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An optimal preventive maintenance policy of parallel-series systems

Keywords

reliability, availability, maintenance optimization

Abstract

This paper presents an algorithm to solve the problem of maintenance management of a two state parallel-series system based on preventive maintenance over the different system components. It is assumed that all components of the system exhibit Weibull hazard function and constant repair rate and that preventive maintenance would bring the system to the as good as new condition. The algorithm calculates the interval of time between preventive maintenance tasks for each component, minimizing the costs, and in such a way that the total downtime, in a certain period of time, does not exceed a predetermined value. It is presented an industrial case study where the algorithm is applied.

1. Introduction

Throughout the years, there has been tremendous pressure on manufacturing and service organizations to be competitive and provide timely delivery of quality products. In many industries, heavily automated and capital intensive, any loss of production due to equipment unavailability strongly impairs the company profit. This new environment has forced managers and engineers to optimise all sectors involved in their organizations.

Maintenance, as a system, plays a key role in achieving organizational goals and objectives. It contributes to reducing costs, minimizing equipment downtime, improving quality, increasing productivity, and providing reliable equipment that are safe and well configured to achieve timely delivery of orders to costumers. In addition, a maintenance system plays an important role in minimizing equipment life cycle cost. To achieve the target rate of return on investment, plant availability and equipment effectiveness have to be maximized.

Grag and Deshmukh had recently review the literature on maintenance management and point out that, next to the energy costs, maintenance costs can be the largest part of any operational budget [6].

Based on a survey on enabling technologies to improve the performance of Flexible Manufacturing Systems (FM S), conducted by a CIRP Working Group on “Flexible Automation-Assessment and Future” in collaboration with the ERC for reconfigurable manufacturing systems, mentioned by [10], it is also revealed that industry considers the cost of maintenance as the second more important critical factor for the success of large FMS. This shows a very low level of industry satisfaction due to the high cost of maintenance of FMS and their disappointment with the low level of availability of the systems, when compared with the expectation when those systems were installed.

Maintenance activities represent an increasingly high cost in any industry or structure. The decision making for effective maintenance is increasing in complexity with the increase in size of the systems and distant locations of customers and also due to the several independent sources of information [10]. Due to need to keep their competitive position customers are demanding improved **system availability**, safety, sustainability, cost-effectiveness and operational flexibility.

In recent years there has been tremendous interest from researchers on modeling policies of preventive maintenance for multi-component systems.

Objective functions, constraints and resolution techniques are quite different in the various models proposed.

Paradoxically the issue of minimizing maintenance costs subject to availability constraints has received much less attention compared to reliability constraints.

Bris developed a single objective optimization model, considering the system components periodically inspected and maintained, aiming to find out the optimal maintenance policy for each element by minimizing the cost function and respecting the availability constraint [1]. The authors propose an algorithm based on the time dependent Birnbaum importance factor and using Monte Carlo simulation and genetic algorithms.

Galante propose an exact algorithm in order to single out the set of components that must be maintained to guarantee a required reliability level up to the next planned stop with the minimum cost [5].

Laggoune present a preventive maintenance approach for a multi-component series system subjected to random failures [7]. They developed an algorithm allowing for combined preventive/corrective/opportunistic replacement of the system components. However the major concern is in economic dependence, so there is no assessment about availability improvement.

Two non-linear mixed-integer optimization models for preventive maintenance and replacement scheduling of multi-components systems are presented by [9]. These models seek to minimize the total cost subject to achieving some minimal reliability and maximize the total reliability of the system subject to a budgetary constraint.

Lin and Wang present a hybrid genetic algorithm to optimize the periodic preventive maintenance model in a series-parallel system [8]. This algorithm is based on the intrinsic properties of periodic preventive maintenance, including the structure of the reliability block diagrams, maintenance priorities of components, and their maintenance periods. The effect on system reliability of a component using the importance measure is used to determine the combination of important components in system. The maintenance periods of the important components are further optimized to minimize total maintenance cost.

A model to minimize the periodic preventive maintenance cost for a series-parallel system using an improved particle swarm optimization is proposed by [13]. The optimal maintenance periods for all components are determined efficiently but the major constraint is still a value for reliability.

Nourelfath have developed an integrated production and preventive planning model for multi-component systems [12]. They developed a non linear mixed programming model taking into account inter-dependence between PM planning and production planning. The integrated objective is to minimize the sum of the total production, and the maintenance costs. Nourelfath and Châtelet extended that model by taking into account the presence of economic dependence and common cause failures in parallel systems [11]. The objective function is a non linear equation still minimizing the sum of maintenance and production costs, while satisfying the demand for all products over the entire horizon. The constraints are related to the dynamics of the inventory and the backorder, the capacity, the setup and the available total maintenance time.

