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Reliability analysis of complex shipyard transportation system with dependent components

Keywords

reliability, multi-state system, ageing, operation process, dependability, complex system, shipyard transportation system

Abstract

In the paper an approach to the reliability analysis of multi-state systems with dependent components operating at variable operation conditions is presented. The multi-state reliability function of complex system is defined and determined for the shipyard rope ship elevator. In developed models, it is assumed that system components have the multi-state exponential reliability functions with interdependent departures rates from the subsets of reliability states.

1. Introduction

Currently, newest trends in the reliability analysis of technical systems are directed to complex systems. These are complex systems that significant features are inside-system dependencies and outside-system dependencies, that in case of damage have significantly destructive influence on the safety of the environment where they are operating. These systems are made of large number of interacting components and even small perturbations can trigger large scale consequences. For above reason, as an extended failure within one of the complex system may result in the critical incapacity or destruction and can significantly damage many aspects of human life, development of suitable tools for their reliability analysis is of great value.

Many technical systems belong to the class of complex systems as a result of large number of interacting components and subsystems they are built of and their complicated operating processes having significant influence on their reliability. This complexity and inside-system and outside-system dependencies and hazards cause that there is a need to develop new comprehensive approaches and general methods of analysis, identification, prediction, improvement and optimization this kind of complex system reliability. We meet complex systems, for instance, in piping transportation of water, gas, oil and various chemical substances and in port, shipyards and maritime transportation systems. Reliability analysis of complex systems' characteristics, considering systems at variable operation conditions and their changing in time reliability structures [8], [12] as well as their among components and subsystems dependability, becomes complicated. Adding to this analysis, the outside of complex systems hazards coming from other systems, from natural cataclysm and from other dangerous events makes the problem essentially difficult to become solved in order to improve and to ensure high level of these systems reliability.

In most reliability analyses, it is assumed that components of a system are independent. For instance, references [8] and [12] describe complex systems with aging components operating at variable operation conditions assuming their components independence. However, in reality, especially in case of complex systems, this assumption is not true, so that dependencies among complex systems' components and subsystems should be assumed and considered. It is a natural assumption, as after decreasing the reliability state by one of components in a subsystem, the inside interactions among the remaining components may cause further components reliability states decrease [1]-[2], [10]-

[11]. In [10]-[11] the authors analyze failure properties of a bundle of fibers assuming equal load sharing (ELS) model, considered in this paper, and local load sharing (LLS) model. The threshold strength of each fiber is determined by the stress value and in ELS model after fiber failure the strength thresholds of fibers are uniformly distributed. In this paper we describe similar model of equal load sharing, however we present multi-state approach to reliability analysis of complex systems with dependent components. This way we link the inside system dependencies between its components with influence on the complex systems' reliability coming from their external dependencies. In contrast to this paper the authors in [10]-[11] describe the breaking dynamics by a recursion relation in discrete time steps. Comparing with results presented in [1]-[2] this paper extends problem of reliability analysis of systems with dependent components adding a component stress proportionality correction coefficient and taking into account variable operation conditions of systems.

To tie results of investigations of complex systems inside-dependences together with results coming from the assumed their outside-dependencies, the semi-Markov model [5], [6], [9], [13]-[16] can be used to describe those systems operation processes. This linking of the inside and outside of complex system dependencies under the assumed their structures multi-state models, is the main idea of those systems reliability analysis methodology.

2. Reliability of multi-state systems

In the multi-state reliability analysis to define a system with degrading components, we assume that:

- n is the number of system components,
- E_i, $i = 1,2,...,n$, are components of a system,
- all components and a system under consideration have the reliability state set $\{0,1,...,z\}$, $z \geq 1$,
- the reliability states are ordered, the reliability state 0 is the worst and the reliability state z is the best,
- $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing lifetimes of components E_i in the reliability state subset $\{u,u+1,...,z\}$, while they were in the reliability state z at the moment $t = 0$,
- $T(u)$ is a random variable representing lifetime of a system in the reliability state subset $\{u,u+1,...,z\}$ while it was in the reliability state z at the moment $t = 0$,
- the system states degrades with time t,
- $E_i(t)$ is a component E_i reliability state at the moment t, $t \in <0,\infty)$, given that it was in the reliability state z at the moment $t = 0$,
- $S(t)$ is a system S reliability state at the moment t, $t \in <0,\infty)$, given that it was in the reliability state z at the moment $t = 0$.

The above assumptions mean that reliability states of the system with degrading components may be changed in time only from better to worse [7], [8], [15]-[16].

Definition 1. A vector

$$R_i(t,\cdot) = [R_i(t,0),R_i(t,1),...,R_i(t,z)], \qquad (1)$$

where

$$R_i(t,u) = P(E_i(t) \geq u \mid E_i(0) = z) = P(T_i(u) > t), \qquad (2)$$
$$t \in <0,\infty),\ u = 0,1,...,z,\ i = 1,2,...,n,$$

is the probability that the component E_i is in the reliability state subset $\{u,u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a component E_i.

Definition 2. A vector

$$\boldsymbol{R}(t,\cdot) = [\boldsymbol{R}(t,0),\boldsymbol{R}(t,1),...,\boldsymbol{R}(t,z)],\ t \in <0,\infty), \qquad (3)$$

where

$$\boldsymbol{R}(t,u) = P(S(t) \geq u \mid S(0) = z) = P(T(u) > t), \qquad (4)$$

for $t \in <0,\infty)$, $u = 0,1,...,z$, is the probability that a system is in the reliability state subset $\{u,u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of this system.

