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Kołowrocki Krzysztof

Soszyńska-Budny Joanna

Torbicki Mateusz
Maritime University, Gdynia, Poland

Reliability of large port grain transportation system

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Abstract

The combination of the results on the reliability of complex multi-state systems related to their operation processes and the results concerning the limit reliability functions of the multistate systems is proposed, to obtain the results on the asymptotic approach to the evaluation of the large complex multi-state systems reliability at the variable operation conditions. The asymptotic approach to the large complex system reliability evaluation and the large complex system limit reliability function are defined. Limit reliability functions of selected large complex systems composed of components having Weibull reliability functions are fixed. The way of using this results is illustrated by their application to the evaluation of reliability characteristics of the large complex port grain transportation subsystem composed of three large multistate non-homogeneous series systems and changing its reliability structure and its components reliability parameters at variable operation conditions.

1. Introduction

In the case of large complex systems, the determination of their exact reliability functions and their risk functions leads us to very complicated formulae that are often useless for reliability practitioners. One of the important techniques that can be useful in this situation is the asymptotic approach [1]-[3], [7]-[9], [11] to that system reliability evaluation. In this approach, instead of the preliminary complex formula for the large complex system reliability function, after assuming that the number of system components tends to infinity and finding the limit reliability of the system, we can obtain its simplified form. Moreover, in the case of large complex systems, the possibility of combining the results of the reliability joint models of complex systems and the results concerning the limit reliability functions of the considered systems is possible [1]-[3], [6]-[9], [11]. This way, the results concerned with asymptotic approach to estimation of multi-state systems at variable operation conditions may be obtained. Main results concerning asymptotic approach to multi-state large system reliability with ageing components in the constant operation conditions are comprehensively presented in the monograph [1]-[3] and some of these results' extensions to the systems operating at the variable conditions can be found in [3]-[4], [7]-[11].

2. Asymptotic approach to reliability of large complex systems

In order to combine the results on the reliability of multi-state systems related to their operation processes and the results concerning the limit reliability functions of the multistate systems, and to obtain the results on the asymptotic approach to the evaluation of the large complex systems reliability, we assume the following definition [10].

Definition 1. A reliability function

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1),..., \boldsymbol{\mathfrak{R}}(t,z)],\ t \in (-\infty,\infty), \qquad (1)$$

where

$$\boldsymbol{\mathfrak{R}}(t,u) = \sum_{b=1}^{v} p_b [\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}, \quad u = 1,2,...,z, \qquad (2)$$

is called a limit reliability function of a large complex multistate system with the reliability function sequence

$$\boldsymbol{R}_n(t,\cdot) = [1, \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)],\ t \in (-\infty,\infty),$$
$$n \in N, \tag{3}$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}_n(t,u)]^{(b)},\ u = 1,2,...,z, \tag{4}$$

if there exist normalizing constants

$$a_n^{(b)}(u) > 0,\ b_n^{(b)}(u) \in (-\infty,\infty),$$
$$u = 1,2,...,z,\ b = 1,2,...,v,$$

such that

$$\lim_{n\to\infty} [\boldsymbol{R}_n(a_n^{(b)}(u)t + b_n^{(b)}(u),u)]^{(b)} = [\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}$$

for all t from the sets of continuity points $C_{[\boldsymbol{\mathfrak{R}}(u)]^{(b)}}$ of the functions $[\boldsymbol{\mathfrak{R}}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,v$.
Hence, for sufficiently large n, the following approximate formulae are valid

$$\boldsymbol{R}_n(t,\cdot) = [1, \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)],\ t \in (-\infty,\infty), \tag{5}$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{\mathfrak{R}}(\frac{t - b_n^{(b)}(u)}{a_n^{(b)}(u)},u)]^{(b)},\ t \in (-\infty,\infty),$$
$$u = 1,2,...,z. \tag{6}$$

3. Limit reliability functions of large complex systems

Definition 2. A multi-state regular series-parallel system is called non-homogeneous if it is composed of a, $1 \le a \le k$, $k \in N$, different types of series subsystems and the fraction of the ith type series subsystem is equal to q_i, where $q_i > 0$, $\sum_{i=1}^{a} q_i = 1$. Moreover, the ith type series subsystem consists of e_i, $1 \le e_i \le l$, $l \in N$, types of components with reliability functions

$$R^{(i,j)}(t,u) = 1 - F^{(i,j)}(t,u),\ t \in (-\infty,\infty),$$
$$j = 1,2,...,e_i,\ u = 1,2,...,z,$$

and the fraction of the jth type component in this subsystem is equal to p_{ij}, where $p_{ij} > 0$ and $\sum_{j=1}^{e_i} p_{ij} = 1$.
The numbers

$$k,\ l,\ a,\ e_i,\ i = 1,2,...,a,$$

and

$$q_i,\ p_{ij},\ i = 1,2,...,a,\ \ j = 1,2,...,e_i,$$

are called the system structure shape parameters.

The scheme of a regular non-homogeneous series-parallel system is shown in *Figure 1*.

