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**Kołowrocki Krzysztof**

**Kuligowska Ewa**
*Maritime University, Gdynia, Poland*

# Monte Carlo simulation application to reliability evaluation of port grain transportation system operating at variable conditions

## Keywords

reliability, operation process, complex system, Monte Carlo simulation, port grain transportation system

## Abstract

This paper presents the Monte Carlo simulation technique applied to the reliability evaluation of systems related to the variable operation conditions. A semi-Markov processes is applied to construct the system operation model and its main characteristics are determined. The method of linking this model with the systems reliability is proposed to get a general reliability model of the complex system operating at the variable conditions. An application of the proposed Monte Carlo simulation based on this method is illustrated in the reliability evaluation of a port grain transportation system. This way obtained results are compared with the results achieved by analytical way.

## 1. Introduction

The complex systems' operation processes and their influence on changing in time the systems' structures and their components' reliability characteristics is often very difficult to identify and to apply in the analysis of their reliability [1]-[10]. The Monte Carlo simulation method is a tool that sometimes allows to simplify solving this problem [11]. The analytical approach to the complex systems reliability analysis is presented and next the background of the computer simulation modelling method for such systems reliability assessment is given. The Monte Carlo method is practically applied to examine the reliability of port grain transportation system at variable operation conditions. The main reliability and operation process characteristics of this system are found and compared with that obtained by the analytical method.

## 2. System operation process

We assume that a system during its operation at the fixed moment $t$, $t \in \langle 0,+\infty)$ may be at one of $\nu$, $\nu \in N$, different operations states $z_b$, $b = 1,2,...,\nu$. Consequently, we mark by $Z(t)$, $t \in \langle 0,+\infty)$ the system operation process, that is a function of a continuous variable $t$, taking discrete values at the set $\{z_1, z_2,..., z_\nu\}$ of the system operation states. We assume a semi-Markov model [2]-[5] of the system operation process $Z(t)$ and we mark by $\theta_{bl}$ its random conditional sojourn times at the operation states $z_b$, when its next operation state is $z_l$, $b, l = 1,2,..., \nu$, $b \neq l$

Consequently, the operation process may be described by the following parameters [5]:

- the vector $[p_b(0)]_{1\times\nu}$, $b = 1,2,...,\nu$, of the initial probabilities of the system operation process $Z(t)$ staying at the particular operation states at the moment $t = 0$;
- the matrix $[p_{bl}]_{\nu\times\nu}$ of the probabilities of the system operation process $Z(t)$ transitions between the operation states $z_b$ and $z_l$, $b, l = 1,2,...,\nu$, $b \neq l$;
- the matrix $[H_{bl}(t)]_{\nu\times\nu}$ of the conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ at the operation states, $b, l = 1,2,...,\nu$, $b \neq l$,

Having identified the probabilities $p_{bl}$ of transitions between the operation states and the distributions of conditional sojourn times $\theta_{bl}$, the limit values $p_b(t)$, of the transient probabilities at the particular operation states, $b = 1,2,...,\nu$ can be determined.

## 3. System reliability

We consider a series system composed of the series-parallel subsystems $S_\upsilon$, $\upsilon = 1, 2, ..., n,$ each composed of components $\left[E_{ij}^{(\upsilon)}\right]^{(b)}$, $i = 1, 2, ..., [k^{(\upsilon)}]^{(b)}$, $j = 1, 2, ..., [l_k^{(\upsilon)}]^{(b)}$, while the system is at the operation state $z_b$. The numbers $[k^{(\upsilon)}]^{(b)}, [l_1^{(\upsilon)}]^{(b)}, [l_2^{(\upsilon)}]^{(b)}, ..., [l_k^{(\upsilon)}]^{(b)}$, $\upsilon = 1, 2, ..., n,$ $b = 1,2,...,\nu,$ are the considered system structure shape parameters (*Figure 1*).