Certa propose a multi-objective approach to find out an optimal periodic maintenance policy for a repairable and stochastically deteriorating multi-component system over a finite time horizon [2]. The aim of this approach is to single out the elements set to replace at each scheduled inspection so that the minimization of both the total maintenance cost and the global unavailability time of the system is ensured. As these authors say these two objective functions are contrasting to each other and therefore, it is not possible to find a single solution corresponding to the best result for all of the two considered objectives but a set of nondominated trade-off solutions.

This paper presents an algorithm to solve the problem of maintenance management of a two state parallel-series system based on preventive maintenance over the different system components. It is assumed that all components of the system exhibit Weibull hazard function and constant repair rate and that preventive maintenance would bring the system to the as good as new condition. The algorithm calculates the interval of time between preventive maintenance actions for each component, minimizing the costs, and in such a way that the total downtime, in a certain period of time, does not exceed a predetermined value.

The paper is organized as follows. In the next section the problem is mathematically formulated. Sections 3 and 4 present a numerical example solved by the proposed algorithm. An industrial case study is presented in section 5 where the results show the effectiveness of the proposed approach. Some possible extensions and remarks are discussed in the conclusion.

2. Reliability and availability of two-state systems

In this paper we are especially interested in a two-state parallel-series system. To define it, we assume that

$$C_{ij},\ i = 1,2,...,n,\ \text{and}\ j = 1,2,...,m,\ m,n \in N,$$

are two-state components of the system having reliability functions

$$R_{ij}(t) = P(T_{ij} > t),\ t \in (-\infty, \infty),$$

where

$$T_{ij},\ i = 1,2,...,n,\ \text{and}\ j = 1,2,...,m$$

are independent random variables representing the lifetimes of components C_{ij} with distribution functions

$$F_{ij}(t) = P(T_{ij} \le t),\ t \in (-\infty, \infty).$$

Definition. We call a two-state parallel-series system if its lifetime T is given by

$$T = \max_{1 \le i \le k_n}\{\min_{1 \le j \le l_i}\{T_{ij}\}.$$

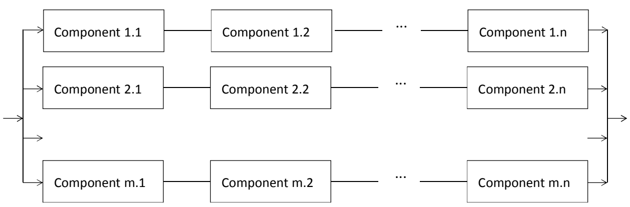

Figure 1. Scheme of a two-state parallel-series system

The reliability function of the two-state parallel-series system is given by

$$\boldsymbol{R}_{m,n}(t) = 1 - \prod_{i=1}^{m}[1 - \prod_{j=1}^{n} R_{ij}(t)],\ t \in (-\infty, \infty), \qquad (3)$$

where m is the number of series subsystems linked in parallel and n are the numbers of components in the series subsystems.

If all components are identical and the reliability of a single unit is $R(t)$, then the reliability of the system becomes

$$\boldsymbol{R}_{m,n}(t) = 1 - (1 - R^n(t))^m.$$

In the papers of Duarte [3], [4], are presented algorithms to determine the interval time between preventive maintenance tasks (assuming that the system is restored to the "as good as new" condition after each maintenance operation) in such a way that the availability of the system is no lesser than A.

The main idea for the solution of this problem consists of determining the time interval during which the increasing hazard rate can be substituted by a constant failure rate in order to guarantee a pre-determinate availability level.

In those papers, another algorithm is developed to solve the problem of maintenance management of a series system based on preventive maintenance over the different system components. It's assumed that all components of the system still exhibit increasing hazard rate and constant repair rate and that preventive maintenance would bring the system to the as good as new condition. It's defined a cost function for maintenance tasks (preventive and corrective) for the system. The algorithm calculates the interval of time between preventive maintenance actions for each component, minimizing the costs, and in such a way that the total downtime, in a certain period of time, does not exceed a predetermined value.

We follow the same approach to solve a similar problem but now applied to a series-parallel system.

We are especially interested in a *k-out-of-n* system.

A *k-out-of-n* redundant system is a parallel configuration where k of the system components, as a minimum, are required to be fully operational at the completion time T of the mission, for the system to "succeed" (for $k = 1$ it reduces to a parallel system; for $k = n$, to a series one).