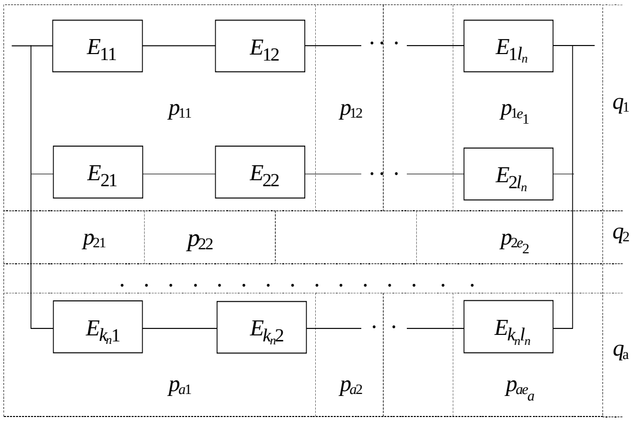

Figure 1. The scheme of a regular non-homogeneous series-parallel system

The proposition concerned with the reliability function of large series-parallel Weibull system operating at the variable operation states is an exemplary result that can be worked out on the basis of the results included in [3] for the considered there large and complex systems. Namely, from Proposition 9.1, Proposition 7.4 and Corollary 5.12 given in [3], we get the following theorem.

Proposition 1. If components of the non-homogeneous, regular multi-state series-parallel complex system with the structure shape parameters

$$k = k^{(b)},\ l = l^{(b)},\ a = a^{(b)},\ e_i = e_i^{(b)},$$
$$i = 1,2,...,a^{(b)},\ b = 1,2,...,v, \tag{7}$$

and

$$q_i = q_i^{(b)},\ p_{ij} = p_{ij}^{(b)},\ i = 1,2,...,a^{(b)},\ j = 1,2,...,e_i^{(b)},$$
$$b = 1,2,...,v, \tag{8}$$

at the operation state z_b, $b = 1,2,...,v$, have Weibull reliability functions

$[R^{(i,j)}(t,\cdot)]^{(b)} = [1, [R^{(i,j)}(t,1)]^{(b)},...,[R^{(i,j)}(t,z)]^{(b)}],$
$t \in (-\infty,\infty),\ b = 1,2,...,v,$

where

$[R^{(i,j)}(t,u)]^{(b)} = \exp[-[\lambda_{ij}(u)]^{(b)} t^{[\beta_{ij}(u)]^{(b)}}]$ for $t \ge 0$,
$\lambda_{ij}(u) > 0$, $\beta_{ij}(u) > 0$, $u = 1,2,\ldots,z$, $i = 1,2,...,a^{(b)}$,
$j = 1,2,...,e_i^{(b)}$, $b = 1,2,...,v$,

and

$$k^{(b)} = \text{constant},\ l^{(b)} \to \infty \text{ as } n \to \infty,\quad b = 1,2,...,v, \tag{9}$$

$$a_n^{(b)}(u) = \left(\frac{1}{[\lambda(u)]^{(b)} l^{(b)}}\right)^{1/[\beta(u)]^{(b)}},\quad b_n^{(b)}(u) = 0,\quad u = 1,2,\ldots,z,\quad b = 1,2,...,v, \tag{10}$$

where

$$[\beta_i(u)]^{(b)} = \min_{1 \le i \le e_i^{(b)}} \{[\beta_{ij}(u)]^{(b)}\}, \tag{11}$$

$$[\beta(u)]^{(b)} = \max_{1 \le i \le a^{(b)}} \{[\beta_i(u)]^{(b)}\}, \tag{12}$$

$$[\lambda_i(u)]^{(b)} = \sum_{\{j:[\beta_{ij}(u)]^{(b)} = [\beta_i(u)]^{(b)}\}} p_{ij}^{(b)} [\lambda_{ij}(u)]^{(b)}, \tag{13}$$

$$[\lambda(u)]^{(b)} = \min_{1 \le i \le a^{(b)}} \{[\lambda_i(u)]^{(b)} : \beta_i(u) = \beta(u)\},$$

$$u = 1,2,\ldots,z,\ i = 1,2,...,a^{(b)},\ b = 1,2,...,v, \tag{14}$$

then

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1),..., \boldsymbol{\mathfrak{R}}(t,z)],\ t \in (-\infty,\infty), \tag{15}$$

where

$$\boldsymbol{\mathfrak{R}}(t,u) = \sum_{b=1}^{v} p_b [\boldsymbol{\mathfrak{R}}(t,u)]^{(b)},\ u = 1,2,...,z, \tag{16}$$

$$[\boldsymbol{\mathfrak{R}}(t,u)]^{(b)} = \boldsymbol{\mathfrak{R}}'_9(t,u) = 1 - \prod_{\{i:[\beta_i(u)]^{(b)} = [\beta(u)]^{(b)}\}} [1 - \exp[-\frac{[\lambda_i(u)]^{(b)}}{[\lambda(u)]^{(b)}} t^{[\beta(u)]^{(b)}}]]^{q_i^{(b)} k^{(b)}}$$

$$\text{for } t \ge 0,\ u = 1,2,\ldots,z, b = 1,2,...,v, \tag{17}$$

is its limit reliability function.

4. Example of large complex system reliability evaluation

4.1. Port grain transportation system operation process

We consider the port grain transportation system, presented in *Figure 2*, assigned to handle the clearing of exported and imported grain.

