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Non-renewal multistate system with Semi-Markov components

Keywords

Semi-Markov process, multistate system, multistate reliability functions, binary representation

Abstract

The paper deals with non-renewal multistate monotone systems consisting of multistate components which are modeled by the semi-Markov processes. In the case of a non-renewal system the multistate reliability functions of the system components and the whole system are discussed. All presented concepts and models are illustrated by simple numerical examples.

1. Introduction

We can find many papers which are devoted to the reliability of multistate monotone systems [1]-[15]. The basic concepts deal with MMS are presented in [1], [4], [5], [6], [13]. Some results of investigation of the multistate monotone system (MMS) with components modelled by the independent semi-Markov processes are presented in this paper. We assume that the states of the system components are modelled by the independent semi-Markov processes. Some characteristics of a semi-Markov process are used as reliability characteristics of the system components. The binary representation of the multistate monotone systems allows to use traditional reliability metothod for analysis of MMS . The concept of a minimal path vector to level *l* is crucial to these considerations. The multistate reliability functions of the system components and the whole system are discussed in the paper. The presented concepts and models are illustrated by some numerical examples.

2. Structure of the system

Consider a system consisting of n components with the index set $C = \{1,...,n\}$.. We suppose that $S_k = \{0,1,..., z_k\}, k \in C$ is the set of the states of the component k and $S = \{0,1,...,s\}$. is the set of the system states. All the states are ordered. States of the system (a component k) denote successive levels of the object technical condition from the perfect functioning level z (z_k) to the complete failure level 0. Therefore the state 0 is the worst and the state z (z_k) is the best.

The function

$$\psi : S_1 x ... x S_n \to S$$

is called the system structure function.

If the system structure function is non-decreasing in each argument and

$$\psi(0,...,0) = 0,\ \psi(z_1,..., z_n) = z.$$

then it is said to be monotone. Formally a multistate system is represented by a sequence of symbols $(C, S, S_1,..., S_n \psi)$. If the system structure function is monotone the system is called multistate monotone *system* (MMS). We assume that the considered in this chapter systems are MMS. The state of a component k at fixed instant t may be described by the random variable $X_k(t)$ taking its value in S_k. The random vector

$$X(t) = (X_1(t),..., X_n(t))$$

represents the states of all system components at fixed moment t. The state of the system at the fixed instant t is completely defined by the states of components through the system structure function ψ

$$Y(t) = \psi(X(t)). \qquad (1)$$

If the parameter t runs the interval $[0, \infty)$, all mentioned above random variables become random processes. Therefore $\{Y(t): t \in [0,\infty)\}$ is a stochastic process with the state space $S = \{0,1,...,z\}$. The process determines a reliability state of the system.

3. Reliability of non-renewal MMS

We suppose that the reliability states of system components are described by the independent semi-Markov processes $\{X_k(t): t \geq 0\}, k \in C$. Unfortunately the random process $\{Y(t): t \geq 0\}$, $Y(t) = \psi(X_1(t),..., X_n(t))$ taking its values from the set $S = \{0,1,...,z\}$ which describes the system reliability state at time $t \in [0,\infty)\}$ is not a semi-Markov process. We have at least two ways to analyse the reliability of the multistate system. The first one is based on the extension of the process $\{Y(t): t \geq 0\}$, to a semi-Markov process by construction the superposition of independent Markov renewal processes associated with the semi-Markov processes $\{X_k(t): t \geq 0\}, k \in C$. [11], [12]. This way needs more advanced mathematical concepts which go beyond the scope of this paper. The second way consists in calculating the reliability characteristics of the multistate system based on the characteristics of its independent components. In this paper we apply the second way.

We suppose that the semi-Markov process representing the reliability state of the component *k* is determined by by a following kernel

$$Q^{(k)}(t)$$

$$= \begin{bmatrix} Q_{00}^{(k)}(t) & 0 & 0 & ... & 0 \\ Q_{10}^{(k)}(t) & 0 & 0 & ... & 0 \\ Q_{20}^{(k)}(t) & Q_{21}^{(k)}(t) & 0 & ... & 0 \\ Q_{30}^{(k)}(t) & Q_{31}^{(k)}(t) & Q_{32}^{(k)}(t) & ... & 0 \\ ... & ... & ... & ... & 0 \\ Q_{z_k 0}^{(k)}(t) & Q_{z_k 1}^{(k)}(t) & ... & Q_{z_k z_{k-1}}^{(k)}(t) & 0 \end{bmatrix} \quad (2)$$

