda(3)}{\lambda(2)[\frac{1}{q(2,0)}+\frac{2}{q(2,1)}]-3(\lambda(3)+\lambda(2))} \cdot[\frac{1}{3(\lambda(3)+\lambda(2))}-\frac{1}{\lambda(2)[\frac{1}{q(2,0)}+\frac{2}{q(2,1)}]}], \qquad (44)$$

$$\mu_{LLS}(3) = \frac{1}{3(\lambda(4)+\lambda(3))}$$
$$+ \frac{2\cdot\lambda(4)}{\lambda(3)[\frac{1}{q(3,0)}+\frac{1}{q(3,1)}+\frac{1}{q(3,2)}] - 3(\lambda(4)+\lambda(3))}$$
$$\cdot[\frac{1}{3(\lambda(4)+\lambda(3))} - \frac{1}{\lambda(3)[\frac{1}{q(3,0)}+\frac{1}{q(3,1)}+\frac{1}{q(3,2)}]}]$$
$$+ \frac{\lambda(4)}{\lambda(3)[\frac{1}{q(3,0)}+\frac{2}{q(3,1)}] - 3(\lambda(4)+\lambda(3))}$$
$$\cdot[\frac{1}{3(\lambda(4)+\lambda(3))} - \frac{1}{\lambda(3)[\frac{1}{q(3,0)}+\frac{2}{q(3,1)}]}], \quad (45)$$

$$\mu_{LLS}(4) = \frac{1}{3\lambda(4)}. \quad (46)$$

Then, for the component intensities given by (36) and substituting the non-increasing coefficients of components' distance, given by (35), (38) and (40), into the formulae (43)-(46), we obtain the expected values of the system lifetimes in the reliability state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}:

$$\mu_{LLS}(1) \cong 1.03, \ \mu_{LLS}(2) \cong 0.61,$$
$$\mu_{LLS}(3) \cong 0.43, \ \mu_{LLS}(4) \cong 0.33. \quad (47)$$

Similarly, form *Corollary 1*, we can determine the standard deviation of the exemplary system sojourn time in the reliability state subset {1,2,3,4}, {2,3,4}, {3,4}, {4}. According to (22), for the component intensities given by (36) and for the non-increasing coefficients of components' distance given by (35), (38) and (40), we obtain:

$$n_{LLS}(1) \cong 1.96, \ n_{LLS}(2) \cong 0.72,$$
$$n_{LLS}(3) \cong 0.36. \quad (48)$$

Then, substituting the results (47) and (48) into formula (21), the standard deviations of the system lifetimes in the reliability state subsets {1,2,3,4}, {2,3,4}, {3,4}, take values:

$$\sigma_{LLS}(1) \cong 0.95, \ \sigma_{LLS}(2) \cong 0.59,$$
$$\sigma_{LLS}(3) \cong 0.42, \quad (49)$$

and from formula (23), the standard deviation of the system lifetimes in the reliability state subset {4} is equal:

$$\sigma_{LLS}(4) \cong 0.33. \quad (50)$$

Further, using (47) and from definitions presented in [12], it follows that the mean values of the system lifetimes in the particular reliability states are:

$$\bar{\mu}_{LLS}(1) \cong 0.42, \ \bar{\mu}_{LLS}(2) \cong 0.18,$$
$$\bar{\mu}_{LLS}(3) \cong 0.10, \ \bar{\mu}_{LLS}(4) \cong 0.33. \quad (51)$$

The expected values and standard deviations of the system lifetimes in the reliability state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, assuming components independence, respectively are:

$$\mu(1) \cong 1.19, \ \mu(2) \cong 0.66,$$
$$\mu(3) \cong 0.44, \ \mu(4) \cong 0.33, \quad (52)$$

$$\sigma(1) \cong 0.10, \ \sigma(2) \cong 0.63,$$
$$\sigma(3) \cong 0.44, \ \sigma(4) \cong 0.33 \quad (53)$$

and the mean values of the system lifetimes in the particular reliability states, assuming components independence, are:

$$\bar{\mu}(1) \cong 0.53, \ \bar{\mu}(2) \cong 0.22,$$
$$\bar{\mu}(3) \cong 0.11, \ \bar{\mu}(4) \cong 0.33. \quad (54)$$