Under those assumptions

$$\mu(u) = \int_0^\infty \boldsymbol{R}(t,u)dt,\ u = 1,2,...,z, \qquad (5)$$

is the mean lifetime of a system in the state subset $\{u,u+1,...,z\}$,

$$\sigma(u) = \sqrt{n(u) - [\mu(u)]^2},\ u = 1,2,...,z, \qquad (6)$$

where

$$n(u) = 2\int_0^\infty t\,\boldsymbol{R}(t,u)dt,\ u = 1,2,...,z, \qquad (7)$$

is the standard deviation of the system lifetime in the reliability state subset $\{u, u+1, ..., z\}$ and moreover

$$\bar{\mu}(u) = \int_0^\infty p(t,u)dt, \; u = 1,2,...,z, \tag{8}$$

is the mean lifetime of a system in the state u while the integrals (5), (7) and (8) are convergent.
Additionally, according to (5) and (8), we get the following relationship

$$\bar{\mu}(u) = \mu(u) - \mu(u+1), \; u = 0,1,..., z-1,$$
$$\bar{\mu}(z) = \mu(z). \tag{9}$$

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid S(0) = z) = P(T(r) \le t), \; t \in \langle 0, \infty),$$

that a system is in subset of reliability states worse than the critical reliability state r, $r \in \{1,...,z\}$ while it was in the reliability state z at the moment t = 0 is called a risk function of the multi-state system [7].
Under this definition, from (4), we have

$$\boldsymbol{r}(t) = 1 - P(S(t) \ge r \mid S(0) = z) = 1 - \boldsymbol{R}(t,r), \tag{10}$$

for $t \in \langle 0, \infty)$, and if τ is the moment when a system risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{11}$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the system risk function $\boldsymbol{r}(t)$.

3. Reliability of multi-state "*m* out of *n*" system with dependent components

One of basic multi-state reliability structures with components aging in time are "*m* out of *n*" systems.
Definition 4. [7] A multi-state system is called "*m* out of *n*" system if its lifetime $T(u)$ in the reliability state subset $\{u, u+1, ..., z\}$ is given by

$$T(u) = T_{(n-m+1)}(u), \; m = 1,2,...,n, \; u = 1,\ldots,z,$$

where $T_{(n-m+1)}(u)$ is the n-m+1-th order statistic in the sequence of the component lifetimes $T_1(u)$, $T_2(u)$, . . ., $T_n(u)$.
The above definition means that the multi-state "*m* out of *n*" system is in the reliability state subset $\{u, u+1, ..., z\}$ if and only if at least m out of its n components are in this reliability state subset.

The multi-state "*m* out of *n*" system is called a multi-state parallel system if m = 1, and it is called a multi-state series system if $m = n$.
Consequently, the multi-state parallel system is in the reliability state subset $\{u, u+1, ..., z\}$ if and only if at least 1 of its n components are in this reliability state subset and the multi-state series system is in the reliability state subset $\{u, u+1, ..., z\}$ if and only if all of its n components are in this reliability state subset.
Definition 5. [7] A multi-state "*m* out of *n*" system is called homogeneous if its component lifetimes $T_i(u)$ in the reliability state subset have an identical distribution function i.e. if its components E_i have the same reliability function

$$R_i(t,\cdot) = [1, R_i(t,1), \ldots, R_i(t,z)] \tag{12}$$

with the coordinates

$$R_i(t,u) = R(t,u) \tag{13}$$
for $t \in \langle 0, \infty)$, $u = 1, \ldots, z$, i = 1,2,...,n.

Similarly as in [8], various reliability structures of the critical infrastructures with dependent components may be defined and their reliability functions determined. As a particular case, the reliability functions of considered complex systems composed of dependent components having exponential reliability functions may be determined. To do this, the following mathematical model of the inside infrastructure dependences between its components can be applied.
One of suggested approaches to reliability analysis of a homogeneous infrastructure with dependent components E_i, $i = 1,2,\ldots,n$, is assumption that after changing the reliability state subset by one of system components to the worse reliability state subset, lifetimes of the remaining system components in this reliability state subsets decrease dependably of the number of components which left that subset of reliability states [1]-[2]. More exactly, we assume that if $\upsilon, \upsilon = 0,1,2,\ldots,n-1$, components of the system are out of the reliability state subset $\{u, u+1, ..., z\}$, the mean values of the lifetimes $T_i'(u)$ in this reliability state subset of the system remaining components are given by

$$E[T_i'(u)] = c(u)[E[T_i(u)] - \frac{\upsilon}{n} E[T_i(u)]$$
$$= c(u)\frac{n-\upsilon}{n} E[T_i(u)], \; i = 1,2,\ldots,n, \tag{14}$$

where $c(u)$ is the component stress proportionality correction coefficient for each u, $u = 1,2,\dots,z$, [6]. The component stress proportionality correction coefficient can be estimated on the basis of behaviour of the component reliability state changing dynamics or assumed a priori. However, in both cases, it should be verified by the actual reliability data analysis and experts' judgment.
Next we consider case when components have the same exponential reliability functions of the form

$$R_i(t,\cdot) = [1, R_i(t,1),\dots, R_i(t,z)] \tag{15}$$

for $t \in <0,\infty)$, $i = 1,2,\dots,n$,

where

$$R_i(t,u) = \begin{cases} 1, & t < 0 \\ \exp[-\lambda(u)t], & \\ t \ge 0, \lambda(u) \ge 0, i = 1,2,...,n & \end{cases} \tag{16}$$

with intensity of departure $\lambda(u)$ from the reliability state subset $\{u, u+1,..., z\}$, $u = 1,2,\dots,z$.

