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Critical infrastructure systems modelling

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Abstract

The article is showing some concept of critical infrastructure system's safety states model. Model construction is basing on popular technical systems' safety states models, and notions specified in acts of law related to crisis management and other studies concerning it. Implementation of crisis management issues and problems into technical systems' safety states model, resulted with formulating of critical infrastructure system's safety states model, illustrating processes concerned with their transitions, related to particular crisis management phases. Then, probabilistic description of critical infrastructure safety states transitions process have been presented, that, if further evolved, can lead to support works connected to critical infrastructure protection.

1. Introduction

One of the most popular safety states model of technical systems is one (*Figure 1*), including following five safety states [1]:

- safety (no-hazards) state,
- sense of hazards state,
- state of emergency,
- state of disaster,
- state of reduced efficiency.

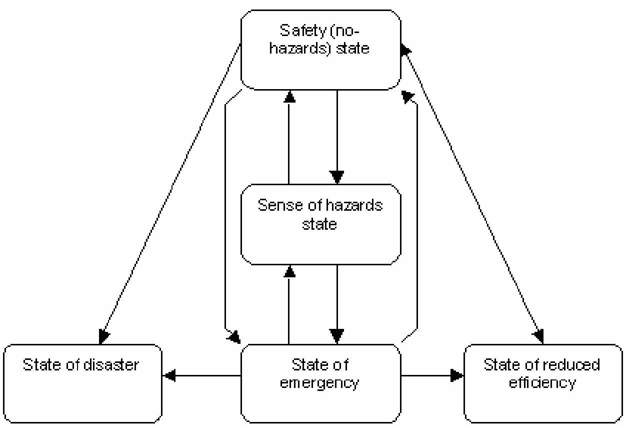

Figure 1. Model of technical systems' safety states.

The safety state is considered as basic system's operation state. State of emergency represents significant increase of system's transition into state of disaster probability. The transition to state of emergency takes place in case of events and incidents that, if not properly responded, can bring the system to state of disaster. The sense of hazards state is one having psychological nature, the difference to state of emergency is fact that events and incidents that can bring the system to state of disaster, are only potential usually caused by lack of information concerning actual situation conditions. The state of disaster begins at the moment of damage to any of system components or natural environment, leading in extreme circumstances also to human loses. State of reduced efficiency is having complementary nature, transition to this state takes place when system partly or completely loses its functional capabilities, but without sense of hazards, emergency or disaster appearance.

Mentioned above notions, defining particular technical systems safety states, are somehow similar to formulae specified in acts of low and other studies concerning crisis management and critical infrastructure protection. Thus, the concept of critical infrastructure system's safety states model, is basing on above.

2. Critical infrastructure system's safety states model

Basing on crisis situation definition included in Act of Law on Crisis Management (2007) [4], the base model of critical infrastructure system's safety states can be seen as two-state one, including (*Figure 2*): no-hazards state and crisis state (crisis situation). Above mentioned act of law is defining crisis situation as one that impacts negatively on the safety of people, property in large sizes or the environment, producing significant restrictions on the operation of the competent authorities of public administration due to the inadequacy of the possessed capabilities and resources.

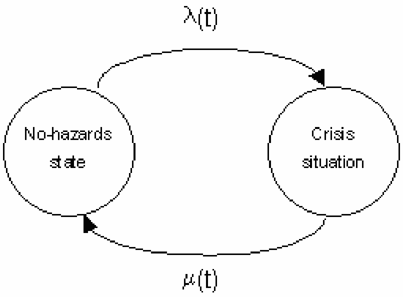

Figure 2. Base model of critical infrastructure systems' safety states.

The no-hazards state corresponds to situation where, according to above mentioned definition, negative impact on safety level, and restrictions on the operation of the competent authorities of public administration in respect to possessed capabilities and resources, do not take place. The transition to crisis state occurs in case of appearance of negative impact on safety level, and significant restrictions to the operation of authorities of public administration, exceeding capabilities and resources being in their possession.

The base disadvantage of introduced above simple two-state model is fact, that situation of hazards zero-level is practically never existing. All human activities are causing non-zero probability of real hazards appearance. The level of hazards can be however acceptable, meaning non exceeding the level resulting transition to crisis situation state, or high enough to impact negatively on safety level, and restricting operation of authorities of public administration.

It is then necessary to expand above simple model, by interpretation of no-hazards state as two states (*Figure 3*), one determined as real zero-level hazards level, and the other one representing increased hazards level, but not exceeding the limit causing transition to crisis situation.