We assume the critical reliability state is $r = 2$. Then, under the definition of a system risk function, presented in [12], we obtain the risk function of the exemplary system with components dependent according to the local load sharing rule of the form

$$\boldsymbol{r}_{LLS}(t) = 1 - \boldsymbol{R}_{LLS}(t,2) = 1 - \exp[-3(\lambda(3)+\lambda(2))t]$$
$$- \sum_{j=1}^{3} \frac{\lambda(3)}{\lambda(2)\sum_{i=1}^{3}\frac{1}{q(2,d_{ij})} - 3(\lambda(3)+\lambda(2))}$$
$$\cdot[\exp[-3(\lambda(3)+\lambda(2))t]$$
$$- \exp[-\lambda(2)\sum_{i=1}^{3}\frac{1}{q(2,d_{ij})}\cdot t]], \ t \ge 0. \quad (55)$$

Hence, the moment when the system risk function exceeds a permitted level, for instance $\delta = 0.1$, is

$$\tau = r^{-1}(\delta) \cong 0.07 \text{ years} \cong 613 \text{ hours}.$$
(56)

The exemplary system risk function is illustrated in *Figure 4*.

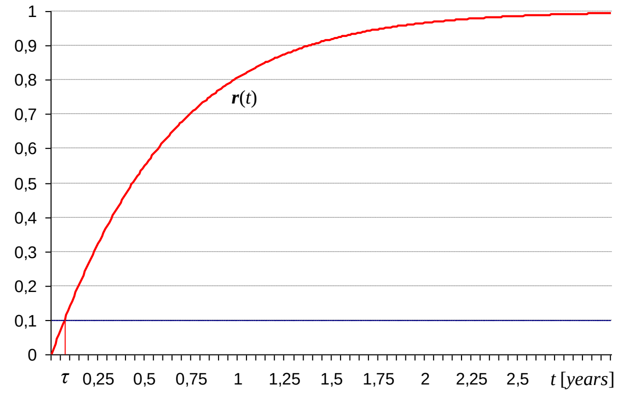

Figure 4. The graph of the risk function $\boldsymbol{r}(t)$ of the exemplary system

From *Corollary 2*, according to (24)-(25), the intensities of departure from the reliability state subsets {1,2,3,4}, {2,3,4}, {3,4}, {4}, of the exemplary system, under assumption that its components are dependent according to the local load sharing rule, are respectively given by

$$\lambda_{LLS}(t,1) = [-0.804 + 0.929 \cdot \exp[1.167t] + 0.625 \cdot \exp[t]] / [-0.357 + 0.857 \cdot \exp[1.167t] + 0.5 \cdot \exp[t]], \quad (57)$$

$$\lambda_{LLS}(t,2) = [-0.461 + 1.244 \cdot \exp[2.05t] + 0.717 \cdot \exp[1.917t]] / [-0.147 + 0.756 \cdot \exp[2.05t] + 0.391 \cdot \exp[1.917t]], \quad (58)$$

$$\lambda_{LLS}(t,3) = [-0.279 + 1.645 \cdot \exp[2.881t] + 0.885 \cdot \exp[2.786t]] / [-0.053 + 0.694 \cdot \exp[2.881t] + 0.359 \cdot \exp[2.786t]], \quad (59)$$

$$\lambda_{LLS}(t,4) = 3 \quad (60)$$

and their graphs are illustrated in *Figure 5*.

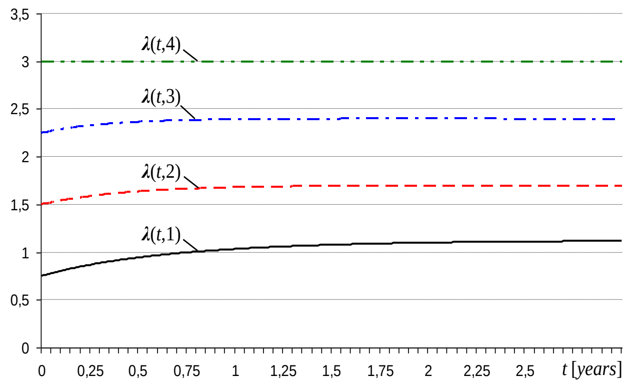

Figure 5. The graph of the exemplary system intensities

5. Conclusion

We have introduced a model of load sharing for multistate series systems with dependent components. The reliability function of a multistate series system under assumption that its components are dependent according to the local load sharing rule is determined and basic reliability characteristics are given in case system components have exponential reliability functions. The obtained theoretical results are illustrated by their application to the reliability evaluation of the exemplary system.

The present study can be extended in the future by linking systems reliability and their operation processes [12]-[13]. Then, a semi-Markov model [9], [15], of the system operation process can be applied to involve interactions between systems' operation processes in the reliability analysis of complex systems with dependent components. This assumption allows to analyze complex systems at their variable operation conditions taking into account their among components dependences that result in changes of their reliability characteristics [4], [5].

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