Hence, we get the following formula for intensities of departure from this reliability state subset of the remaining components

$$\lambda^{(\upsilon)}(u) = \frac{1}{c(u)}\frac{n}{n-\upsilon}\lambda(u) \tag{17}$$

for $\upsilon = 0,1,2,\dots,n-1$, $u = 1,2,\dots,z$.
This simple approach to the inside complex systems dependencies may be developed for the selected critical homogeneous reliability infrastructures and the analytical solutions for their reliability characteristics can be found. Unfortunately, in case of non-homogeneous infrastructures the analytical solutions are generally difficult to obtain and have to be supported by Monte Carlo simulation methods.
On the assumption of components' dependencies, described by (14), based on Markov processes, we can prove following theorem.
Proposition 1. [2], [6] If in a homogeneous multi-state "*m* out of *n*" system
(i) components have exponential reliability function given by (15)-(16),
(ii) components are dependent,
(iii) intensities of departure from the reliability state subsets of components are given by (17),
then the multi-state system reliability function is given by the formula

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1),\dots, \boldsymbol{R}(t,z)], \tag{18}$$

where

$$\boldsymbol{R}(t,u) = \sum_{\upsilon=0}^{n-m} \frac{[\frac{n\lambda(u)}{c(u)}t]^{\upsilon}}{\upsilon!}\exp[-\frac{n\lambda(u)}{c(u)}t] \tag{19}$$

for $t \ge 0$, $u = 1,\dots,z$.
The theoretical result presented in the form of *Proposition 1* is a generalization of the results given in [2] and the proof of this proposition can be found in [4].
Next, from *Proposition 1* we obtain following corollary.
Corollary 1. If in a homogeneous multi-state series system
(i) components have exponential reliability functions given by (15)-(16),
(ii) components are dependent;
(iii) intensities of departures of components from the reliability state subsets are given by (17),
then the system reliability function is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1),\dots, \boldsymbol{R}(t,z)], \tag{20}$$

where

$$\boldsymbol{R}(t,u) = \exp[-\frac{n\lambda(u)}{c(u)}t],\ t \ge 0,\ u = 1,\dots,z. \tag{21}$$

4. Reliability of multi-state "*m* out of *l*"-series system with dependent components

To define a "*m* out of *l*" – series system, assume that [8]:
– k is the number "*m* out of *l*" subsystems of a system,
– l is the numbers of components of "*m* out of *l*" subsystems,
– E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l$, $k, l \in N$, are components of a system,
– all components E_{ij} have the same reliability state set as before $\{0,1,...,z\}$,
– $T_{ij}(u)$, $i = 1,2,...,k$, $j = 1,2,...,l$, $k, l \in N$, are random variables representing lifetimes of components E_{ij} in the reliability state subset $\{u, u+1,..., z\}$, while they were in the reliability state z at the moment $t = 0$,
– $s_{ij}(t)$ is a component E_{ij} reliability state at the moment t, $t \in <0,\infty)$, while they were in the reliability state z at the moment $t = 0$.

Definition 6. A vector

$$R_{ij}(t,\cdot) = [R_{ij}(t,0), R_{ij}(t,1), ..., R_{ij}(t,z)], \quad (22)$$

where

$$R_{ij}(t,u) = P(S_{ij}(t) \geq u \mid S_{ij}(0) = z) = P(T_{ij}(u) > t), \quad (23)$$

for $t \in <0,\infty)$, $u = 0,1,...,z$, $i = 1,2,...,k$, $j = 1,2,...,l$, is the probability that a component E_{ij} is in the reliability state subset $\{u, u+1,..., z\}$ at the moment t, $t \in <0,\infty)$, while it was in the reliability state z at the moment $t = 0$, called the multi-state reliability function of a component E_{ij}.

Definition 7. [7] A multi-state system is called an "m out of l"-series system if its lifetime $T(u)$ in the reliability state subset $\{u, u+1,..., z\}$ is given by

$$T(u) = \min_{1 \leq i \leq k} T_{i(l-m+1)}(u), \; m = 1,2,...,l, \; u = 1,2,...,z,$$

where $T_{i(l-m+1)}(u)$ are the $l-m+1$ order statistics in the set of random variables

$$T_{i1}(u), \; T_{i2}(u), \; ..., \; T_{il}(u), \;\; i = 1,2,...,k, \;\; u = 1,2,...,z.$$

The above definition means that the multi-state "m out of l"-series system is composed of k subsystems that are multi-state "m out of l" systems and it is in the reliability state subset $\{u, u+1,..., z\}$ if all its "m out of l" subsystems are in this reliability state subset. In this definition l denote the numbers of components in the "m out of l" subsystems. The numbers k, m and l are called the system structure shape parameters.

Definition 8. [7] A multi-state "m out of n"-system is called homogeneous if its components E_{ij} have the same reliability function

$$R_{ij}(t,\cdot) = [1, R_{ij}(t,1), \ldots, R_{ij}(t,z)]$$

with the coordinates

$$R_{ij}(t,u) = R(t,u) \text{ for } t \in <0,\infty),$$
$$u = 1,2,\ldots,z, \;\; i = 1,2,\ldots,k, \;\; j = 1,2,\ldots,l.$$

Then we can consider a multi-state "m out of l"-series system and its multi-state reliability function in case its components are dependent. To this end, we assume similarly as in Section 3 that if $\upsilon, \upsilon = 0,1,2,\ldots,l-1$, components of each "$m$ out of l" subsystem of a system are out of the reliability state subset $\{u, u+1,..., z\}$, the mean values of lifetimes $T_{ij}'(u)$ in the reliability state subset $\{u, u+1,..., z\}$ of this subsystem remaining components are given by

$$E[T_{ij}'(u)] = c(u)\left[E[T_{ij}(u)] - \frac{\upsilon}{l} E[T_{ij}(u)]\right]$$
$$= c(u)\frac{l-\upsilon}{l} E[T_{ij}(u)]$$

for $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l$, $u = 1,2,\ldots,z$, where $c(u)$ are component stress proportionality correction coefficients [6].