The port grain transportation system function is loading railway trucks with grain. The railway truck loading is performed in the following successive grain transportation system steps:

- gravitational passing of grain from the storage placed on the 8th elevator floor through 45 hall to horizontal conveyors placed in the elevator basement,

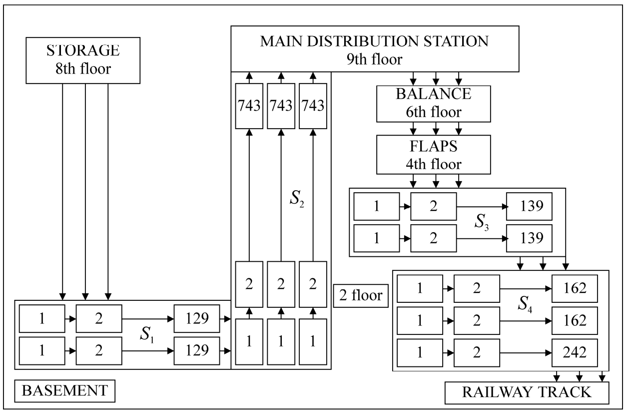

Figure 2. The scheme of the port grain transportation system structure at the operation state z_1

- transport of grain through horizontal conveyors to vertical bucket elevators transporting grain to the main distribution station placed on the 9th floor,
- gravitational dumping of grain through the main distribution station to the balance placed on the 6th floor,
- dumping weighed grain through the complex of flaps placed on the 4th floor to horizontal conveyors placed on the 2nd floor,
- dumping of grain from horizontal conveyors to worm conveyors,
- dumping of grain from worm conveyors to railway trucks.

In loading the railway trucks with grain the following presented in *Figures 2-3* transportation subsystems take part:

S_1 – horizontal conveyors of the first type,
S_2 – vertical bucket elevators,
S_3 – horizontal conveyors of the second type,
S_4 – worm conveyors,

the main distribution station and the balance.

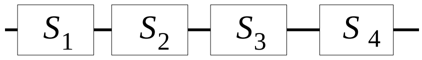

Figure 3. The general scheme of the port grain transportation system reliability structure

The main distribution station is the system of dumping channels in the form of a steel box composed of dividing walls, which direct the grain from bucket conveyors to the balance. Its executive elements are composed of three steel sleeves and pneumatic elements in the form of three servomotors. The electronic balance weighs the dumped grain with electronic indicators. Its executive elements during loading and unloading with grain are flaps, which are opened and closed by five pneumatic servomotors. In further analysis, we omit these two subsystems and will deal with the reliability of the subsystems S_1, S_2, S_3 and S_4 only.
The transportation subsystems S_1, S_2, S_3 and S_4 have steel covers and they are provided with drives in the form of electrical engines with gears. In their reliability analysis we omit their drives as they are different types mechanisms. We also omit their covers as they have a high reliability and, practically, do not fail.
Taking into account the operation process of the considered transportation system, described by its operators, we distinguish its following $\nu = 3$ operation states:

- z_1 – the system operation with the largest efficiency when all components of the subsystems S_1, S_2, S_3 and S_4 are used (*Figure 2*),
- z_2 – the system operation with less efficiency system when the first conveyor of subsystem S_1, the first and second elevators of subsystem S_2, the first conveyor of subsystem S_3 and the first and second conveyors of subsystem S_4 are used (*Figure 4*),
- z_3 – the system operation with least efficiency when only the first conveyor of subsystem S_1, the first elevator of subsystem S_2, the first conveyor of subsystem S_3 and the first conveyor of subsystem S_4 are used (*Figure 5*).

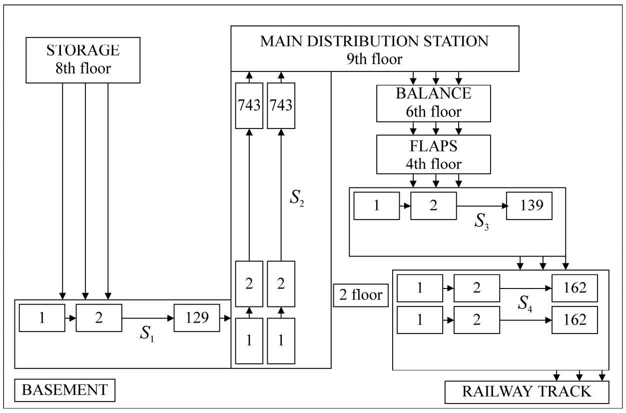

Figure 4. The scheme of the port grain transportation system structure at the operation state z_2

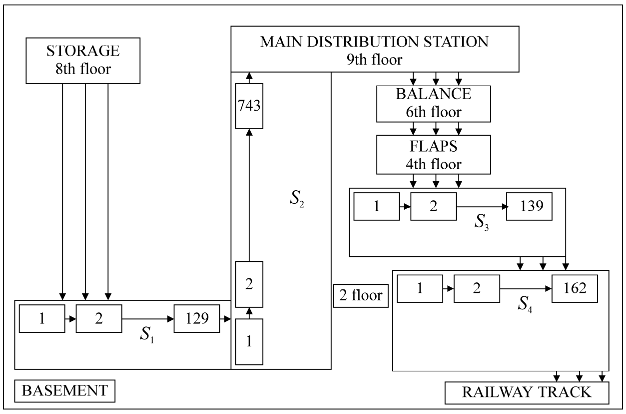

Figure 5. The scheme of the port grain transportation system structure at the operation state z_3