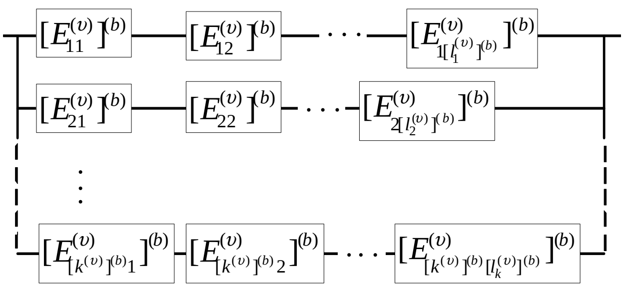


*Figure 1.* The scheme of a series-parallel subsystem

We assume that the changes of the system operation process states have an influence on the system components reliability and on the system structure as well [5]. Thus, we denote the conditional reliability function of the system component $\left[E_{ij}^{(\upsilon)}\right]^{(b)}$ while the system is at the operation state $z_b$ $b = 1,2,...,\nu,$ by

$$\left[R_{ij}^{(\upsilon)}(t)\right]^{(b)} = P(\left[T_{ij}^{(\upsilon)}\right]^{(b)} > t \,|\, Z(t) = z_b), \tag{1}$$

where $\left[T_{ij}^{(\upsilon)}\right]^{(b)}$ are the system components conditional lifetimes at the operation states $z_b$, for $t \in \langle 0,+\infty)$, $b = 1,2,...,\nu$, $\upsilon = 1, 2, ..., n,$ $i = 1, 2, ..., [k^{(\upsilon)}]^{(b)}$, $j = 1, 2, ..., [l_k^{(\upsilon)}]^{(b)}$.
Further, we denote the subsystem conditional reliability function while the system is at the operation state $z_b$, $b = 1,2,...,\nu,$ by

$$\left[\boldsymbol{R}^{(\upsilon)}_{[k^{(\upsilon)}]^{(b)},[l_1^{(\upsilon)}]^{(b)},[l_2^{(\upsilon)}]^{(b)},...,[l_k^{(\upsilon)}]^{(b)}}(t)\right]^{(b)}$$
$$= P([T^{(\upsilon)}]^{(b)} > t \,|\, Z(t) = z_b),$$

$$= 1 - \prod_{i=1}^{k}\left[1 - \prod_{j=1}^{l_i}(1 - \left[R_{ij}^{(\upsilon)}(t)\right]^{(b)})\right] \tag{2}$$

where $[T^{(\upsilon)}]^{(b)}$ are the subsystem $S_\upsilon$, $\upsilon = 1, 2, ..., n,$ lifetimes at the operation states $z_b$, for $t \in \langle 0,+\infty)$, $b = 1,2,...,\nu$, given by

$$[T^{(\upsilon)}]^{(b)} = \max_{1 \le i \le [k^{(\upsilon)}]^{(b)}}\{ \min_{1 \le j \le [l_i^{(\upsilon)}]^{(b)}}\{[T_{ij}^{(\upsilon)}]^{(b)}\}\}. \tag{3}$$

We denote the system conditional reliability function while the system is at the operation state $z_b$, $b = 1,2,...,\nu,$ by

$$\left[\boldsymbol{R}(t)\right]^{(b)} = P([T^{(\upsilon)}]^{(b)} > t \,|\, Z(t) = z_b),$$

$$= \prod_{\upsilon=1}^{n}\left[1 - \prod_{i=1}^{[k^{(\upsilon)}]^{(b)}}\left[1 - \prod_{j=1}^{[l_i^{(\upsilon)}]^{(b)}}\left[R_{ij}^{(\upsilon)}(t)\right]^{(b)}\right]\right], \tag{4}$$

where the system lifetimes at the operation states $z_b$, for $t \in \langle 0,+\infty)$, $b = 1,2,...,\nu$, are given by

$$T^{(b)} = \min_{1 \le \upsilon \le n}\{[T^{(\upsilon)}]^{(b)}\}. \tag{5}$$

We denote the system unconditional lifetime by T and the system unconditional reliability function by

$$\boldsymbol{R}(t) = P(T > t),\ t \ge 0.$$

The approximate formula for the unconditional system reliability function, for large operation time, takes the form [3], [5]

$$\boldsymbol{R}(\mathrm{t}) \cong \sum_{b=1}^{\nu} p_b\left[\boldsymbol{R}(\mathrm{t})\right]^{(b)}. \tag{6}$$

The unconditional mean value of the system lifetimes is given by

$$\mu = \int_0^\infty \boldsymbol{R}(t)dt, \tag{7}$$

where $\boldsymbol{R}(t)$ is given according to (7).

## 4. Monte Carlo simulation approach to the system operation process modelling

We denote by $z_b(q)$, $b = 1,2,...,\nu,$ the realization of the system operation process initial operation state at the moment $t = 0$ generated from the distribution

$[p_b(0)]_{1\times\nu}$. This realization is generated according to the formula

$$z_b(q) = \begin{cases} z_1, & 0 \le q < p_1(0), \\ z_2, & p_1(0) \le q < p_1(0) + p_2(0), \\ \vdots & \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_i(0) \le q \le 1, \end{cases} \quad (8)$$

where $q$ is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$.