Hence, in case subsystem components have exponential reliability functions with intensity of departure $\lambda(u)$ from the reliability state subset $\{u, u+1,..., z\}$, according to the well known relationship between the lifetime mean value in this reliability state subset and the intensity of departure from this reliability state subset we get following formula for intensities of departure from this reliability state subset of subsystem remaining components

$$\lambda^{(\upsilon)}(u) = \frac{1}{c(u)}\frac{l}{l-\upsilon}\lambda(u) \quad (24)$$

for $\upsilon = 0,1,2,\ldots,l-1$, $u = 1,2,\ldots,z$.

Considering results for a "m out of n" system with dependent components given in *Proposition 1* and the reliability function of a series system presented in *Corollary 1*, we can obtain formula for the reliability function of a "m out of l"-series system in the form of following proposition.

Proposition 2.

If in a homogeneous multi-state "m out of l"-series system

(i) components have exponential reliability function given by

$$R_{ij}(t,\cdot) = [1, R_{ij}(t,1), \ldots, R_{ij}(t,z)] \quad (25)$$

where

$$R_{ij}(t,u) = \begin{cases} 1, & t < 0, \\ \exp[-\lambda(u)t], & t \geq 0, \end{cases} \quad (26)$$

for $u = 1,2,\ldots,z$, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l$, with intensity $\lambda(u) \geq 0$ of departure from the reliability state subset $\{u, u+1,..., z\}$,

(i) components are dependent,

(ii) intensities of departure from the reliability state subsets of components are given by (24),

then the multi-state system reliability function is given by the formula

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \ldots, \boldsymbol{R}(t,z)], \quad (27)$$

where

$$\boldsymbol{R}(t,u) = [\sum_{\upsilon=0}^{l-m} \frac{[\frac{l\lambda(u)}{c(u)} t]^{\upsilon}}{\upsilon!} \exp[-\frac{l\lambda(u)}{c(u)} t]]^{k} \quad (28)$$

for $t \geq 0,\ u = 1, \ldots, z.$

5. Reliability of multi-state "*m* out of *l*"- series system with dependent components at variable operation conditions

We assume that a system during its operation process is taking $\nu_o, \nu_o \in N$, different operation states $z_1, z_2, \ldots, z_{\nu_o}$. We define the system operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_{\nu_o}\}$. Further, we assume that the system operation process *Z*(*t*) is a semi-Markov process [5], [8]-[12].

We assume that changes of the system operation process $Z(t)$ states have an influence on the reliability of system multi-state components and reliability structure of a system as well. We mark by $T_1^{(b)}(u),\ T_2^{(b)}(u),\ \ldots, T_n^{(b)}(u)$ conditional lifetimes in the reliability states subset $\{u, u+1, \ldots, z\}$ of system components $E_1, E_2, \ldots, E_n$ and by $T^{(b)}(u)$ conditional lifetime of a system in the reliability states subset $\{u, u+1, \ldots, z\}$, $u = 1,2,\ldots,z$, while a system is at the operation state z_b, $b = 1,2,\ldots,\nu_o$. Further, we define the conditional reliability function of system's multi-state component E_i, $i = 1,2,\ldots,n$, while a system is at the operation state z_b, $b = 1,2,\ldots,\nu_o$, by the vector [8], [12]

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, \ldots, [R_i(t,z)]^{(b)}], \quad (29)$$

where

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t \mid Z(t) = z_b) \quad (30)$$

for $t \in <0,\infty)$, $u = 1,2,\ldots,z$, $i = 1,2,\ldots,n$, $b = 1,2,\ldots,\nu_o$, and the conditional reliability function of a multi-state system while a system is at the operation state z_b, $b = 1,2,\ldots,\nu_o$, by the vector [8], [12]

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1, [\boldsymbol{R}(t,1)]^{(b)}, \ldots, [\boldsymbol{R}(t,z)]^{(b)}, \quad (31)$$

where

$$[\boldsymbol{R}(t,u)]^{(b)} = P(T^{(b)}(u) > t \mid Z(t) = z_b) \quad (32)$$

for $t \in <0,\infty)$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu_o$.

The system conditional lifetimes

$$T^{(b)}(u) = T(T_1^{(b)}(u), T_2^{(b)}(u), \ldots, T_n^{(b)}(u))$$

defined for $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu_o$, are dependent on the conditional lifetimes $T_1^{(b)}(u),\ T_2^{(b)}(u),\ \ldots, T_n^{(b)}(u)$, of system components at the operation state z_b and coordinates of the system conditional multi-state reliability functions

$$[\boldsymbol{R}(t,u)]^{(b)} = [\boldsymbol{R}([R_1(t,u)]^{(b)}, \ldots, [R_n(t,u)]^{(b)})]^{(b)}$$

defined for $t \in <0,\infty)$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu_o$, are dependent on the conditional reliability function $[R_1(t,u)]^{(b)}, [R_2(t,u)]^{(b)}, \ldots, [R_n(t,u)]^{(b)}$ of components at the operation state z_b.