Our goal is to calculate k vectors

$$[\tau_{i1} \quad \tau_{i2} \quad \tau_{i3} \quad \cdots \quad \tau_{in}]^T,\ i = 1, ..., m,$$

in such a way that the total down time of each of k parallel branches in a certain period of time does not exceed a predetermined value, that is to say, that it guarantees the specified service level and simultaneously minimizes the maintenance costs.

We assume that each component has a linearly increasing hazard-rate function,

$$h_{ij}(t) = a_{ij}t, a_{ij} > 0,$$

and a constant repair rate

$$m_{ij}(t) = m_{ij}.$$

The cost of each preventive maintenance task is cmp_{ij} and the cost of each corrective maintenance task is cmc_{ij}.

Since the availability of the system consisting of m components in parallel requires that at least k units must be available (assuming that components' failures are independent), system availability A_{ij} is

$$A = \prod_{i=1}^{n} A_i$$

where A_{ij} is the availability of component ij.

Applying proposition presented in section 3 we can write that the availability of each component ij is A_{ij} over the interval

$$\left[0, \frac{2}{a_{ij}} \frac{m_{ij}\left(1-A_{ij}\right)}{A_{ij}}\right].$$

and its hazard function can be approximated by the constant function

$$h_{ij}(t) = \frac{m_{ij}\left(1-A_{ij}\right)}{A_{ij}}.$$

Then, the expected number of failures in that time interval is

$$\frac{2}{a_{ij}} \frac{m_{ij}\left(1-A_{ij}\right)}{A_{ij}} \times \frac{m_{ij}\left(1-A_{ij}\right)}{A_{ij}} = \frac{2}{a_{ij}} \frac{m_{ij}^{\;2}\left(1-A_{ij}\right)^2}{A_{ij}^{\;2}}$$

The objective function for each i (i=1,…,m) parallel branch (defined as a cost function per unit time) is

$$c_i(A_{i1}, A_{i2}, \dots, A_{in}) =$$
$$= \sum_{j=1}^{n} \left[\frac{cmp_{ij}}{\frac{2}{a_{ij}} \frac{m_{ij}\left(1-A_{ij}\right)}{A_{ij}} + TTP} + \frac{\left(\frac{2}{a_{ij}} \frac{m_{ij}^{\;2}\left(1-A_{ij}\right)^2}{A_{ij}^{\;2}}\right) cmc_{ij}}{\frac{2}{a_{ij}} \frac{m_{ij}\left(1-A_{ij}\right)}{A_{ij}} + TTP} \right]$$

subject to

$$\begin{cases} \prod_{j=1}^{n} A_{ij} \geq A, \\ 0 < A_{ij} < 1,\; j = 1,2,\dots,n. \end{cases}$$

Minimizing these functions and sorting them in ascending order of cost value, the first k will define what components should be active. The other ones could be on standby and their maintenance plan must be reassessed.

3. A numerical example

The model described on section 2 was implemented to a parallel-series system. In this example there are *4* series subsystems linked in parallel and *3* are the numbers of components in the series subsystems.

Data is presented on *Table 1*.

Table 1. Initial conditions

			Coefficient of hazard function	Preventive maintenance cost	Corrective maintenance cost	Time to repair	Constant repair rate m(t)=m constante	Time to perform preventive maintenance
			a	PMC	CMC	TTR	m	TTP
Parallel 1	Series components	1	5,00E-07	2000	4000	100	0,010	10
		2	5,70E-07	2500	5000	50	0,020	40
		3	7,97E-06	1000	2000	80	0,013	10
Parallel 2	Series components	1	3,50E-06	1200	2000	40	0,025	10
		2	4,53E-07	1600	3000	80	0,013	20
		3	5,00E-07	1750	3200	10	0,100	4
Parallel 3	Series components	1	5,70E-07	2500	5000	50	0,020	40
		2	4,53E-07	1600	3000	80	0,013	20
		3	2,50E-06	2000	4000	100	0,010	15
Parallel 4	Series components	1	3,75E-07	2000	4000	35	0,029	10
		2	4,53E-06	1600	3000	70	0,014	12
		3	4,50E-07	1750	3200	10	0,100	4

The target for availability is 90%.

We have applied the tool "*SOLVER*" of Excel to solve the optimization problem and the solution we got**Błąd! Nie można odnaleźć źródła odwołania.** is presented in *Table 2*.