We denote by $z_{bl}(g)$, $l = 1,2,...,\nu$, $b \ne l$, the sequence of the realizations of the system operation process consecutive operation states generated from the distribution defined by $[p_{bl}]_{\nu\times\nu}$. Those realizations are generated according to the formula

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < p_{12}, \\ z_3, & p_{12} \le g < p_{12} + p_{13}, \\ \vdots & \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_{bi} \le g \le 1, \end{cases} \quad (9)$$

$$z_{bl}(g) = \begin{cases} z_1, & 0 \le g < p_{b1}, \\ \vdots & \vdots \\ z_{b-1}, & \sum_{i=1}^{b-2} p_{bi} \le g < \sum_{i=1}^{b-1} p_{bi}, \\ z_{b+1}, & \sum_{i=1}^{b-1} p_{bi} \le g < \sum_{i=1}^{b+1} p_{bi}, \\ \vdots & \vdots \\ z_\nu, & \sum_{i=1}^{\nu-1} p_{bi} \le g \le 1, \end{cases} \quad (10)$$

for $b = 2, 3, ..., \nu$,

$$z_{\nu l}(g) = \begin{cases} z_1, & 0 \le g < p_{\nu 1}, \\ z_2, & p_{\nu 1} \le g < p_{\nu 1} + p_{\nu 2} \\ \vdots & \vdots \\ z_{\nu-1}, & \sum_{i=1}^{\nu-2} p_{\nu i} \le g \le 1, \end{cases} \quad (11)$$

where $g$ is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$.

We denote by $\theta_{bl}^{(i)}$, $b, l = 1,2,...,\nu$, , $i = 1,2,..., n_{bl}$, $b \ne l$, the realizations of the conditional sojourn time $\theta_{bl}$ of the system operation process generated from the distribution $H_{bl}$, where $n_{bl}$ is the number of those sojourn time realizations during the experiment time $\tilde{\theta}$. Those realizations are generated according to the formulae

$$\theta_{bl} = H_{bl}^{-1}(h), \; b, l = 1,2,...,\nu, b \ne l, \quad (12)$$

where $H_{bl}^{-1}(h)$ is the inverse function of the distribution function $H_{bl}(t)$ and $h$ is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$, which in the case of exponential distribution

$$H_{bl}(t) = 1 - \exp[-\alpha_{bl}\, t], \; t \ge 0, \quad (13)$$

takes the following form

$$\theta_{bl} = -\frac{1}{\alpha_{bl}} \ln(1-h), \; b, l = 1,2,...,\nu, b \ne l. \quad (14)$$

Having the realisations the conditional sojourn times of the system operation process

$$\theta_{bl}^{(1)}, \theta_{bl}^{(2)}, \ldots, \theta_{bl}^{(n_{bl})}, \; b, l = 1,2,...,\nu, b \ne l, \quad (15)$$

it is possible to determine approximately the total sojourn time at the operation state $z_b$ during the time of the experiment $\tilde{\theta}$ applying the formula

$$\tilde{\theta}_b = \sum_{\substack{l=1 \\ l \ne b}}^{\nu} \sum_{j=1}^{n_{bl}} \theta_{bl}^{(i)}. \quad (16)$$

Further, the limit transient probabilities $p_b(t)$ can be approximately obtained using the formula

$$p_b = \frac{\tilde{\theta}_b}{\tilde{\theta}}, \quad (17)$$

where

$$\tilde{\theta} = \sum_{b=1}^{\nu} \tilde{\theta}_b, \; b = 1,2,...,\nu. \quad (18)$$

## 5. Monte Carlo approach to the system reliability modelling

The realizations $t_{ij}^{(b)}$ of the component conditional lifetimes $T_{ij}^{(b)}$, $b = 1,2,...,\nu$, $i = 1,2,...,[k^{(\upsilon)}]^{(b)}$, $j = 1, 2, ..., l_k^{(\upsilon)}$ are generated according to the distributions corresponding to the reliability functions (1), i.e. they are generated by the sampling formulae:

$$t_{ij}^{(b)} = \left(F_{ij}^{(b)}(f)\right)^{-1} = \left(1 - R_{ij}^{(b)}(f)\right)^{-1}, \qquad (19)$$

where $\left(F_{ij}^{(b)}(f)\right)^{-1}$ is the inverse function of the distribution function