Consequently, we mark by *T*(*u*) the system unconditional lifetime in the reliability states subset {*u*,*u*+1,...,*z*}, *u* = 1,2,…,*z*, and we define the system unconditional reliability function by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \ldots, \boldsymbol{R}(t,z)], \quad (33)$$

where

$$\boldsymbol{R}(t,u) = P(T(u) > t) \quad (34)$$

for $t \in <0,\infty)$, $u = 1,2,\ldots,z$.

In case system operation time θ is large enough, the system unconditional reliability function is given by [8], [12]

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{\nu_o} p_b [\boldsymbol{R}(t,u)]^{(b)},\ t \geq 0, \quad (35)$$

where $[\boldsymbol{R}(t,u)]^{(b)}$, $u = 1,2,\ldots,z$, $b = 1,2,\ldots,\nu_o$, are the coordinates of the system conditional reliability functions defined by (31)-(32) and p_b are limit transient probabilities of the system operation process at the operation state z_b, $b = 1,2,\ldots,\nu_o$, defined in [8].

In reliability analysis of multi-state system with dependent components at variable operation conditions we assume that both intensity of departure $\lambda(u)$ from the reliability state subset $\{u,u+1,...,z\}$ and component stress proportionality correction coefficients $c(u)$, $u = 1,2,\dots,z$, are influenced by changes of the system operation process states and their values can differ at various operation states z_b, $b = 1,2,...,\nu_o$.

Proposition 3. If in a homogeneous multi-state "*m* out of *n*"-series system

(i) components have exponential reliability function given by

$$[R_{ij}(t,\cdot)]^{(b)} = [1, [R_{ij}(t,1)]^{(b)}, ..., [R_{ij}(t,z)]^{(b)}] \quad (36)$$

where

$$[R_{ij}(t,u)]^{(b)} = \begin{cases} 1, & t < 0, \\ \exp[-[\lambda(u)]^{(b)} t], & t \ge 0, \end{cases} \quad (37)$$

for $u = 1,2,...,z$, $i = 1,2,\dots,k$, $j = 1,2,\dots,l$, $b = 1,2,...,\nu_o$, with intensity of departure $[\lambda(u)]^{(b)} \ge 0$ from the reliability state subset $\{u, u+1,...,z\}$,

(ii) components are dependent,

(iii) intensities $[\lambda(u)]^{(b)}$ of departure from the reliability state subsets of components at the operation states z_b are given by (24), i.e.

$$[\lambda^{(\upsilon)}(u)]^{(b)} = \frac{1}{[c(u)]^{(b)}} \frac{l}{l-\upsilon} [\lambda(u)]^{(b)} \quad (38)$$

for $\upsilon = 0,1,2,\dots,l-1$, $u = 1,2,\dots,z$, $b = 1,2,...,\nu_o$,

then the multi-state system reliability function is given by the formula

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1),\dots,\boldsymbol{R}(t,z)], \quad (39)$$

where

$$\boldsymbol{R}(t,u) = \sum_{b=1}^{\nu_o} p_b [\sum_{\upsilon=0}^{l-m} \frac{(\frac{l[\lambda(u)]^{(b)}}{[c(u)]^{(b)}} t)^{\upsilon}}{\upsilon!} \cdot \exp[-\frac{l[\lambda(u)]^{(b)}}{[c(u)]^{(b)}} t]]^k \quad (40)$$

for $t \ge 0$, $u = 1,\dots,z$.

6. Reliability of a shipyard rope transportation system

Ship-rope elevators are used to dock and undock ships coming to shipyards for repairs. The elevator utilized in shipyard, with the scheme presented in *Figure 1*, is composed of a steel platform-carriage placed in its syncline (hutch).

The platform is moved vertically with 10 rope-hoisting winches fed by separate electric motors. Motors are equipped in ropes with diameter 47 mm each rope having a maximum load of 300 tones. During ship docking the platform, with a ship settled in special supporting carriages on the platform, is raised to the wharf level. During undocking, the operation is reversed. While a ship is moving into or out of a syncline and while stopped in the upper position the platform is held on hooks and loads in ropes are relieved. Since the platform-carriage and electric motors are highly reliable in comparison to ropes, which work in extremely aggressive conditions, in our further analysis we will discuss reliability of the rope system only.

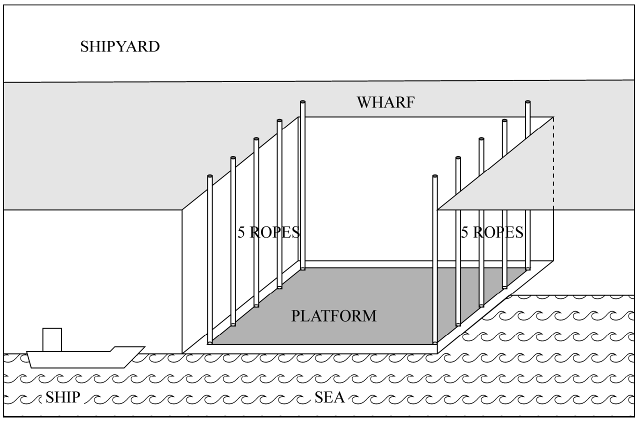

Figure 1. The scheme of the ship-rope elevator

Considering the tonnage of docked and undocked ships by the rope elevator we can divide system's load into six groups and due to fact that the rope elevator system depends mainly on the tonnage of docking ships we can distinguish following operation states of the rope elevator system operation process [3]:

- z_1 – without loading (the system is not working),
- z_2 – loading over 0 up to 500 tones,
- z_3 – loading over 500 up to 1000 tones,
- z_4 – loading over 1000 up to 1500 tones,
- z_5 – loading over 1500 up to 2000 tones,
- z_6 – loading over 2000 up to 2500 tones.