At all system operation states, subsystems S_1, S_2, S_3 and S_4 become either non-homogeneous series-parallel systems or non-homogeneous series systems. This way, the changes of the grain transportation system reliability structure at different operation states are defined. Its components reliability parameters at different operation states will be defined in this example continuation in Section 4.2.
Considering the system operators opinion, we assume the matrix of the probabilities of transitions between the states are given by [3]

$$[p_{bl}]_{3x3} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{4}{9} & 0 & \frac{5}{9} \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} \quad (18)$$

Moreover, we assume the following matrix of the conditional distribution functions of the system sojourn times θ_{bl}, $b, l = 1, 2, 3,$ at the operation states [3]

$$[H_{bl}(t)]_{3x3} = \begin{bmatrix} 0 & 1-e^{-5t} & 1-e^{-10t} \\ 1-e^{-40t} & 0 & 1-e^{-50t} \\ 1-e^{-10t} & 1-e^{-20t} & 0 \end{bmatrix}.$$

Further, using (1) given in [5], we fix the conditional mean values $M_{bl} = E[\theta_{bl}],\ b,l = 1,2,3,4,$ of the exemplary system sojourn times at the particular operation states as follows:

$$M_{12} = 0.20,\ M_{13} = 0.10,\ M_{21} = 0.025,$$
$$M_{23} = 0.020,\ M_{31} = 0.10,\ M_{32} = 0.05. \quad (19)$$

This way, the exemplary system operation process is defined and we may find its main characteristics. Namely, using (6), (7) and applying (2) given in [5], the unconditional mean sojourn times at the particular operation states are given by:

$$M_1 = 0.133,\ M_2 = 0.022,\ M_3 = 0.067. \quad (20)$$

Next, according to (4) given in [5], the system of equations after considering (6), we find the steady probabilities

$$\pi_1 \cong 0.279,\ \pi_2 \cong 0.344,\ \pi_3 \cong 0.377. \quad (21)$$

After considering the result (8) and (9), according to (3) given in [5], the limit values of the exemplary system operation process transient probabilities $p_b(t)$ at the operation states z_b are given by

$$p_1 \cong 0.530,\ p_2 \cong 0.109,\ p_3 \cong 0.361. \quad (22)$$

4.2. Port grain transportation system reliability

Taking into account the efficiency of the considered port grain transportation system we distinguish the following three reliability states of the systems and its components:

state 2 – the state ensuring the largest efficiency of the system and its conveyors,

state 1 – the state ensuring less efficiency of the system caused by throwing grain off the system conveyors,

state 0 – the state involving failure of the system.

The considered here transportation system analysis of its varying in time operation process was performed in Section 4.1, where it was assumed that the system reliability structure and its subsystems and components reliability depend on its changing in time operation states. Considering that, we assume that its subsystems $S_\upsilon,\ \upsilon = 1,2,3.4,$ are composed of three-state, i.e. z = 3, components $E_{ij}^{(\upsilon)},\ \upsilon = 1,2,3,4,$ having the conditional reliability functions given by the vector

$$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1,[R_{ij}^{(\upsilon)}(t,1)]^{(b)},[R_{ij}^{(\upsilon)}(t,2)]^{(b)}],$$
$$b = 1,2,3,$$

with the Weibull co-ordinates

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)} t^{[\beta_{ij}^{(\upsilon)}(1)]^{(b)}}],$$
$$[R_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)} t^{[\beta_{ij}^{(\upsilon)}(2)]^{(b)}}],$$

different at various operation states $z_b,\ b = 1,2,3.$

The influence of the system operation states changing on the changes of the system reliability structure and its components reliability functions is as follows.

At the system operation state z_1, the system is a series system with the structure showed in *Figure 3*, composed of four series-parallel subsystems S_1, S_2, S_3 and S_4 illustrated respectively in *Figure 2*.

Further, we consider only a part of the port grain transportation system composed of three large multistate non-homogeneous series-parallel subsystems linked in series changing its reliability structures and its components reliability parameters at variable operation conditions. Namely, we analyse the reliability of the subsystem S_4.

At the system operation state z_1, the subsystem S_4 with the structure showed in *Figure 2*, consists of three chain conveyors forming series subsystems ($k^{(1)} = 3$), each composed of a wheel driving the belt, a reversible driving wheel and 160, 160 and 240 links respectively. Thus, two conveyors have 162 components and the remaining one has 242 components ($l_1^{(1)} = 162$, $l_2^{(1)} = 162$, $l_3^{(1)} = 242$) what means that the subsystem is a non-homogeneous non-regular three-state series-parallel system with the Weibull reliability functions. In two series subsystems of the subsystem S_4 there are respectively:

- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 1,2$, $j = 1,2$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.005t],$$
$$[R_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.006t],\ t \geq 0,$$
$$i = 1,2,\ j = 2; \qquad (23)$$

- 160 links marked by $E_{ij}^{(4)}$, $i = 1,2$, $j = 3,4,...,162$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.012t],$$
$$[R_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.014t],\ t \geq 0,$$
$$i = 1,2,\ j = 3,4,...,162. \qquad (24)$$