$$F_{ij}^{(b)}(t) = 1 - R_{ij}^{(b)}(t)$$

of the component conditional lifetime $T_{ij}^{(b)}$ and $R_{ij}^{(b)}(t)$ is defined by (1) which in the case of exponential distribution takes the following form

$$F_{ij}^{(b)}(t) = 1 - \exp[-\lambda_{ij}^{(b)}\, t], \qquad (20)$$

for $t \ge 0$, $b = 1,2,...,\nu$. In the case of the above exponential distribution the realisations of the component conditional lifetimes take the following form

$$t_{ij}^{(b)} = -\frac{1}{\lambda_{ij}^{(b)}} \ln(1 - f),\ b = 1,2,...,\nu. \qquad (21)$$

where $\lambda_{ij}^{(b)}$, are the failure rates and $f$ is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$.

The realizations $t_{\upsilon}^{(b)}$ of the subsystem $S_{\upsilon}$ lifetimes $T_{\upsilon}^{(b)}$, $\upsilon = 1, 2, ..., n,$ according to (3), are generated by the sampling formula

$$t_{\upsilon}^{(b)} = \max_{1 \le i \le [k^{(\upsilon)}]^{(b)}} \{ \min_{1 \le j \le [l_i^{(\upsilon)}]^{(b)}} \{t_{ij}^{(b)}\}\}.$$

The realizations $t^{(b)}$ of the system lifetimes $T^{(b)}$ are generated by the sampling formula

$$t^{(b)} = \min_{1 \le \upsilon \le n} \{t_{\upsilon}^{(b)}\}. \qquad (22)$$

The procedure of Monte Carlo simulation is repeated $N$ times. The approximate mean value of the system lifetimes is an arithmetic mean of all system lifetime realizations for $N$ iterations

$$[t^{(b)}]^{(i)},\ i = 1, 2, ..., N,$$

i.e.

$$\bar{t}^{(b)} = \frac{1}{N} \sum_{i=1}^{N} [t^{(b)}]^{(i)},\ b = 1,2,...,\nu.$$

## 6. Port grain transportation system reliability evaluation

### 6.1 Port grain transportation system description

The grain elevator, presented in *Figure 2*, is the basic structure in the Baltic Grain Terminal of the Port of Gdynia assigned to handle the clearing of exported and imported grain. Elevator technological potentialities allow us to join different loading and unloading relations of ships, cars and railway trucks. Its output in the grain unloading process is 400 ton/hour and in grain loading is 360 ton/hour. The whole technological process is controlled electronically. A computer station delivers full visual information about the grain stream (flow) and its balance and the elevator's working state.

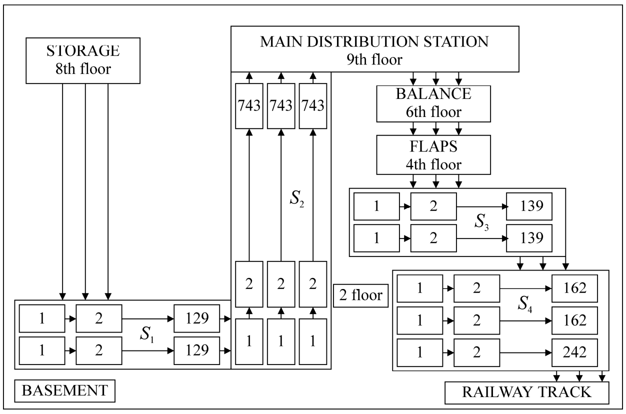


*Figure 2.* The scheme of the grain transportation system structure

One of the basic elevator functions is loading railway trucks with grain. The railway truck loading is performed in the following successive elevator operation steps [3], [7], [8]:

- gravitational passing of grain from the storage placed on the 8th elevator floor through 45 hall to horizontal conveyors placed in the elevator basement,
- transport of grain through horizontal conveyors to vertical bucket elevators transporting grain to the main distribution station placed on the 9th floor,
- gravitational dumping of grain through the main distribution station to the balance placed on the 6th floor,

- dumping weighed grain through the complex of flaps placed on the 4th floor to horizontal conveyors placed on the 2nd floor,
- dumping of grain from horizontal conveyors to worm conveyors,
- dumping of grain from worm conveyors to railway trucks.

In loading the railway trucks with grain the following elevator transportation subsystems take part (*Figure 3*):

$S_1$ – horizontal conveyors of the first type,
$S_2$ – vertical bucket elevators,
$S_3$ – horizontal conveyors of the second type,
$S_4$ – worm conveyors,
the main distribution station and the balance.