Let

$$T_{[l]}^{(k)} = \inf\{t : X_k(t) \in A_{[l]}^{(k)}\} \quad (3)$$

where

$$A_{[l]}^{(k)} = \{0,..., l-1\} \quad (4)$$

and

$$A'^{(k)}_{[l]} = S - A_{[l]}^{(k)} = \{l,..., z_k\}.$$

The function

$$\Phi_{i[l]}^{(k)}(t) = P(T_{[l]}^{(k)} \leq t \setminus X(0) = i,\ i \in A'^{(k)}_{[l]} \quad (5)$$

represents the cumulative distribution function (CDF) of the first passage time from the state $i \in A'^{(k)}_{[l]}$ to the subset $A_{[l]}^{(k)}$ for the process $\{X_k(t): t \geq 0\}$. If $X(0) = s_k$ then the random variable $T_{[l]}^{(k)}$ represents the *l*-level lifetime of the component *k*. A corresponding reliability function is

$$R_{zk[l]}^{(k)} = 1 - \Phi_{zk[l]}^{(k)}(t). \quad (6)$$

The Laplace-Stieltjes transforms of the CDF's $\Phi_{i[l]}^{(k)}(t), i \in A'^{(k)}_{[l]}$ satisfy the integral system of equations [5], [6].

$$\tilde{\varphi}_{i[l]}^{(k)}(s) = \sum_{j \in A_{[l]}^{(k)}} \tilde{q}_{ij}^{(k)}(s) + \sum_{j \in A'_k(u)} \tilde{q}_{ij}^{(k)}(s) + \tilde{\varphi}_{j[u]}^{(k)}(s), (7)$$

$$i \in A'^{(k)}_{[l]},$$

where

$$\tilde{\varphi}_{i[l]}^{(k)}(s) = \int_0^\infty e^{-st} d\Phi_{i[l]}^{(k)}(t), \quad (8)$$

$$\tilde{q}_{ij}^{(k)}(s) = \int_0^\infty e^{-st} dQ_{ij}^{(k)}(t), \quad (9)$$

The Laplace transform

$$\tilde{R}_{zk[l]}^{(k)}(s) = \int_0^\infty e^{-st} R_{zk[l]}^{(k)}(t), \quad (10)$$

of the *k*-th component reliability function to level *l* is given by the formula

$$\tilde{R}_{sk[l]}^{(k)}(s) = \frac{1 - \tilde{\varphi}_{k[u]}^{(k)}(s)}{s}. \quad (11)$$

On the other hand

$$R_{zk[l]}^{(k)}(t) = P(T_{k[l]} > t \setminus X(0) = z_k)$$

$$= P(\forall u \in [0,t] \quad (12)$$

$$X_k(u) \in A'_{k[l]} \setminus X(0) = z_k.$$

As components of the system are unrepairable then we have

$$R_{zk[l]}^{(k)}(t) = P(\forall u \in [0,t]$$

$$X_k(u) \in A'^{(k)}_{[l]} \setminus X(0) = z_k)$$

$$= P(X_k(t) \in A'^{(k)}_{[l]} \setminus X(0) = z_k). \tag{13}$$

Finally we get

$$R_{zk[l]}^{(k)}(t) = \sum_{j=1}^{z_k} P(X(t)\, j \setminus X(0) = z_k)$$
$$= \sum_{j=1}^{z_k} P_{zkj}^{(k)}(t). \tag{14}$$

Applying the equations (14.02) from [5] we obtain a linear system of equations for the Laplace transforms of the reliability functions to level l for the system components:

$$\tilde{R}_{i[l]}^{(k)}(s) = \frac{1}{s} - \tilde{G}_i^{(k)}(s) + \sum_{j \in A'^{(k)}_{[l]}} \tilde{q}_{ij}^{(k)}(s) \tilde{R}_{j[l]}^{(k)}(s), \quad i \in A'^{(k)}_{[l]}, \tag{15}$$

where

$$\tilde{G}_i^{(k)}(s) = \int_0^\infty e^{-st} G_i^{(k)}(t)dt,$$
$$\tilde{R}_{i[l]}^{(k)}(s) = \int_0^\infty e^{-st} R_{i[l]}^{(k)}(t)dt, \tag{16}$$

are the Laplace transforms of the functions $G_i^{(k)}(t)$ and $R_{i[l]}^{(k)}(t)$ $t \geq 0$. Passing to the matrix notation we get