In the third series subsystems of the subsystem S_4 there are respectively:

- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 3$, $j = 1,2$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.210\sqrt{t}\ ],$$
$$[R_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.219\sqrt{t}\ ],\ t \geq 0,$$
$$i = 3,\ j = 2; \qquad (25)$$

- 240 links marked by $E_{ij}^{(4)}$, $i = 3$, $j = 3,4,...,242$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(1)} = \exp[-0.261\sqrt{t}\ ],$$
$$[R_{ij}^{(4)}(t,2)]^{(1)} = \exp[-0.283\sqrt{t}\ ],\ t \geq 0,$$
$$i = 3,\ j = 3,4,...,242. \qquad (26)$$

At the system operation state z_2, the subsystem S_4 with the structure showed in *Figure 4*, consists of three identical chain conveyors forming series subsystems ($k^{(2)} = 2$), each composed of a wheel driving the belt, a reversible driving wheel and 160 links ($l_1^{(2)} = 162$, $l_2^{(2)} = 162$) what means that the subsystem is a non-homogeneous non-regular three-state series-parallel system with the Weibull reliability functions. In the series subsystems of the subsystem S_4 there are respectively:

- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 1,2$, $j = 1,2$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(2)} = \exp[-0.002t],$$
$$[R_{ij}^{(4)}(t,2)]^{(2)} = \exp[-0.004t],\ t \geq 0,$$
$$i = 1,2,\ j = 2; \qquad (27)$$

- 160 links marked by $E_{ij}^{(4)}$, $i = 1,2$, $j = 3,4,...,162$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(2)} = \exp[-0.008t],$$
$$[R_{ij}^{(4)}(t,2)]^{(2)} = \exp[-0.010t],\ t \geq 0,$$
$$i = 1,2,\ j = 3,4,...,162. \qquad (28)$$

At the system operational state z_3, the subsystem S_4 with the structure showed in *Figure 5*, consists of on chain conveyor forming a series system ($k^{(3)} = 1$), composed of a wheel driving the belt, a reversible driving wheel and 160 links ($l_1^{(3)} = 162$) with the Weibull reliability functions. In the series system of the subsystem S_4 there are respectively:

- 2 two driving wheels marked by $E_{ij}^{(4)}$, $i = 1$, $j = 1,2$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(3)} = \exp[-0.001t],$$
$$[R_{ij}^{(4)}(t,2)]^{(3)} = \exp[-0.003t],\ t \geq 0,$$
$$i = 1,\ j = 2; \qquad (29)$$

- 160 links marked by $E_{ij}^{(4)}$, $i = 1$, $j = 3,4,...,162$, with a reliability function co-ordinates

$$[R_{ij}^{(4)}(t,1)]^{(3)} = \exp[-0.007t],$$
$$[R_{ij}^{(4)}(t,2)]^{(3)} = \exp[-0.009t],\ t \geq 0,$$
$$i = 1,\ j = 3,4,...,162. \qquad (30)$$

The subsystem S_4 at the operation state z_1 is a non-regular series-parallel system composed of three chain conveyors forming series systems. To make it regular, we conventionally complete its two first conveyors that have 162 components with 80 components that do not fail, i.e. we assume that the additional components have "reliability functions" with the following coordinates

$$[R_{ij}^{(4)}(t,1)]^{(1)} = \exp[-[\lambda_{ij}^{(4)}(1)]^{(1)} t)],$$
$$[R_{ij}^{(4)}(t,2)]^{(1)} = \exp[-[\lambda_{ij}^{(4)}(2)]^{(1)} t)] \text{ for } t \geq 0,$$
$$i = 1,2,\ j = 163,164,...,242, \qquad (31)$$

where

$$[\lambda_{ij}^{(4)}(1)]^{(1)} = 0\ [\lambda_{ij}^{(4)}(2)]^{(1)} = 0,$$
$$i = 1,2,\ j = 163,164,...,242. \qquad (32)$$

After that modification, applying *Proposition 1*, since its structure shape parameters defined by (7)-(8) are

$$k^{(1)} = 3, \quad l^{(1)} = 242, \quad a^{(1)} = 2, \quad e_1^{(1)} = 3, \quad e_2^{(1)} = 2,$$

and

$$q_1^{(1)} = \frac{2}{3}, \; p_{11}^{(1)} = \frac{2}{242}, \; p_{12}^{(1)} = \frac{16}{242}, \; p_{13}^{(1)} = \frac{80}{242},$$

$$q_2^{(1)} = \frac{1}{3}, \; p_{21}^{(1)} = \frac{2}{242}, \; p_{22}^{(1)} = \frac{240}{242},$$

the number of reliability states is $z = 2$ and by (9)-(14) and considering the values of its components' intensities of departure from the reliability state subsets defined by (23)-(26) and by (31)-(32):

$$[\beta_1(1)]^{(1)} = \min\{1\} = 1,$$
$$[\beta_2(1)]^{(1)} = \min\{1\} = 1,$$
$$[\beta_3(1)]^{(1)} = \min\{0.5\} = 0.5,$$
$$[\beta(1)]^{(1)} = \max\{1, \; 0.5\} = 1,$$