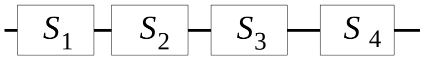


*Figure 3.* The scheme of the grain transportation system reliability structure

The main distribution station is the system of dumping channels in the form of a steel box composed of dividing walls, which direct the grain from bucket conveyors to the balance. Its executive elements are composed of three steel sleeves and pneumatic elements in the form of three servomotors. The electronic balance weighs the dumped grain with electronic indicators. Its executive elements during loading and unloading with grain are flaps, which are opened and closed by five pneumatic servomotors.

The transporting subsystems have steel covers and they are provided with drives in the form of electrical engines with gears. In their reliability analysis we omit their drives as they are different types mechanisms. We also omit their covers as they have a high reliability and, practically, do not fail.

## 6.2. Parameters of port grain transportation system operation process and its components reliability

Taking into account the operation process of the considered transportation system, there are distinguished the following $v = 3$ (*Table 1*) operation states as the system three tasks:

$z_1$ – the system operation with the largest efficiency when all components of the subsystems $S_1$, $S_2$, $S_3$ and $S_4$ are used (*Figure 2*),

$z_2$ – the system operation with less efficiency system when the first conveyor of subsystem $S_1$, the first and second elevators of subsystem $S_2$, the first conveyor of subsystem $S_3$ and the first and second conveyors of subsystem $S_4$ are used (*Figure 4*),

$z_3$ – the system operation with least efficiency when only the first conveyor of subsystem $S_1$, the first elevator of subsystem $S_2$, the first conveyor of subsystem $S_3$ and the first conveyor of subsystem $S_4$ are used (*Figure 5*).

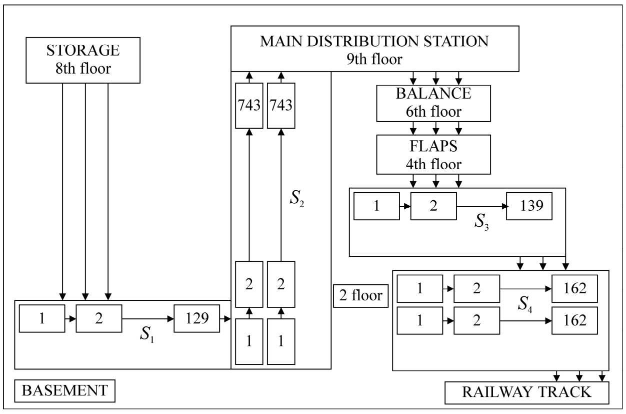


*Figure 4.* The scheme of the grain transportation system structure at operation state $z_2$

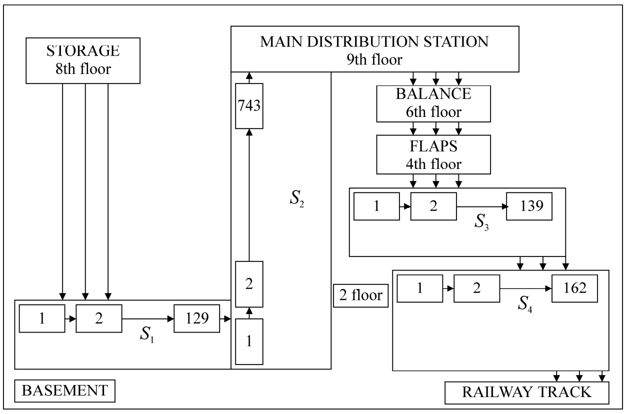


*Figure 5.* The scheme of the grain transportation system structure at operation state $z_3$

*Table 1.* List of operation states.

| State | Sub-system | Components | Failure rate |
|---|---|---|---|
| $z_1$ | $S_1$ | 2 identical belt conveyors (the 1st type) | |
| | | 1 belt | 0,0125 |
| | | 2 drums | 0,0015 |
| | | 117 channelled rollers | 0,005 |
| | | 9 supporting rollers | 0,004 |
| | $S_2$ | 3 identical vertical bucket elevators | |
| | | 1 belt | 0,025 |
| | | 2 drums | 0,0015 |
| | | 740 buckets | 0,03 |