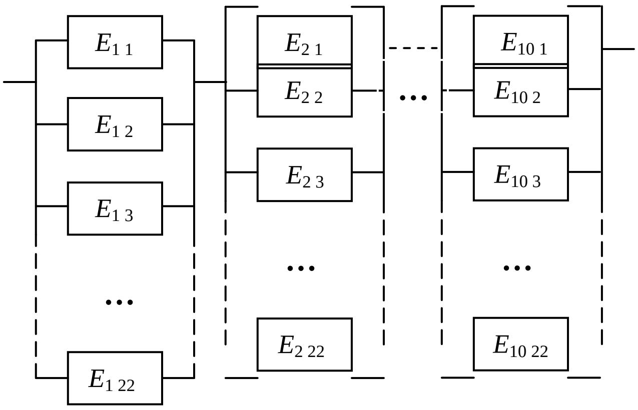

Figure 2. The scheme of the shipyard rope transportation system reliability structure

In all six operational states system has the same structure. Considered shipyard rope transportation system is composed of k = 10 subsystems i.e. ropes linked in series and each rope is composed of l = 22 parallel-linked strands.

The assumption that ropes satisfy technical conditions when at least one of its strands satisfies these conditions is not always true. In reality it is said that a rope is changing its reliability state subset after some number of strands change their reliability state subsets. Therefore better, closer in reality approach to the system reliability evaluation is assumption that a rope is "*m* out of *n*" system. Further, on the basis of rope's parameters given in its technical certificate and experts' opinions, we assume that m = 5 and a rope is "5 out of 22" system. Considering strands as basic components of a system E_{ij}, $i = 1,2,\ldots,10$, $j = 1,2,...,22$, we conclude that the ship-rope elevator is a regular "5 out of 22"-series system with the reliability structure presented in *Figure 2*.

From the operation process analysis and its statistical identification presented in [3] we obtain limit values of transient probabilities $p_b(t)$ at particular operational states z_b, $b = 1,...,6$, respectively:

$$p_1 = 0.9810,\ p_2 = 0.0032,\ p_3 = 0.0021,$$
$$p_4 = 0.0083,\ p_5 = 0.0028,\ p_6 = 0.0026. \qquad (41)$$

According to rope reliability data given in their technical certificates and experts' opinions based on the nature of strand failures following four reliability states have been distinguished:

- a reliability state 3 – a strand is new, without any defects,
- a reliability state 2 –number of broken wires in a strand is greater than 0% and less than 25% of all its wires, or corrosion of wires is greater than 0% and less than 25%,
- a reliability state 1 –number of broken wires in a strand is greater than or equal to 25% and less than 50% of all its wires, or corrosion of wires is greater than or equal to 25% and less than 50%,
- a reliability state 0 – otherwise (a strand is failed).

We fix the critical reliability state r = 2.

Moreover, we assume that components E_{ij}, $i = 1,2,\ldots,10$, $j = 1,2,...,22$, of the ship-rope elevator i.e. strands have four-state reliability functions at the operation state z_b, $b = 1,...,6$, with following exponential conditional reliability functions co-ordinates at the operational state z_1:

$$[R_{ij}(t,1)]^{(1)} = \exp[-0.1613t] \text{ 1/year},$$
$$[R_{ij}(t,2)]^{(1)} = \exp[-0.2041t] \text{ 1/year},$$
$$[R_{ij}(t,3)]^{(1)} = \exp[-0.2326t] \text{ 1/year},\ t \geq 0,$$

at the operational state z_2:

$$[R_{ij}(t,1)]^{(2)} = \exp[-0.2041t] \text{ 1/year},$$
$$[R_{ij}(t,2)]^{(2)} = \exp[-0.2564t] \text{ 1/year},$$
$$[R_{ij}(t,3)]^{(2)} = \exp[-0.2941t] \text{ 1/year},\ t \geq 0,$$

at the operational state z_3:

$$[R_{ij}(t,1)]^{(3)} = \exp[-0.2222t] \text{ 1/year},$$
$$[R_{ij}(t,2)]^{(3)} = \exp[-0.2857t] \text{ 1/year},$$
$$[R_{ij}(t,3)]^{(3)} = \exp[-0.3226t] \text{ 1/year},\ t \geq 0,$$

at the operational state z_4:

$$[R_{ij}(t,1)]^{(4)} = \exp[-0.2702t] \text{ 1/year},$$
$$[R_{ij}(t,2)]^{(4)} = \exp[-0.3508t] \text{ 1/year},$$
$$[R_{ij}(t,3)]^{(4)} = \exp[-0.4167t] \text{ 1/year},\ t \geq 0,$$

at the operational state z_5:

$$[R_{ij}(t,1)]^{(5)} = \exp[-0.3333t] \text{ 1/year},$$
$$[R_{ij}(t,2)]^{(5)} = \exp[-0.4762t] \text{ 1/year},$$
$$[R_{ij}(t,3)]^{(5)} = \exp[-0.5882t] \text{ 1/year},\ t \geq 0,$$

and at the operational state z_6:

$$[R_{ij}(t,1)]^{(6)} = \exp[-0.4348t] \text{ 1/year},$$
$$[R_{ij}(t,2)]^{(6)} = \exp[-0.7143t] \text{ 1/year},$$
$$[R_{ij}(t,3)]^{(6)} = \exp[-0.9091t] \text{ 1/year},\ t \geq 0,$$

for $i = 1,2,\ldots,10$, $j = 1,2,...,22$.