$$[\beta_1(2)]^{(1)} = \min\{1\} = 1,$$
$$[\beta_2(2)]^{(1)} = \min\{1\} = 1,$$
$$[\beta_3(2)]^{(1)} = \min\{0.5\} = 0.5,$$
$$[\beta(2)]^{(1)} = \max\{1, \; 0.5\} = 1,$$

$$[\lambda_i(1)]^{(1)} = \frac{2}{242}0.005 + \frac{160}{242}0.012 + \frac{80}{242}0.0$$
$$= 1.93/242,$$
$$[\lambda_i(2)]^{(1)} = \frac{2}{242}0.006 + \frac{160}{242}0.014 + \frac{80}{242}0.0$$
$$= 2.252/242, \quad i = 1,2,$$

$$[\lambda_3(1)]^{(1)} = \frac{2}{242}0.21 + \frac{240}{242}0.261 = 63.06/242,$$

$$[\lambda_3(2)]^{(1)} = \frac{2}{242}0.219 + \frac{240}{242}0.283 = 68.358/242,$$

$$[\lambda(1)]^{(1)} = \min\{1.93/242\} = 1.93/242,$$

$$[\lambda(2)]^{(1)} = \min\{2.252/242\} = 2.252/242,$$

$$a_n^{(1)}(1) = \frac{1}{(1.93/242)\cdot 242} = \frac{1}{1.93}, \quad b_n^{(1)}(1) = 0,$$

$$a_n^{(1)}(2) = \frac{1}{(2.252/242)\cdot 242} = \frac{1}{2.252}, \quad b_n^{(1)}(2) = 0,$$

and according to (15)-(17) and considering (6), the approximate formula for the subsystem S_4 reliability function takes the form

$$[\boldsymbol{R}^{(4)}(t,\cdot)]^{(1)} \cong \boldsymbol{\mathfrak{R}}^{(4)}((t - b_n^{(1)}(u))/a_n^{(1)}(u), \cdot)$$
$$= [[\boldsymbol{\mathfrak{R}}^{(4)}(t,1)]^{(1)}, [\boldsymbol{\mathfrak{R}}^{(4)}(t,2)]^{(1)}] \qquad (33)$$

for $t \geq 0$, where

$$[\boldsymbol{\mathfrak{R}}^{(4)}(t,1)]^{(1)} \cong \boldsymbol{\mathfrak{R}}'_9 \left(\frac{t - b_n^{(1)}(1)}{a_n^{(1)}(1)}, 1\right)$$
$$= 1 - [1 - \exp[-1.93t]]^2$$
$$= 2\exp[-1.93t] - \exp[-3.86t], \qquad (34)$$

$$[\boldsymbol{\mathfrak{R}}^{(4)}(t,2)]^{(1)} = \boldsymbol{\mathfrak{R}}'_9 \left(\frac{t - b_n^{(1)}(2)}{a_n^{(1)}(2)}, 2\right)$$
$$= 1 - [1 - \exp[-2.252]]^2$$
$$= 2\exp[-2.252t] - \exp[-4.504t]. \qquad (35)$$

The subsystem S_4 at the operation state z_2 is a regular series-parallel system composed of two identical chain conveyors forming series systems, then applying *Proposition 1*, since its structure shape parameters defined by (7)-(8) are

$$k^{(2)} = 2, \quad l^{(2)} = 162, \quad a^{(2)} = 1, \quad e_1^{(2)} = 2,$$

and

$$q_1^{(2)} = 1, \; p_{11}^{(2)} = \frac{2}{162}, \; p_{12}^{(2)} = \frac{160}{162},$$

the number of reliability states is $z = 2$ and by (9)-(14) and considering the values of its components' intensities of departure from the reliability state subsets defined by (27)-(28):

$$[\beta_i(1)]^{(2)} = \min\{1\} = 1,$$
$$[\beta(1)]^{(2)} = \max\{1\} = 1,$$
$$[\beta_i(2)]^{(2)} = \min\{1\} = 1,$$
$$[\beta(2)]^{(2)} = \max\{1\} = 1,$$
$$[\lambda_i(1)]^{(2)} = \frac{2}{162}0.002 + \frac{160}{162}0.008 = 1.284/162,$$

$$[\lambda_i(2)]^{(2)} = \frac{2}{162}0.004 + \frac{160}{162}0.010 = 1.608/162,$$
$$i = 1,2,$$

$$[\lambda(1)]^{(2)} = \min\{1.284/162\} = 1.284/162,$$

$$[\lambda(2)]^{(2)} = \min\{1.608/162] = 1.608/162,$$

$$a_n^{(2)}(1) = \frac{1}{(1.284/162)\cdot 162} = \frac{1}{1.284}, \ \ b_n^{(2)}(1) = 0,$$

$$a_n^{(2)}(2) = \frac{1}{(1.608/162)\cdot 162} = \frac{1}{1.608}, \ \ b_n^{(2)}(2) = 0,$$

and according to (15)-(17) and considering (6), the approximate formula for the subsystem S_4 reliability function takes the form

$$[\boldsymbol{R}^{(4)}(t,\cdot)]^{(1)} \cong \mathfrak{R}^{(4)}((t-b_n^{(1)}(u))/a_n^{(1)}(u),\cdot) = [[\mathfrak{R}^{(4)}(t,1)]^{(1)}, [\mathfrak{R}^{(4)}(t,2)]^{(1)}] \quad (36)$$