| | | | |
|---|---|---|---|
| | $S_3$ | 2 identical belt conveyors (the 2nd type) | |
| | | 1 belt | 0,0125 |
| | | 2 drums | 0,0015 |
| | | 117 channelled rollers | 0,005 |
| | | 19 supporting rollers | 0,004 |
| | $S_4$ | 3 chain worm conveyors | |
| | | 2 conveyors of the 1st type | |
| | | 1 wheel driving a belt | 0,005 |
| | | 1 reversible wheel | 0,005 |
| | | 160 links | 0,012 |
| | | 1 conveyor of the 2nd type | |
| | | 1 wheel driving a belt | 0,022 |
| | | 1 reversible wheel | 0,022 |
| | | 240 links | 0,034 |
| $z_2$ | $S_1$ | 1 belt conveyor (the 1st type) | |
| | $S_2$ | 2 identical vertical bucket elevators | |
| | $S_3$ | 1 belt conveyor (the 2nd type) | |
| | $S_4$ | 2 chain worm conveyors (the 1st type) | |
| $z_3$ | $S_1$ | 1 belt conveyor (the 1st type) | |
| | $S_2$ | 1 vertical bucket elevator | |
| | $S_3$ | 1 belt conveyor (the 2nd type) | |
| | $S_4$ | 1 chain worm conveyor (the 1st type) | |

To illustrate the problem we simplify our considerations by assuming that all system components have exponential distributions of their lifetimes of the form

$$R_{ij}^{(b)}(t) = \exp[-\lambda_{ij}^{(b)} t], \tag{23}$$

for $t \in \langle 0, +\infty)$ where $\lambda_{ij}^{(b)}$ are the failure rates at the operation states $z_b$, $b = 1,2,...,v$, $i = 1,2,...,k$, $j = 1,2,...,l_n$ given in [3] and presented in *Table 1*.
At all system operational states, subsystems $S_1$, $S_2$, $S_3$ and $S_4$ become a non-homogeneous regular series-parallel systems with parameters given in [3] and presented in *Table 1*. At the operation state $z_1$, the subsystem $S_4$ consists of three conveyors. Two of them have 162 components and the remaining one has 242 components. Thus it is a non-homogeneous non-regular multi-state series-parallel system.

## 6.3. Analytical evaluation of port grain transportation system operation process

Since the system tasks are disjoint then its operation states belong to the set

$$Z = \{z_1, z_2, z_3\}. \tag{24}$$

We arbitrarily assume the vector of realisations

$$[p_b(0)]_{1\times v} = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right], \tag{25}$$

of the initial probabilities $p_b(0)$, $b = 1, 2, 3,$ of the system operation process stay at the particular states $z_b$ at the time $t = 0$ and the following matrix of the conditional distribution functions of the system sojourn times $\theta_{bl}$, $b, l = 1, 2, 3,$

$$[H_{bl}(t)] = \begin{bmatrix} 0 & 1-e^{-5t} & 1-e^{-10t} \\ 1-e^{-40t} & 0 & 1-e^{-50t} \\ 1-e^{-10t} & 1-e^{-20t} & 0 \end{bmatrix}. \tag{26}$$

Moreover, we assume that the probabilities of transitions between the states are given by

$$[p_{bl}] = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{4}{9} & 0 & \frac{5}{9} \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} \tag{27}$$

Hence and from (26), the limit values of the transient probabilities $p_b(t)$ at the operational states $z_b$, are given by [3]

$$p_1 = \frac{34}{64} \cong 0.531, \quad p_2 = \frac{7}{64} \cong 0.109,$$
$$p_3 = \frac{23}{64} \cong 0.360. \tag{28}$$

## 6.4. Analytical evaluation of port grain transportation system reliability

Asymptotic approach is considered in [3]. The results based on *Table 3* are as follows:

- the reliability function at the operational state $z_1$ is given by

$$\begin{aligned}\boldsymbol{R}^{(1)}(t) &\cong \boldsymbol{R}^{(1)}_{2,129}(t)\,\boldsymbol{R}^{(1)}_{3,743}(t)\,\boldsymbol{R}^{(1)}_{2,139}(t)\,\boldsymbol{R}^{(1)}_{3,242}(t) \\ &= 24\exp[-25.471t] - 24\exp[-47.699t] \\ &\quad - 12\exp[-26.1075t] - 12\exp[-27.401t] \\ &\quad + 12\exp[-48.3355t] + 12\exp[-49.629] \\ &\quad - 12\exp[-26.1475t] + 12\exp[-48.3755t]\end{aligned}$$

$$+\ 8\exp[-69.927t] + 6\exp[-28.0375t]$$
$$-\ 6\exp[-50.2655t] - 6\exp[-50.3055t]$$
$$+\ 6\exp[-26.784t] + 6\exp[-28.0775t]$$
$$-\ 6\exp[-49.012t] - 4\exp[-70.6035t]$$
$$-\ 4\exp[-70.5635t] - 4\exp[-71.857t]$$
$$+\ 3\exp[-50.942t] - 3\exp[-28.714t]$$
$$+\ 2\exp[-72.4935t] + 2\exp[-71.24t]$$
$$+\ 2\exp[-72.5335t] - \exp[-73.17t], \quad (29)$$

for $t \in \langle 0,+\infty)$;