$$(I - q_{[l]}^{(k)}(s))R_{[l]}^{(k)}(s) = G_{[l]}^{(k)}(s). \tag{17}$$

The function

$$R_{[l]}^{(k)}(t) = P(T_{[l]}^{(k)} > t) = P(X_k(t) \in A'^{(k)}_{[l]}) \tag{18}$$

means the reliability function to level l of a k-th system component.
The vector function

$$R^{(k)}(t) = [R_{[0]}^{(k)}(t), R_{[1]}^{(k)}(t), ..., R_{[zk]}^{(k)}(t)] \tag{19}$$

is said to be the multistate reliability function of the k-th component of the system. Let us notice that

$$A'^{(k)}_{[0]} = S_k \supset A'^{(k)}_{[1]} \supset ... \supset A'^{(k)}_{[sk]}.$$

From the well known property of probability we have

$$1 = P(X_k(t) \in S_k) \geq P(X_k(t) \in A'^{(k)}_{[1]})$$
$$\geq ... P(X_k(t) \in A'^{(k)}_{[z_k]}).$$

It means that

$$1 = R_{[0]}^{(k)}(t) \geq R_{[1]}^{(k)}(t) \geq ... \geq R_{[z_k]}^{(k)}(t).$$

The equation (6) enables to calculate the Laplace transform of the multistates reliability function of the k-th component.

$$R^{(k)}(s) = [\tilde{R}_{z_k[0]}^{(k)}(s), \tilde{R}_{z_k[1]}^{(k)}(s), ..., \tilde{R}_{z_k[z_k]}^{(k)}(s)]. \tag{20}$$

Its inverse Laplace transform is equal to the vector of functions.

4. Binary representation of MMS

A vector $y = (y_1, y_2, ..., y_n) \in S_1 \times ... \times S_n$ is called a path vector to level (of level) l of the multi-state monotone system if $\psi(y) \geq l$.
The path vector y is said to be a minimal path vector to level l if in addition the inequality $x < y$ implies $\psi(y) < l$. The inequality $x < y$ means that $x_i \leq y_i$ for $i = 1,2,...,n$ and $x_i < y_i$ for some i. We denote the set of all minimal path vectors to level l by U_l, $l = 1,..., z(z_k)$ and $U_0 = \{0\}$, where $\{0\} = (0,0,...,0)$.
In reliability analysis of the multistate monotone systems we may use their binary representation. This approach was presented among other in papers of Block and Savits [3] and Korczak [13]. We define the binary random variables $\{X_{kr}(t) : t \geq 0\}, k \in C, r \in S_k$:

$$X_{kr}(t) = \begin{cases} 1 \; for \; X_k(t) \geq r \\ 0 \; for \; X_k(t) < r. \end{cases} \tag{21}$$

We determine the system level indicators $\psi_j, j \in \{1,..., z\}$:

$$\psi_j(X(t)) = \begin{cases} 1 \; for \; \psi(X(t)) \geq j \\ 0 \; for \; \psi(X(t)) < j. \end{cases} \tag{22}$$

We will use symbols introduced by Barlow and Proshan [2] which denote the binary operations:

$$\coprod_{i=1}^n x_k = 1 - \prod_{i=1}^n (1 - x_k), x_k \in \{0,1\}$$

$$x_1 \coprod x_2 = 1 - (1 - x_1)(1 - x_2), \; x_1, x_2 \in \{0,1\}.$$

From (21), (22) and definition of the minimal path vectors we obtain a following binary representation of the stochastic process describing evolution of the multistate monotone system

$$\psi_l(X(t)) = \prod_{y\in U_l} \prod_{k\in C, y_k>0}$$

$$X_{ky_k}(t) = 1 - \prod_{y\in U_l}(1 - \prod_{k\in C, y_k>0} X_{ky_k}(t)). \qquad (23)$$