Table 2. Results of Solver optimization

			Time between two consecutive preventive maintenance tasks - τ	Optimized availability	Cost function	Cost function	Series system availability
Parallel 2	Series components	1	66	96,13%	4,03	**8,4**	93,22%
		2	255	97,33%	2,06		
		3	1852	99,63%	2,36		
Parallel 4	Series components	1	705	98,95%	2,43	**11,1**	91,72%
		2	29	93,02%	6,43		
		3	2058	99,65%	2,24		
Parallel 3	Series components	1	325	98,31%	3,67	**12,0**	90,00%
		2	255	97,69%	2,08		
		3	37	93,71%	6,23		
Parallel 1	Series components	1	185	97,58%	2,98	**12,8**	90,00%
		2	325	98,49%	3,72		
		3	15	93,65%	6,11		

If we are dealing with a *2-out-of-4* system the maintenance plan for the 2 operational series (parallel 2 and parallel 4) is defined. Maintenance plans for the others parallels should be reassessed.

4. A case study

The model we have presented was also applied to a subsystem of a production line of a factory of textile industry. *Figure 2* shows a winder system, from which it was extracted the parallel-series subsystem. The number of components connected in series is 10 whereas the number of parallel paths is 6.

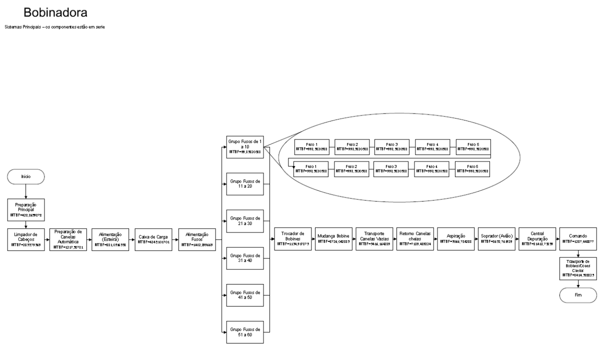

Figure 2. The winder system

It's assumed that each component has the following hazard function

$$h_{ij}(t) = \frac{\beta_{ij}}{\theta_{ij}}\left(\frac{t}{\theta_{ij}}\right)^{\beta_{ij}-1}, \theta_{ij} > 0, \beta_{ij} > 0, t \geq 0,$$
$$i = 1,\dots,6,\ j = 1,\dots,10$$

The values of β parameter are

$$\beta_{ij} = 1, \begin{cases} i = 1,\dots,6 \\ j = 1,\dots,10. \end{cases}$$

This means that the hazard function is linear, of the type

$$h_{ij}(t) = a_{ij}t, a_{ij} > 0, \begin{cases} i = 1,\dots,6 \\ j = 1,\dots,10. \end{cases}$$

It's also assumed that the repair rate, $m_{ij}(t)$, of each component is constant and equal to m_{ij}.
To achieve the desired output level (according to the nominal equipment rate) this subsystem must guarantee an availability of 90%.
Although the system is composed of a set of six sub-systems in parallel, they must all be in a state of good functioning. Indeed, the failure of any of the branches does not prevent nor the proper functioning of the system or the level of output. However it requires that the other branches of the parallel, work on a highest rate, with all the negative consequences resulting therefrom, in particular, the increase of their rate of degradation.
So, the first constraint to the objective function will be

$$\begin{cases} \prod_{j=1}^{10} A_{ij} \geq 0.90, i = 1,\dots,6 \\ 0 < A_{ij} < 1, \begin{cases} i = 1,\dots,6 \\ j = 1,\dots,10 \end{cases} \end{cases}.$$

Data is presented on *Table 3*. The nomenclature is as follows.
a_{ij} – coefficient of hazard function.
TTR_{ij} – Mean Time to Repair (corrective maintenance).
TTP_{ij} – Time to perform one preventive maintenance task.
PMC_{ij} – Preventive maintenance mean cost.
CMC_{ij} – Corrective maintenance mean cost.
τ_{ij} - time between two consecutive preventive maintenance tasks.

Table 3. Input data

			Coefficient of hazard function	Preventive maintenance cost	Corrective maintenance cost	Time to repair	Constant repair rate m(t)=m constante	Time to perform preventive maintenance	Time between two consecutive preventive maintenance tasks
			a	PMC	CMC	TTR	m	TTP	τ
Parallel 1	Series components	1	1,60E-06	2000	4000	40	0,025	10	1000
		2	2,45E-06	1600	3000	80	0,013	20	1000
		3	1,94E-06	1750	3200	10	0,100	4	1000
		4	2,17E-06	1700	3400	30	0,033	12	1000
		5	3,30E-06	1100	1800	20	0,050	8	750
		6	1,74E-06	2500	4500	25	0,040	8	1500
		7	6,28E-06	450	750	12	0,083	4	750
		8	4,09E-06	750	1250	12	0,083	5	750
		9	2,79E-06	1000	1750	18	0,056	6	1250
		10	1,28E-05	200	350	12	0,083	3	500

The target for availability is set to 90%.
The tool "SOLVER" of Excel was also applied to solve the optimization problem and the solution we got is presented in *Table 4*. It must be notice that in this case maintenance plan is the same for all branches in parallel.