- the reliability function at the operational state $z_2$ is given by

$$\boldsymbol{R}^{(2)}(t) = \boldsymbol{R}^{(2)}_{1,129}(t)\,\boldsymbol{R}^{(2)}_{2,743}(t)\,\boldsymbol{R}^{(2)}_{1,139}(t)\,\boldsymbol{R}^{(2)}_{2,242}(t)$$
$$= 4\exp[-25.471t]$$
$$-\ 2\exp[-27.401t]$$
$$-2\exp[-47.699t]$$
$$+\ \exp[-49.629t], \quad (30)$$

for $t \in \langle 0,+\infty)$

- the reliability function at the operational state $z_3$ is given by

$$\boldsymbol{R}^{(3)}(t) = \boldsymbol{R}^{(3)}_{1,129}(t)\,\boldsymbol{R}^{(3)}_{1,743}(t)\,\boldsymbol{R}^{(3)}_{1,139}(t)\,\boldsymbol{R}^{(3)}_{1,242}(t)$$
$$= \exp[-25.471t] \quad (31)$$

for $t \in \langle 0,+\infty)$

Finally, considering (28) and according to (6) the system unconditional reliability is given by

$$\boldsymbol{R_a}(t) \cong \frac{34}{64}\boldsymbol{R}^{(1)}(t) + \frac{7}{64}\boldsymbol{R}^{(2)}(t) + \frac{23}{64}\boldsymbol{R}^{(3)}(t), \quad (32)$$

where $\boldsymbol{R}^{(1)}(t)$, $\boldsymbol{R}^{(2)}(t)$ and $\boldsymbol{R}^{(3)}(t)$ respectively are given by (29), (30) and (31). Hence, applying (7) we get the mean value of the system unconditional lifetime

$$\mu \cong 0.0638 \text{ year} \cong 23.29 \text{ days}. \quad (33)$$

### 6.5. Monte Carlo evaluation of port grain transportation system operation process

The first step of the simulation is to select the initial operation state $z_b(g)$, $b = 1, 2, 3, 4,$ at the moment $t = 0,$ using formula (8), which is given by

$$z_b(q) = \begin{cases} z_1, & 0 \le q < \frac{1}{3} \\ z_2, & 0 \le q < \frac{2}{3} \\ z_3, & 0 \le q \le 1, \end{cases}$$

where $q$ is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$. The next operation state $z_l$, $l = 1,2,3,4,$ is generated according to (9)-(11), from $z_{bl}(g)$, $b = 1,2,3,4,$ defined as

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < \frac{1}{3} \\ z_3, & \frac{1}{3} \le g \le 1, \end{cases}$$

$$z_{2l}(g) = \begin{cases} z_1, & 0 \le g < \frac{4}{9} \\ z_3, & \frac{4}{9} \le g \le 1, \end{cases}$$

$$z_{3l}(g) = \begin{cases} z_1, & 0 \le g < \frac{1}{3} \\ z_2, & \frac{1}{3} \le g \le 1. \end{cases}$$

Applying (26), the realizations of the empirical conditional sojourn times are generated according to the formulae

$$\theta_{12}(h) = -0.2\ln[1-h],$$
$$\theta_{13}(h) = -0.1\ln[1-h],$$
$$\theta_{21}(h) = -0.025\ln[1-h],$$
$$\theta_{23}(h) = -0.02\ln[1-h],$$
$$\theta_{31}(h) = -0.1\ln[1-h],$$
$$\theta_{32}(h) = -0.05\ln[1-h],$$

where $h$ is a randomly generated number from the uniform distribution on the interval $\langle 0, 1\rangle$.

The system operation process characteristics are calculated using the Monte Carlo method with time of the experiment fixed as $\tilde{\theta} = 18\,250$ days.