Consider a three components multistate system $(C, S, S_1, S_2, S_3, \psi)$ where $C = \{1,2,3\}$, $S = \{0,1,2\}$, $S_1 = \{0,1,2\}$, $S_2 = \{0,1,2\}$, $S_3 = \{0,1\}$ and the system structure function is determined by the formulae:

$$\psi(x) = 0 \text{ for } x = (x_1, x_2, x_3) \in D_0,$$
$$\psi(x) = 1 \text{ for } x = (x_1, x_2, x_3) \in D_1,$$
$$\psi(x) = 2 \text{ for } x = (x_1, x_2, x_3) \in D_2,$$

where

$$D_0 = \{(0,0,0),(0,0,1),(0,1,0),(1,0,0),(1,1,0),$$
$$(2,0,0),(2,2,0),(0,2,0),(1,2,0),(0,2,1)\},$$
$$D_1 = \{(1,0,1),(0,1,1),(1,1,1),(2,0,1),(0,2,1)\},$$
$$D_2 = \{(2,1,1),(1,2,1),(2,2,1)\}.$$

First we have to determine the set U_l of all minimal hth vectors to the level l for $l = 1,2.$ We take under consideration the set D_1. The vector $y = (1,0,1)$ is a minimal path vectors to level 1, because according to definition $\psi(y) = 1 \geq 1$ and there exists a vector $x = (0,0,1)$ such that $x < y$ and $\psi(x) = 0 < 1$. The vector $y = (0,2,1)$ is not a minimal path vectors to level 1, because $\psi(y) = 11$ and for $x = (0,1,1)$ we have $x < y$ and $\psi(x) = 1$. Also the vector $y = (2,0,1)$ is not a minimal path vectors to level 1, because $\psi(y) = 1 \geq 1$ and for $x = (1,0,1)$ is $x < y$ and $\psi(x) = 1$.

Analysing all vectors from D_1 we get a set of the minimal path vectors of the level $l = 1$ which is denoted as U_l:

$$U_1 = \{(1,0,1),(0,1,1)\}.$$

In the similar manner we get U_2:

$$U_2 = \{(2,1,1),(1,2,1)\}.$$

From (23) we have

$$\psi_l(x) = \prod_{y\in U_l} \prod_{k\in C, y_k>0} x_{ky_k} \qquad (24)$$
$$= 1 - \prod_{y\in U_l}(1 - \prod_{k\in C, y_k>0} x_{ky_k}).$$

Applying this equality we have

$$\psi_1(x) = x_{11}x_{31} \coprod x_{21}x_{31} = x_{11}x_{31}$$
$$x_{11}x_{31} + x_{21}x_{31} - x_{11}x_{21}x_{31}. \qquad (25)$$

In a similar way, using an equality

$$x_{kr}x_{kp} = x_{k\max\{r,p\}} \qquad (26)$$

we get

$$\psi_2(x) = x_{12}x_{21}x_{31} + x_{11}x_{22}x_{31} - x_{12}x_{22}x_{31}.$$

5. Reliability of unrepairable system

We suppose that the semi-Markov processes $X_1(t): t \geq 0\}, ..., \{X_n(t): t \geq 0\}$ are independent. A stochastic process $\{Y(t): t \geq 0\}$,

$$Y(t) = \psi(X(t)) = \psi(X_1(t), ..., X_n(t)) \qquad (27)$$

taking its values in a state space $S = \{0,1,...,z\}$ describes a reliability state for $t \in [0,\infty).$ It is not a semi-Markov process. Let $A'_{[l]} = \{l, l+1, ..., z\}$ and $A_{[l]} = S - A'_{[l]} = \{0,1,...,l-1\}$. A random variable

$$T_{[l]} = \inf\{t : S((t) \in A_{[l]}\} \qquad (28)$$

denotes the time to failure to level (of level) l of the system. A reliability function to level l of the system is determined by the rule

$$R_{[l]}(t) = P(T_{[l]} > t). \qquad (29)$$

We have at least two ways of calculating it. The first one consists in applying distributions of the processes which describe the reliability evolution of the system components. The l level reliability function of the system may be computed according to the rule

$$R_{[l]}(t) = \sum_{j\in A'_{[l]}} P_j(t), \qquad (30)$$

where

$$P_j(t) = P(S(t) = j) = P(X(t) \in D_j)$$
$$= \sum_{(x_1,...,x_n)\in D_j} P^1_{x_1}(t)...P^n_{x_n}(t),\ D_j = \psi^{-1}(j).$$

The second way leads through the computation of the components of reliability functions to level *l*. Applying (24) we have

$$R_{[l]}(t) = E\{[\psi_l(X(t))]$$
$$= 1 - \prod_{y \in U_l} (1 - \prod_{k \in C, y_k > 0} E[X_{ky_k}(t))]. \qquad (31)$$

The vector function

$$R(t) = [1, R_{[1]}(t), ..., R_{[z]}(t)] \qquad (32)$$

is called the multistate reliability function of the system. The vector

$$m = [1, m_{[1]}, ..., m_{[z]}], \qquad (33)$$

$$m_{[l]} = \int_0^\infty R_{[l]}(t), l = 1, ..., z, \qquad (34)$$

is said to be the multistate mean time to failure of the system.