for $t \ge 0$, where

$$[\mathfrak{R}^{(4)}(t,1)]^{(2)} \cong \mathfrak{R}'_9(\frac{t-b_n^{(2)}(1)}{a_n^{(2)}(1)},1) = 1-[1-\exp[-1.284t]]^2 = 2\exp[-1.284t]-\exp[-2.568t], \quad (37)$$

$$[\mathfrak{R}^{(4)}(t,2)]^{(2)} = \mathfrak{R}'_9(\frac{t-b_n^{(2)}(2)}{a_n^{(2)}(2)},2) = 1-[1-\exp[-1.608t]]^2 = 2\exp[-1.608t]-\exp[-3.216t]. \quad (38)$$

The subsystem S_4 at the operation state z_3 is a series system composed of one chain conveyor forming series system, then applying *Proposition 1*, since its structure shape parameters defined by (7)-(8) are

$$k^{(3)} = 1, \ \ l^{(3)} = 162, \ \ a^{(3)} = 1, \ \ e_1^{(3)} = 2,$$

and

$$q_1^{(3)} = 1, \ p_{11}^{(3)} = \frac{2}{162}, \ p_{12}^{(3)} = \frac{160}{162},$$

the number of reliability states is $z = 2$ and by (9)-(14) and considering the values of its components' intensities of departure from the reliability state subsets defined by (29)-(30):

$$[\beta_i(1)]^{(3)} = \min\{1\} = 1,$$

$$[\beta(1)]^{(3)} = \max\{1\} = 1,$$

$$[\beta_i(2)]^{(3)} = \min\{1\} = 1,$$

$$[\beta(2)]^{(3)} = \max\{1\} = 1,$$

$$[\lambda_i(1)]^{(3)} = \frac{2}{162}0.001 + \frac{160}{162}0.007 = 1.122/162,$$

$$[\lambda_i(2)]^{(3)} = \frac{2}{162}0.003 + \frac{160}{162}0.009 = 1.446/162,$$

$i = 1$,

$$[\lambda(1)]^{(3)} = \min\{1.122/162\} = 1.122/162,$$

$$[\lambda(2)]^{(3)} = \min\{1.446/162] = 1.446/162,$$

$$a_n^{(3)}(1) = \frac{1}{(1.122/162)\cdot 162} = \frac{1}{1.122}, \ \ b_n^{(3)}(1) = 0,$$

$$a_n^{(3)}(2) = \frac{1}{(1.446/162)\cdot 162} = \frac{1}{1.446}, \ \ b_n^{(3)}(2) = 0,$$

and according to (15)-(17) and considering (6), the approximate formula for the subsystem S_4 reliability function takes the form

$$[\boldsymbol{R}^{(4)}(t,\cdot)]^{(3)} \cong \mathfrak{R}^{(4)}((t-b_n^{(3)}(u))/a_n^{(3)}(u),\cdot) = [[\mathfrak{R}^{(4)}(t,1)]^{(3)}, [\mathfrak{R}^{(4)}(t,2)]^{(3)}] \quad (39)$$

for $t \ge 0$, where

$$[\mathfrak{R}^{(4)}(t,1)]^{(3)} \cong \mathfrak{R}'_9(\frac{t-b_n^{(3)}(1)}{a_n^{(3)}(1)},1) = 1-[1-\exp[-1.122t]]^1 = \exp[-1.122t], \quad (40)$$

$$[\mathfrak{R}^{(4)}(t,2)]^{(3)} = \mathfrak{R}'_9(\frac{t-b_n^{(3)}(2)}{a_n^{(3)}(2)},2) = 1-[1-\exp[-1.446t]]^1 = \exp[-1.446t]. \quad (41)$$

Finally, considering (6) and (22), the approximate formula for the subsystem S_4 unconditional reliability function takes the form

$$\boldsymbol{R}^{(4)}(t,\cdot) = [\mathfrak{R}^{(4)}(t,1), \mathfrak{R}^{(4)}(t,2)] \text{ for } t \ge 0, \quad (42)$$

where

$$\mathfrak{R}^{(4)}(t,1) = 0.530[\mathfrak{R}^{(4)}(t,1)]^{(1)} + 0.109[\mathfrak{R}^{(4)}(t,1)]^{(2)} + 0.361[\mathfrak{R}^{(4)}(t,1)]^{(3)}, \quad (43)$$

$$[\mathfrak{R}(t,2)] = 0.530[\mathfrak{R}^{(4)}(t,2)]^{(1)} + 0.109[\mathfrak{R}^{(4)}(t,2)]^{(2)}$$

$$+\ 0.361[\mathfrak{R}^{(4)}(t,2)]^{(3)}, \quad (44)$$

where $[\mathfrak{R}^{(4)}(t,1)]^{(1)}$, $[\mathfrak{R}^{(4)}(t,1)]^{(2)}$, $[\mathfrak{R}^{(4)}(t,1)]^{(3)}$ are given by (34), (37), (40) and $[\mathfrak{R}^{(4)}(t,2)]^{(1)}$, $[\mathfrak{R}^{(4)}(t,2)]^{(2)}$, $[\mathfrak{R}^{(4)}(t,2)]^{(3)}$ are given by (35), (38), (41).