Applying (16)-(18) the approximate limit values of the system operation process transient probabilities at the operation states $z_b$ are as follows:

$$p_1 = 0.532, \quad p_2 = 0.112, \quad p_3 = 0.358. \quad (34)$$

### 6.6. Monte Carlo evaluation of port grain transportation system reliability

The realizations of the port grain transportation system components lifetimes $t_{ij}^{(b)}$, $b = 1,2,3,4,$ are generated from the exponential distribution according to (21) and *Table 1*. The realizations $t^{(b)}$ of the system lifetimes $T^{(b)}$, $b = 1,2,3,4,$ are generated using the sampling formula (22). The histogram of the system lifetimes is illustrated in *Figure 6*.

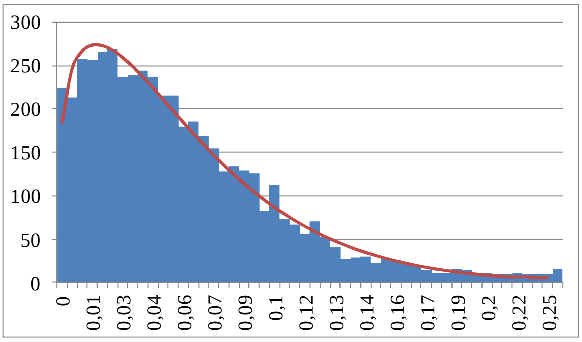


*Figure 6*. The graph of the histogram of port grain transportation system lifetime and the Weibull distribution density function (35)

After analyzing and comparing the histogram with the graph of exponential distribution density function, we formulate the null hypothesis:
$H_0$: The lifetime of the system has Weibull distribution with the density function

$$f(t) = \begin{cases} 0, & t < 0 \\ \alpha\beta t^{\beta-1} \exp[-\alpha t^{\beta}], & t \geq 0, \end{cases} \tag{35}$$

where $\alpha, \beta \in \langle 0, +\infty)$.
Further, we estimate the unknown parameters $\alpha, \beta$ of the density function (35) of the hypothetical Weibull distribution and we obtain

$$\alpha = 31.3852, \quad \beta = 1.2602.$$

Hence, we get the following form of the system unconditional reliability function

$$\boldsymbol{R}(t) \cong \begin{cases} 0, & t < 0 \\ \exp[-31.382 t^{1.2602}], & t \geq 0. \end{cases} \tag{36}$$

To verify the hypothesis, we find the realization of the $\chi^2$ (chi-square)-Pearson's, calculated according to the formula given in [5], which amounts $u_n \cong 62.39$. Assuming the significance level $\alpha = 0.05$ for $\bar{r} - l - 2 = 50 - 1 - 2 = 47$ degrees of freedom, from the tables of the $\chi^2$-Pearson's distribution we find the value $u_\alpha = 64.00$. The obtained value $u_n$ belongs to the acceptance domain, i.e.

$$u_n = 62.39 \leq u_\alpha = 64.00.$$

Therefore, at the significance level $\alpha = 0.05$, we do not reject the hypothesis $H_0$ stating that the system unconditional reliability function is Weibull of the form (36).
The mean value of the system unconditional lifetime $T$ obtained by using Monte Carlo method is given by

$$\mu = \int_0^{+\infty} \boldsymbol{R}(t)\, dt \approx 22.03 \text{ days.} \tag{37}$$

## 7. Results comparison

The results of application obtained by the analytical method and the Monte Carlo simulation method differ not too much. That concerned with the port grain transportation system operation process main characteristics and given by (28) and (34) are almost the same. Whereas, that concerned with the port grain transportation system reliability main characteristic and given by (33) and (37) differs more. The last difference may follow from the fact that the both formulae (32) and (36) expressing the port grain transportation reliability are approximate.

## 8. Conclusions

The analytical method and the Monte Carlo simulation methods are proposed to reliability evaluation of complex systems operating at variable conditions. Those methods are applied to the port grain transportation system reliability evaluation. The achieved results i.e. the approximate limit values of the system operation process transient probabilities at the particular operation states and the mean values of the system unconditional lifetimes may be very useful in system reliability analysis and improvement. The results obtained and presented in this paper lead to the conclusion that the Monte Carlo method is a useful tool in modeling objects' reliability, however, further analysis of this approach is necessary due to the differences between the analytical and simulation results.
The reliability data concerned with the operation process and component reliability functions of the

port grain transportation system are not precise. They come from experts and are concerned with the mean lifetimes of the system components and with the conditional sojourn times of the system in the operation states under the arbitrary assumption that their distributions are exponential. By improving the system operation and reliability data and further development of the proposed method it seems to be possible to obtain more precise results useful in the complex technical systems and their operation processes and reliability evaluation, improvement and optimisation.

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