5. Conclusion

This paper deals with a maintenance optimization problem for a parallel-series system. Based on an algorithm previously developed we have developed another one to optimize maintenance management of a parallel-series system based on preventive maintenance over the different system components.

Table 4. Results of Solver optimization

			Time between two consecutive preventive maintenance tasks - T - hours	Optimized availability	Cost function	Cost function	Series system availability
Model 1	Critical components	1	424	96.66%	5.9	46.6	[illegible]
		2	[illegible]	[illegible]	[illegible]		
		3	867	99.48%	1.8		
		4	377	96.79%	[illegible]		
		5	374	96.96%	1.3		
		6	520	96.66%	[illegible]		
		7	197	99.26%	[illegible]		
		8	296	99.26%	3.2		
		9	[illegible]	[illegible]	[illegible]		
		10	97	99.26%	2.2		

We assume that all components of the system still exhibit linearly increasing hazard rate and constant repair rate and that preventive maintenance would bring the system to the as good as new condition. We define a cost function for maintenance tasks (preventive and corrective) for the system. The algorithm calculates the interval of time between preventive maintenance actions for each component, minimizing the costs, and in such a way that the total downtime, in a certain period of time, does not exceed a predetermined value. The maintenance interval of each component depends on factors such as failure rate, repair and maintenance times of each component in the system. In conclusion, the proposed analytical method is a feasible technique to optimize preventive maintenance scheduling of each component in a parallel-series system.

Currently we are developing a software package for the implementation of the algorithm presented in this paper.

Extensions of this approach to series-parallel systems are also under consideration.

References

[1] Bris, R., Chãtelet, E. & Yalaoui, F. (2003). New method to minimize the preventive maintenance cost of series-parallel systems. *Reliability Engineering and System Safety* 82, 247-255.

[2] Certa, A., Enea, M., Galante, G. & Lupo, T. (2012). A multi-objective approach to optimize a periodic maintenance policy. *International Journal of Reliability, Quality and Safety Engineering* Vol.19, No 6, 1240002 (16 pages).

[3] Duarte, J.C., Craveiro, J.T. & Trigo, T. (2006). Optimization of the preventive maintenance plan of a series components system. *International Journal of Pressure Vessels and Piping* 83, 244-248.

[4] Duarte, J.C. & Soares, C.G. (2007). Optimization of the preventive maintenance plan of a series components system with Weibull hazard function. Proc. of SSARS 2007 – European Safety and Reliability Conference, Tri City, Poland.

[5] Galante, G. & Passannanti, G. (2009). Na exact algorithm for preventive maintenance planning of series-parallel systems. *Reliability Engineering and System Safety* 94, 1517-1525.

[6] Garg, A. & Deshmukh, S.G. (2006). Maintenance management: literature review and directions. *Journal of Quality in Maintenance Engineering*. 12 No. 3, 205–238.

[7] Laggoune, R., Chateauneuf, A. & Aissani, D. (2009). Opportunistic policy for optimal preventive maintenance of a multi-component system in continuous operating units. *Computers and Chemical Engineering* 33, 1499-1510.

[8] Lin, W. & Wang, H. (2012). A hybrid genetic algorithm to minimize the periodic preventive maintenance cost in a series-parallel system. *Journal of Intelligent Manufacturing* 23, 1225-1236.

[9] Moghaddam, K.S. & Usher, J.S. (2011). Preventive maintenance and replacement scheduling for repairable and maintainable systems using dynamic programming. *Computers & Industrial Engineering* 60, 654-665.

[10] Ni, J. & Jin, X. (2012). Decision support systems for effective maintenance operations. *CIRP Annals - Manufacturing Technology* 61, 411 – 414.

[11] Nourelfath, M. & Châtelet, E. (2012). Integrating production, inventory and maintenance planning for a parallel system with dependent components. *Reliability Engineering and System Safety* 101, 59-66.

[12] Nourelfath, M., Fitouhi, M. & Machani, M. (2010). An Integrated Model for Production and Preventive Maintenance Planning in Multi-State Systems. *IEEE Transactions on Reliability*, Vol. 59, No 3, 496-506.

[13] Wang, H. & Lin, W. (2011). Improved particle swarm optimization to minimize periodic preventive maintenance cost for series-parallel systems. *Expert Systems with Applications* 38, 8963-8969.