The results given by (42)-(44) are not much different from the exact results given in [3], what is illustrated in *Table 1*, presenting the differences between the values of the port grain transportation subsystem S_4 exact reliability function the values of the system approximate reliability function.

The approximate expected value of the port grain transportation subsystem S_4 unconditional lifetime in the reliability state subset $\{1,2\}$, calculated according to (17) given in [5] from the result given by (43) and (22), is

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1)$$
$$= 0.530 \cdot 1.5/1.93 + 0.109 \cdot 1.5/1.284$$
$$+ 0.361 \cdot 0.891 \cong 0.861 \text{ year.} \quad (45)$$

The approximate expected value of the port grain transportation subsystem S_4 unconditional lifetime in the reliability state subset $\{2\}$, calculated according to (17) from the result given by (44) and (22), is

$$\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2)$$
$$= 0.530 \cdot 1.5/2.252 + 0.109 \cdot 1.5/1.608$$
$$+ 0.361 \cdot 0.692 \cong 0.705 \text{ year.} \quad (46)$$

Further, considering (45) and (46) and applying (19) given in [5], the approximate mean values of the subsystem S_4 unconditional lifetimes in the particular reliability states 1, 2, respectively are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.156 \text{ year},$$
$$\bar{\mu}(2) = \mu(2) = 0.705 \text{ year.} \quad (47)$$

The approximate values of the unconditional subsystem S_4 lifetimes given by (45)-(47) are almost identical with their exact values determined in [3]. Thus, the accuracy of the approximation of the subsystem exact reliability function by its asymptotic form is very good.

Table 1. The differences between the values of the coordinates of the port grain transportation subsystem S_4 exact and approximate reliability function

t	$\boldsymbol{R}^{(4)}(t,1) - \mathfrak{R}^{(4)}(t,1)$	$\boldsymbol{R}^{(4)}(t,2) - \mathfrak{R}^{(4)}(t,2)$
0	0	0
0.005	$1.3443 \cdot 10^{-6}$	$5.2874 \cdot 10^{-7}$
0.01	$1.2021 \cdot 10^{-6}$	$2.8242 \cdot 10^{-7}$
0.015	$8.5490 \cdot 10^{-7}$	$1.3521 \cdot 10^{-7}$
0.02	$5.7471 \cdot 10^{-7}$	$6.5094 \cdot 10^{-8}$
0.025	$3.8080 \cdot 10^{-7}$	$3.2133 \cdot 10^{-8}$
0.03	$2.5225 \cdot 10^{-7}$	$1.6310 \cdot 10^{-8}$
0.035	$1.6799 \cdot 10^{-7}$	$8.5020 \cdot 10^{-9}$
0.04	$1.1273 \cdot 10^{-7}$	$4.5416 \cdot 10^{-9}$
0.045	$7.6285 \cdot 10^{-8}$	$2.4804 \cdot 10^{-9}$
0.05	$5.2068 \cdot 10^{-8}$	$1.3822 \cdot 10^{-9}$
0.055	$3.5839 \cdot 10^{-8}$	$7.8445 \cdot 10^{-10}$
0.06	$2.4869 \cdot 10^{-8}$	$4.5266 \cdot 10^{-10}$
0.065	$1.7391 \cdot 10^{-8}$	$2.6521 \cdot 10^{-10}$
0.07	$1.2251 \cdot 10^{-8}$	$1.5757 \cdot 10^{-10}$
0.075	$8.6907 \cdot 10^{-9}$	$9.4833 \cdot 10^{-11}$
0.08	$6.2058 \cdot 10^{-9}$	$5.7760 \cdot 10^{-11}$
0.085	$4.4592 \cdot 10^{-9}$	$3.5573 \cdot 10^{-11}$
0.09	$3.2231 \cdot 10^{-9}$	$2.2137 \cdot 10^{-11}$
0.095	$2.3428 \cdot 10^{-9}$	$1.3909 \cdot 10^{-11}$
0.1	$1.7120 \cdot 10^{-9}$	$8.8193 \cdot 10^{-12}$
0.105	$1.2574 \cdot 10^{-9}$	$5.6399 \cdot 10^{-12}$
0.11	$9.2797 \cdot 10^{-10}$	$3.6360 \cdot 10^{-12}$
0.115	$6.8800 \cdot 10^{-10}$	$2.3617 \cdot 10^{-12}$
0.12	$5.1232 \cdot 10^{-10}$	$1.5454 \cdot 10^{-12}$
0.125	$3.8311 \cdot 10^{-10}$	$1.0179 \cdot 10^{-12}$
0.13	$2.8763 \cdot 10^{-10}$	0
0.135	$2.1678 \cdot 10^{-10}$	0
0.14	$1.6398 \cdot 10^{-10}$	0
0.145	$1.2448 \cdot 10^{-10}$	0

5. Conclusion

The result concerned with the limit reliability of large complex multi-state non-homogeneous regular series-parallel system related to its operation process was applied practically to the approximate evaluation of the exact reliability function of the port grain transportation subsystem. The differences between the subsystem exact and approximate reliability characteristics are very small what justifies the sensibility of the asymptotic approach practical application to reliability evaluation of large complex systems.

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