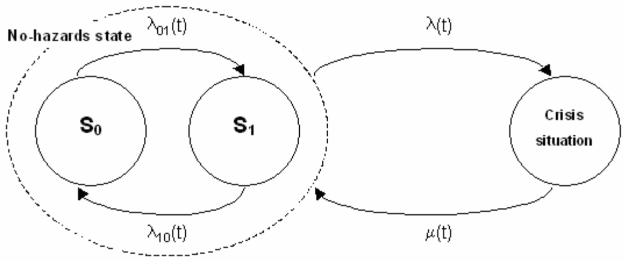

Figure 3. Model of critical infrastructure systems' safety states, including aggregated no-hazards state.

It can be assumed, that states S_0 and S_1 are corresponding to safety state and sense of hazards state of model shown in *Figure 1*. S_0 state (hazards zero-level) is the intentional state of the system. All actions aiming to reach this state are understood as continuous and dynamic process of responding to hazards, representing the transition from sense of hazards state to safety state. The perception of S_0 and S_1 states can lead to their aggregation. The stay of system at one of mentioned states can be determined as stay at no-hazards state. The aggregated no-hazards state can be interpreted as one covering intensive activities of crisis management resources aiming to respond to hazards, meaning increasing transition rate from state S_1 to S_0.

The activities mentioned above correspond to following crisis management phases [5]:

- *Prevention* – analysis of potentially possible crisis situations, and undertaking activities lowering probability of their appearance,
- *Preparedness* – planning of actions (procedures), to be carried out in case of appearing of foreseen crisis situations.

Crisis management services efforts, undertaken when crisis situation occurs, aiming to move system from crisis situation state into no-hazards state, are usually named *Responding*:

- *Responding* – undertaking of previously planned, coordinated activities, leading to stop crisis situation expanding, support casualties, and restrict damages and losses.

All mentioned above phases are indicated in *Figure 4*.

It can be a subject for further considerations, if possibility of "direct" transition from S_0 state (hazards zero-level) into crisis situation state, should be also predicted. The base model (*Figure 1*) assumes such possibility (sudden incident causing immediate transition to state of disaster). For the crisis management purposes it has been however assumed, that every crisis situation is preceded by increase of hazards level, that is why proposed model of critical infrastructure system's safety states is not including such a possibility.

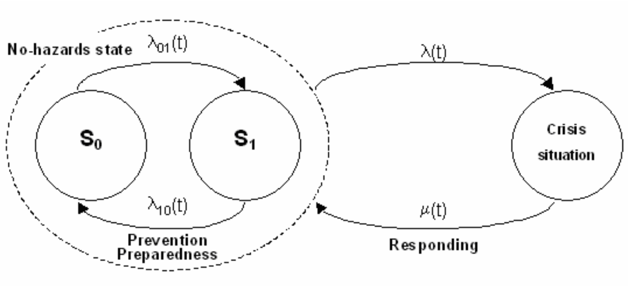

Figure 4. Model of critical infrastructure systems' safety states, indicating crisis management phases: Prevention, Preparedness and Responding.

Crisis management services activities performed within responding to crisis situations not always bring desired results, meaning restricting damages and losses for critical infrastructure objects and systems. In some circumstances damage or loss of whole system or its elements takes place, and it is necessary then to start actions aiming to restore them. This makes necessary further expand of model constructed – by interpreting crisis situation state as two states (*Figure 5*): one (S_2) representing hazards level causing system's transition to crisis situation, but not resulting with damages and losses for critical infrastructure objects, and the other (S_3), taking place when mentioned damages and losses happen.

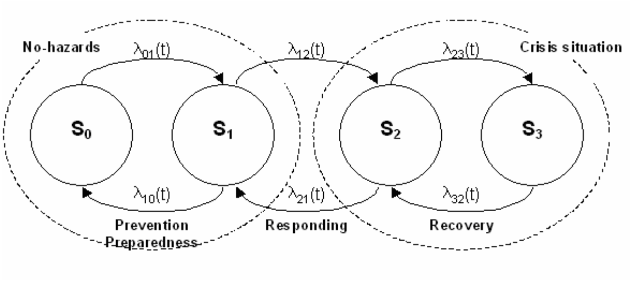

Figure 5. Expanded critical infrastructure systems' safety states model, with aggregated no-hazards state and crisis situation state.