Then assuming system components' dependence defined by (38) and applying directly the formulae (39)-(40), we get the system reliability function

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2), \boldsymbol{R}(t,3)],\ t \geq 0, \qquad (42)$$

where

$$\boldsymbol{R}(t,1) \cong \sum_{b=1}^{6} p_b \left[\sum_{\upsilon=0}^{17} \frac{[(l[\lambda(1)]^{(b)} / [c(1)]^{(b)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(l[\lambda(1)]^{(b)} / [c(1)]^{(b)})t]\right]^{10}$$

$$= 0.9810 \left[\sum_{\upsilon=0}^{17} \frac{[(3.5486/[c(1)]^{(1)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(3.5486/[c(1)]^{(1)})t]\right]^{10}$$

$$+ 0.0032 \left[\sum_{\upsilon=0}^{17} \frac{[(4.4902/[c(1)]^{(2)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(4.4902/[c(1)]^{(2)})t]\right]^{10}$$

$$+ 0.0021 \left[\sum_{\upsilon=0}^{17} \frac{[(4.8884/[c(1)]^{(3)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(4.8884/[c(1)]^{(3)})t]\right]^{10}$$

$$+ 0.0083 \left[\sum_{\upsilon=0}^{17} \frac{[(5.9444/[c(1)]^{(4)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(5.9444/[c(1)]^{(4)})t]\right]^{10}$$

$$+ 0.0028 \left[\sum_{\upsilon=0}^{17} \frac{[(7.3326/[c(1)]^{(5)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(7.3326/[c(1)]^{(5)})t]\right]^{10}$$

$$+ 0.0026 \left[\sum_{\upsilon=0}^{17} \frac{[(9.5656/[c(1)]^{(6)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(9.5656/[c(1)]^{(6)})t]\right]^{10}$$

for $t \geq 0$, (43)

$$\boldsymbol{R}(t,2) \cong \sum_{b=1}^{6} p_b [\sum_{\upsilon=0}^{17} \frac{[(l[\lambda(2)]^{(b)} / [c(2)]^{(b)})t]^{\upsilon}}{\upsilon!} \exp[-(l[\lambda(2)]^{(b)} / [c(2)]^{(b)})t]]^{10}$$

$$= 0.9810 \left[\sum_{\upsilon=0}^{17} \frac{[(4.4902/[c(2)]^{(1)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(4.4902/[c(2)]^{(1)})t]\right]^{10}$$

$$+ 0.0032 \left[\sum_{\upsilon=0}^{17} \frac{[(5.6408/[c(2)]^{(2)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(5.6408/[c(2)]^{(2)})t]\right]^{10}$$

$$+ 0.0021 \left[\sum_{\upsilon=0}^{17} \frac{[(6.2854/[c(2)]^{(3)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(6.2854/[c(2)]^{(3)})t]\right]^{10}$$

$$+ 0.0083 \left[\sum_{\upsilon=0}^{17} \frac{[(7.7176/[c(2)]^{(4)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(7.7176/[c(2)]^{(4)})t]\right]^{10}$$

$$+ 0.0028 \left[\sum_{\upsilon=0}^{17} \frac{[(10.4764/[c(2)]^{(5)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(10.4764/[c(2)]^{(5)})t]\right]^{10}$$

$$+ 0.0026 \left[\sum_{\upsilon=0}^{17} \frac{[(15.7146/[c(2)]^{(6)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(15.7146/[c(2)]^{(6)})t]\right]^{10}$$

for $t \geq 0$, (44)

$$\boldsymbol{R}(t,3) \cong \sum_{b=1}^{6} p_b [\sum_{\upsilon=0}^{17} \frac{[(l[\lambda(3)]^{(b)} / [c(3)]^{(b)})t]^{\upsilon}}{\upsilon!} \exp[-(l[\lambda(3)]^{(b)} / [c(3)]^{(b)})t]]^{10}$$

$$= 0.9810 \left[\sum_{\upsilon=0}^{17} \frac{[(5.1172/[c(3)]^{(1)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(5.1172/[c(3)]^{(1)})t]\right]^{10}$$

$$+ 0.0032 \left[\sum_{\upsilon=0}^{17} \frac{[(6.4702/[c(3)]^{(2)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(6.4702/[c(3)]^{(2)})t]\right]^{10}$$

$$+ 0.0021 \left[\sum_{\upsilon=0}^{17} \frac{[(7.0972/[c(3)]^{(3)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(7.0972/[c(3)]^{(3)})t]\right]^{10}$$

$$+ 0.0083 \left[\sum_{\upsilon=0}^{17} \frac{[(9.1674/[c(3)]^{(4)})t]^{\upsilon}}{\upsilon!} \cdot \exp[-(9.1674/[c(3)]^{(4)})t]\right]^{10}$$

$$+ 0.0028 \left[\sum_{\upsilon=0}^{17} \frac{[(12.9404/[c(3)]^{(5)})t]^{\upsilon}}{\upsilon!}$$

$$\cdot \exp[-(12.9404/[c(3)]^{(5)})t]\Big]^{10}$$

$$+0.0026\Big[\sum_{\upsilon=0}^{17}\frac{[(20.0002/[c(3)]^{(6)})t]^{\upsilon}}{\upsilon!}$$

$$\cdot \exp[-(20.0002/[c(3)]^{(6)})t]\Big]^{10}$$

for $t \geq 0$. (45)

Approximate graphs of coordinates of the complex system reliability function are presented in *Figure 3*.