6. Numerical illustrative example

To explain and illustrate presented above concepts we will construct a simple reliability model of the multistate system with the semi-Markov components. We assume that the multistate reliability system consists of three components reliability evolution of which are modelled by independent semi-Markov processes $\{X_1(t): t \geq 0\}$, $\{X_2(t): t \geq 0\}$, $\{X_3(t): t \geq 0\}$, the state spaces $S_1 = S_2 = \{0,1,2\}$, $S_3 = \{0,1\}$. We assume that the kernels of the processes 1 and 2 are the same:

$$Q^{(k)}(t) = \begin{bmatrix} Q_{00}^{(k)}(t) & 0 & 0 \\ Q_{10}^{(k)}(t) & 0 & 0 \\ Q_{20}^{(k)}(t) & Q_{21}^{(k)}(t) & 0 \end{bmatrix}, \qquad (35)$$

where

$$Q_{00}^{(k)}(t) = 1 - e^{-\alpha t},$$
$$Q_{10}^{(k)}(t) = 1 - (1 + \beta t)e^{-\beta t},$$
$$Q_{20}^{(k)}(t) = a[1 - (1 + \lambda t)e^{-\gamma t}],$$
$$Q_{21}^{(k)}(t) = b[1 - (1 + \lambda t)e^{-\gamma t}],$$
$$t \geq 0, \alpha > 0, \ b > 0, \beta > 0, \gamma > 0,$$

Suppose that the initial distributions are

$$P(X^{(k)}(0) = 2) = 1, \ k = 1,2.$$

Assume that a kernel of the last process is of the form

$$Q^{(3)}(t) = \begin{bmatrix} Q_{00}^{(3)}(t) & 0 \\ Q_{10}^{(3)}(t) & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 - e^{-\kappa t} & 0 \\ 1 - (1 + \lambda t)e^{-\lambda t} & 0 \end{bmatrix}, \qquad (36)$$

where $t \geq 0, \kappa > 0, \lambda > 0$.

Now we illustrate the second way of calculation of the system multistate reliability function.

The second method of computing the system mulistate reliability function needs to calculate the reliability functions of its components to level *l*.. Applying (25) we have

$$R_{[1]}(t) = E[\psi_1(X(t))] = E[X_{11}(t)X_{31}(t)]$$
$$+ E[X_{21}(t)X_{31}(t)] - E[X_{11}(t)X_{22}(t)X_{31}(t)]. \qquad (37)$$

Hence, using the independence of the processes discussed here we get the reliability function of the system to level 1:

$$R_{[1]}(t) = R_{[1]}^{(1)}(t)R_{[1]}^{(3)}(t) + R_{[1]}^{(2)}(t)R_{[1]}^{(3)}(t)$$
$$- R_{[1]}^{(1)}(t)R_{[1]}^{(2)}(t)R_{[1]}^{(3)}(t) \qquad (38)$$

In the same way, according to (26) we have

$$R_{[2]}(t) = R_{[2]}^{(1)}(t)R_{[1]}^{(2)}(t)R_{[1]}^{(3)}(t) + R_{[1]}^{(1)}(t)R_{[2]}^{(2)}(t)R_{[1]}^{(3)}$$
$$- R_{[2]}^{(1)}(t)R_{[2]}^{(2)}(t)R_{[1]}^{(3)}(t) \qquad (39)$$

The reliability functions of the component to level $l = 1,2$ we evaluate applaying (17).

In this case $S_k = \{0,1,2\}$, $k = 1,2$. Hence

$$A_{[1]} = \{0\}, \ A'_{[1]} = \{1,2\},$$
$$A_{[2]} = \{0,1\} \ A_{[2]} = \{2\}.$$

For $l = 1$ the matrices from equations (17) take the form

$$I - q_{A'[l]}^{(k)}(s) = \begin{bmatrix} 1 & 0 \\ -\tilde{q}_{21}^{(k)}(s) & 1 \end{bmatrix}, \qquad (40)$$

$$G_{A'[l]}^{(k)}(s) = \frac{1}{s} \begin{bmatrix} 1 - \tilde{q}_{10}^{(k)}(s) \\ 1 - \tilde{q}_{20}^{(k)}(s) - \tilde{q}_{21}^{(k)}(s) \end{bmatrix}. \qquad (41)$$