Mentioned above S_2 and S_3 states are corresponding to state of emergency and state of disaster illustrated by base model (*Figure 1*). Fundamental difference however is fact that the base model is assuming state of disaster as an "absorbing" one, meaning irreversible. Analysis of critical infrastructure systems' safety states model must however assume restoration possibility of critical infrastructure elements and objects, thus constructed model is reflecting return transition from disaster state. The transition relates to the fourth, not mentioned previously phase of crisis management – *Recovery* (*Reconstruction*):

– *Recovery (Reconstruction)* – restoration of previous conditions of critical infrastructure elements and systems.

Final model (*Figure 5*) reflects all four crisis management phases, comparing to base model (*Figure 1*) – does not reflect one of states included in it - state of reduced efficiency. The state of reduced efficiency, according to its definition in [1] – has complementary nature – system transition to this state is caused by partial or complete lose of its functional capabilities, but without sense of hazards, emergency or disaster appearance.
One of problematic issues concerning the aim to a model most suitable to critical infrastructure systems' safety states, is to consider (or not) possibility of transitions between states other than the "neighbouring" ones (i.eg. from S_1 to S_3). For the purposes of this article it has been assumed that only possible transitions are ones between neighbouring states, however, the issue concerning consideration of other transitions, and consequently – appropriate corrections of the model and associated formulae is of course opened.

3. Probabilistic description of the safety states transitions' process

According to outcome of chapter 1 above, critical infrastructure safety states transitions process $S(t)$, $t \in \langle 0,+\infty)$, can stay at one of four particular safety states S_0, S_1, S_2, S_3, already defined. Furthermore, it can be assumed that critical infrastructure safety states transitions process $S(t)$ is a semi-Markov process [2], [3], with the conditional sojourn times T_{ij} at the operation states S_i when its next operation state is S_j, $i, j = 0, 1, 2, 3$ $i \neq j$.
The critical infrastructure safety states transitions process can be described by its following basic parameters:

– the vector $[p_i(0)]_{1x4}$ of the initial probabilities

$$p_i(0) = P(S(0) = S_i), \quad i = 0,1,2,3, \tag{1}$$

of the critical infrastructure safety states transitions process $S(t)$ staying at particular safety states at the moment $t = 0$;

– the matrix $[p_{ij}]_{4x4}$ of probabilities p_{ij}, $i, j = 0, 1, 2, 3$ $i \neq j$, of the critical infrastructure safety states transitions process $S(t)$ transitions between the safety states S_i and S_j;

– the matrix $[F_{ij}(t)]_{4x4}$ of conditional distribution functions

$$F_{ij}(t) = P(T_{ij} < t),\ i,\ j = 0,1,2,3,\ \ i \neq j, \qquad (2)$$

of the critical infrastructure safety states transitions process $S(t)$ conditional sojourn times T_{ij} at the operation states, and the corresponding matrix of the density functions $[f_{ij}(t)]_{4x4}$, where

$$f_{ij}(t) = \frac{d}{dt}[F_{ij}(t)],\ \ i,\ j = 0,1,2,3,\ \ i \neq j; \qquad (3)$$

By means of above mentioned parameters following characteristics of critical infrastructure safety states transitions process can be determined:

– mean values of the critical infrastructure safety states transitions process $S(t)$ conditional sojourn times T_{ij}, at the particular safety states:

$$M_{ij} = E[T_{ij}] = \int_0^\infty t dF_{ij}(t) = \int_0^\infty t f_{ij}(t),$$
$$i,\ j = 0,1,2,3,\ i \neq j; \qquad (4)$$

– rates of critical infrastructure safety states transitions process $S(t)$ between the safety states:

$$\lambda_{ij}(t) = \frac{f_{ij}(t)}{1 - F_{ij}(t)},\ i,\ j = 0,1,2,3,\ \ i \neq j; \qquad (5)$$

– unconditional distribution functions of the critical infrastructure safety states transitions process $S(t)$ stay time T_i at particular safety states:

$$F_i(t) = \sum_{j=0}^{3} p_{ij} F_{ij}(t),\ \ i = 0,1,2,3; \qquad (6)$$

– the mean values of the critical infrastructure safety states transitions process $S(t)$ unconditional sojourn times T_i at the safety states:

$$M_i = E[T_i] = \sum_{j=0}^{3} p_{ij} M_{ij},\ i = 0,1,2,3, \qquad (7)$$

where M_{ij} is given by (4);