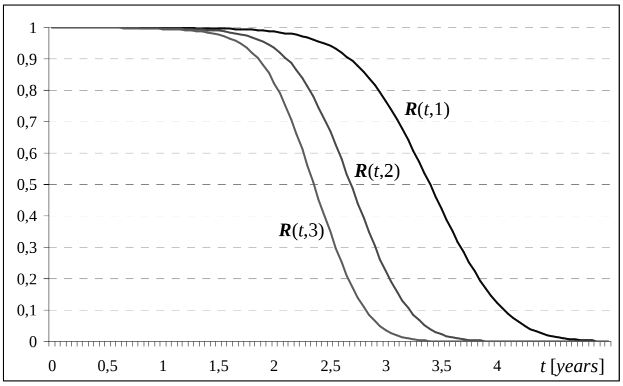

Figure 3. The graph of the ship-rope elevator reliability function $\boldsymbol{R}(t,\cdot)$ coordinates for $c(u)=1$.

The expected values and standard deviations of the shipyard rope transportation system unconditional lifetimes in the reliability state subsets {1,2,3}, {2,3}, {3}, calculated from results given by (42)-(45), according to (5)-(7), for $[c(u)]^{(b)}=1$, $u=1,2,3$ $b=1,...,6$, respectively are:

$$\mu(1) \cong 3.377,\ \sigma(1) \cong 0.557 \text{ years}, \quad (46)$$

$$\mu(2) \cong 2.667,\ \sigma(2) \cong 0.445 \text{ years}, \quad (47)$$

$$\mu(3) \cong 2.340,\ \sigma(3) \cong 0.393 \text{ years}, \quad (48)$$

and further, considering (9) and (46)-(48), the mean values of unconditional lifetimes in the particular reliability states 1, 2, 3, for $[c(u)]^{(b)}=1$, $u=1,2,3$ $b=1,...,6$, respectively are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.710 \text{ years},$$

$$\bar{\mu}(2) = \mu(2) - \mu(3) = 0.327 \text{ years},$$

$$\bar{\mu}(3) = \mu(3) = 2.340 \text{ years}. \quad (49)$$

Since the critical reliability state is r = 2, then the system risk function, according to (10), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,2)$$

where system unconditional reliability function coordinate $\boldsymbol{R}(t,2)$ is given by (44).

Hence, by (11), the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, for $[c(2)]^{(b)}=1$, $b=1,...,6$, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 1.929 \text{ years} \cong 1 \text{ year } 339 \text{ days}. \quad (50)$$

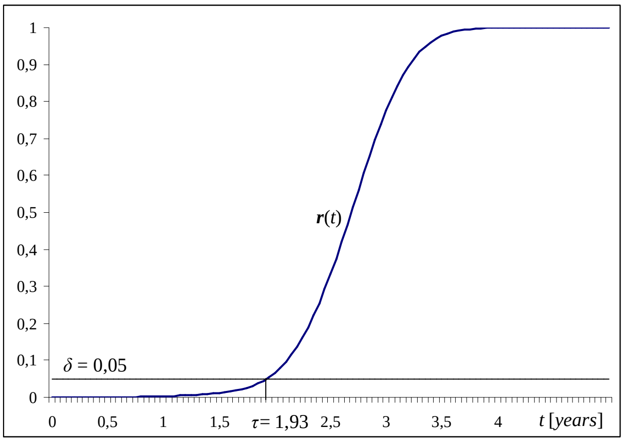

Figure 4. The graph of the ship-rope elevator risk function $\boldsymbol{r}(t)$ for $[c(2)]^{(b)}=1$, $b=1,...,6$.

The expected values and standard deviations of the shipyard rope transportation system unconditional lifetimes in the reliability state subsets {1,2,3}, {2,3}, {3}, for other values of components stress proportionality correction coefficients $[c(u)]^{(b)}$, $u=1,2,...,z$, $b=1,...,6$, are given in *Table 1*.

Table 1. The expected values and standard deviations of the shipyard rope transportation system.

$[c(u)]^{(b)}$, $u=1,2,...,z$, $b=1,...,6$,	$\mu(1)$ $\sigma(1)$ (years)	$\mu(2)$ $\sigma(2)$ (years)	$\mu(3)$ $\sigma(3)$ (years)
0,5	1,688 0,278	1,334 0,222	1,170 0,196
0,6	2,026 0,334	1,600 0,266	1,404 0,235
0,7	2,364 0,390	1,867 0,311	1,638 0,274
0,8	2,701 0,445	2,134 0,355	1,872 0,314
0,9	3,039 0,501	2,400 0,400	2,106 0,353
1,1	3,714 0,613	2,934 0,489	2,573 0,432
1,2	4,052 0,668	3,201 0,534	2,807 0,471
1,3	4,388 0,719	3,467 0,578	3,041 0,511
1,4	4,715 0,755	3,734 0,623	3,275 0,550
1,5	5,018 0,761	4,001 0,667	3,509 0,589

7. Conclusions

The main purpose of this paper was linking inside and outside of complex system dependencies that can have significant influence on system reliability. Theoretical results of reliability analysis of complex multi-state systems with dependent components in variable operation conditions are applied to the shipyard ship-rope elevator. We analyze the shipyard transportation system in case components have exponential reliability functions with intensity of departure from the reliability state subset and with components stress proportionality correction coefficients different in various operation states. Obtained results illustrate that after decreasing a reliability state by one of components in a subsystem, inside interactions among the remaining components may cause further components reliability states decrease.

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