The element $\tilde{R}_{2[1]}^{(k)}(s)$ of the solution of (15) is

$$\tilde{R}^{(k)}_{2[1]}(s) = \frac{1-\tilde{q}^{(k)}_{20}(s) - \tilde{q}^{(k)}_{21}(s) q^{(k)}_{10}(s)}{s} \qquad (42)$$

For $l = 2$ the matrices from equations (17) take the form

$$I - q^{(k)}_{A'[2]}(s) = [1],$$

$$G^{(k)}_{A'[l]}(s) = \frac{1}{s}[1 - \tilde{q}^{(k)}_{20}(s) - \tilde{q}^{(k)}_{21}(s)].$$

The solution of (15) is

$$\tilde{R}^{(k)}_{2[[2]}(s) = \frac{1-\tilde{q}^{(k)}_{20}(s) - \tilde{q}^{(k)}_{21}(s)}{s}, \qquad (43)$$

The Laplace-Stielties transforms of elements from (42) and (43) are

$$\tilde{q}^{(k)}_{10}(s) = \frac{\beta^2}{(s+\beta)^{2'}}, \ \tilde{q}^{(k)}_{20}(s) = \frac{a\gamma^2}{(s+\gamma)^{2'}}$$

$$\tilde{q}^{(k)}_{21}(s) = \frac{b\gamma^2}{(s+\gamma)^2}$$

for $k = 1,2.$ For parameters

$$\alpha = 0.1, \beta = 0.02, \gamma = 0.01, a = 0.2,$$
$$b = 0.8, \eta = 0.01, \kappa = 0.1.$$

The rules (41) and (42) are

$$\tilde{R}^{(k)}_{2[1]}(s) = \frac{0.0000832 + 0.00488s + 0.12s^2 + s^3}{(0.02+s)^2(0.04+s)^2},$$

$$\tilde{R}^{(k)}_{2[2]}(s) = \frac{0.8+s}{(0.04+s)^2} \text{ for } k = 1,2.$$

We get the reliability functions of the system components as the inverse Laplace transforms of these functions. Thus we obtain

$$R^{(k)}_{[1]}(t) = 4.2e^{-0.04t} - 3.2e^{-0.02t} 0.04e^{-0.04t}$$
$$+ 0.064e^{-0.02t}t$$

$$R^{(k)}_{[2]}(t) = 0.04e - 0.04t(25+t) \text{ for } k = 1,2.$$

For $k = 3$ we have

$$R^{(3)}_{[1]}(t) = 1 - Q^{(3)}_{10}(t) = (1+\lambda t)e^{-\lambda t}$$

Using equalities (37) and (38) we obtain elements of the multistate reliability function of the system:

$$R_{[1]}(t) = 2e^{-0.01t}(1+0.01t)(4.2e^{-0.04t}$$
$$- 3.2e^{-0.02t} + 0.04te^{-0.04t} + 0.064te^{-0.02t})$$
$$- e^{-0.01t}(1+0.01t)(4.2e^{-0.04t} - 3.2e^{-0.02t}$$
$$+ 0.04te^{-0.04t} + 0.064te^{0.02t})^2,$$

$$R_{[2]}(t) = -0.0016e^{-0.09t}(1+0.01t)(25+t)^2 +$$
$$+ 0.08e^{-0.05t}(1+0.01t)(25+t)(4.2e^{-0.04t}$$
$$- 3.2e^{-0.02t} + 0.04te^{-0.04t} + 0.064te^{-0.02t}).$$

The multistate reliability function can be written as a vector function

$$R(t) = [1, R_{[1]}(t), R_{[2]}(t)].$$

Conclusions

In many real-life situations the binary models seem to be not sufficient for describing reliability of the system, because in addition to "down" state (0) and "up" state (1) the system may be capable on different levels from perfect functioning to complete failure. Then the multistate models are more adequate. The decomposition method of the multitate unrepairable system to binary systems allows to apply well known methods of classical reliability theory in multinary cases. Semi-Markov processes are very useful as reliability models of the multistate system components. The semi-Markov process theory provides some concepts and theorems which enable to construct the appropriate probability models of the multistate reliability system. Unfortunately all these models are constructed under the assumption of independence of processes describing the reliability of the system components.

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