– the limit values of the critical infrastructure safety states transitions process $S(t)$ transient probabilities at the particular safety states

$$p_i(t) = P(Z(t) = z_i),\ t \in <0,+\infty),\ i = 0,1,2,3, \quad (8)$$

are given by [2], [3]:

$$p_i = \lim_{t \to \infty} p_i(t) = \frac{\pi_i M_i}{\sum_{j=1}^{3} \pi_j M_j},\ i = 0,1,2,3, \qquad (9)$$

where M_i, i = 0, 1, 2, 3, are given by (7), while the steady probabilities π_i of the vector $[\pi_i]_{1x4}$ satisfy the system of equations

$$\begin{cases} [\pi_i] = [\pi_i][p_{ij}] \\ \sum_{j=1}^{j} \pi_j = 1; \end{cases} \qquad (10)$$

Other interesting characteristics of the process $S(t)$ possible to obtain are:

– total sojourn times T_i at the particular safety states S_i, i = 0, 1, 2, 3, during the fixed system opetation time Θ, having approximately normal distributions with the expected value given by

$$\hat{M}_i = E[\hat{T}_i] = p_i \theta,\ i = 0,1,2,3, \qquad (11)$$

where p_i, i = 0, 1, 2, 3, are given by (8);

– the total cost (loss) $\hat{C}$ concerned with critical infrastructure exploitation at fixed exploitation time Θ, that are aproximately

$$\hat{C} = \sum_{i=0}^{3} p_i C_i \theta, \qquad (12)$$

where p_i, i = 0, 1, 2, 3, are given by (8), while C_i, i = 0, 1, 2, 3, are average costs (losses) of exploitation at particular safety states S_i, i = 0, 1, 2, 3, within the time frame, at which exploitation time Θ is measured.

In special circumstances, when critical infrastructure safety states transitions process conditional sojourn times T_{ij} at the particular safety states, are having Weibull's distribution with the density function

$$f_{ij}(t) = \begin{cases} 0, & t < x_{ij} \\ \alpha_{ij}\beta_{ij}(t - x_{ij})^{\beta_{ij}-1} \exp[-\alpha_{ij}(t - x_{ij})^{\beta_{ij}}], & t \geq x_{ij}, \end{cases} \qquad (13)$$

where $0 \le \alpha_{ij} < +\infty$, $0 \le \beta_{ij} < +\infty$, $i, j = 0,1,2,3$, $i \ne j$, its two main characteristics given by (4) and (5) are:

– the mean values of critical infrastructure safety states transitions process $S(t)$ conditional sojourn times T_{ij} at the particular safety states

$$M_{ij} = E[T_{ij}] = x_{ij} + \alpha_{ij}^{-\frac{1}{\beta_{ij}}} \Gamma(1 + \frac{1}{\beta_{ij}}), \tag{14}$$

where $\Gamma(u) = \int_0^{+\infty} t^{u-1} e^{-t} dt, \quad u > 0,$ is the gamma function;

– rates of critical infrastructure safety states transitions process $S(t)$ between the safety states:

$$\lambda_{ij}(t) = \alpha_{ij} \beta_{ij} (t - x_{ij})^{\beta_{ij}-1},\ t > x_{ij},$$
$$i, j = 0,1,2,3,\ i \ne j \tag{15}$$

If the conditional sojourn times T_{ij} at the particular safety states of the safety states transitions process *S(t)*, are having exponential distribution, meaning $\beta_{ij} = 1$, then

$$\lambda_{ij}(t) = \alpha_{ij} = \lambda_{ij} = \text{constant} \tag{16}$$

this means the critical infrastructure safety states transitions process $S(t)$ is the Markov process.

4. Conclusions

Critical infrastructure systems' safety states model proposed in the article, and relations formulated on its basis, describing critical infrastructure safety states transitions processes, and their characteristics, can lead, in case of their further evaluations, to:

– automation of crisis situations (transition of system from safety state into crisis situation) diagnostic and detection processes;
– supporting of analysing of different factors and parameters influence on states transitions between safety (no-hazards) and crisis (crisis situation) states;
– supporting of activities leading to development of proper crisis management procedures – influencing on critical infrastructure systems safety states transitions rates;
– investigations on influence of crisis management procedures on critical infrastructure systems safety states transitions rates.

Relations specified above can be modified, in case of i.eg., considering other transitions between particular safety states, than ones specified in the paper. These modifications however would not have significant impact on main research target, which is the description of critical infrastructure systems safety states transitions processes.

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