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The fuzzy method in the compound conformity criteria of concrete strength as quality and safety control

Keywords

quality, safety, fuzzy method, operating-characteristic curve, concrete, compliance criteria

Abstract

Non-random uncertainty character connected with the evaluation and classification of concrete compressive strength, and also faults and disadvantages of statistical compliance criteria, were reasons to formulate a new fuzzy criterion of concrete strength assessment and classification procedure of produced concrete. Membership functions of mean value of compressive strength, included in conformity criteria, have been determined in the contribution on the basis of fuzzy-statistical experiment.

1. Introduction

The quality of built-in concrete decides about safety, durability and time of operational use of concrete engineering objects. The development of contemporary concrete technology and application of admixtures and additives for concrete offer great opportunities of fulfilling diverse and high requirements put to this material. But it also brings new difficulties and problems, which undoubtedly increase the construction costs. New concrete structures are expected to be safe and durable, and the terms of the first repair should be maximally removed in time.

The PN-EN 206-1:2003 [3] code defines the properties of concrete and fresh concrete and verification methods of these parameters, as well as production control procedures. There are two areas where quality requirements have been established [1]:

- hardened concrete
- fresh concrete.

The decision on the conformity or inconformity of strength is based on a comparison of the samples test results with the compliance criteria. The ascertainment of conformity of the tested compressive strength with the accepted criterion may be based on [1, 2, 3]:

- accepted statistical quality control plan (SQCP),
- fuzzy-statistical and fuzzy methods.

Quantitative criteria, used to assess the conformity of concrete compressive strength, recommended in the code [3] for continuous production have the following form:

- for the small sample size n = 3:

$$f_{cm} \geq f_{ck} + 4 \;, \quad f_{ci} \geq f_{ck} - 4 \tag{1}$$

- for sample size n ≥ 15:

$$f_{cm} \geq f_{ck} + 1{,}48\sigma \;, \quad f_{ci} \geq f_{ck} - 4 \tag{2}$$

Compound compliance criteria of type (1) raises many questions, so the performed analysis with use of fuzzy-statistical methods refer to the sample size n = 3. The paper presents analysis and assessment of the quality and safety associated with use of codes compliance criteria for compressive strength of single kinds of ordinary concrete. The Monte Carlo random simulation method and fuzzy sets theory with use of fuzzy-statistical methods have been applied for the analysis.

2. Fuzzy-statistical compliance criteria

Compressive strength of concrete f_c, fulfilling the compound compliance criterion can be inscribed by using a fuzzy set:

$$f_c = [\mu_{f,C}(f_{cm}), f_c], \;\; \mu_{f,C} : F_c \to [0,1] \tag{3}$$

Where: $\mu_{f,C}(f_{cm})$ is the membership function assigning degree of the fuzzy set membership f_c (from the interval [0,1]) to each element of the strenght set $f_c \in F_c$

The code compound criterion of conformity of produced concrete lot with projected class may have the following form:

- for sample size n = 3:

$$\left.\begin{aligned} f_{cm} &\geq f_{ck} + 4 \\ f_{ci} &\geq f_{ck} - 4 \end{aligned}\right\} = K \tag{4}$$

- for sample size n = 15:

$$\left.\begin{aligned} f_{cm} &\geq f_{ck} + 1{,}48\sigma \\ f_{ci} &\geq f_{ck} - 4 \end{aligned}\right\} = K \tag{5}$$

Where: K is the fuzzy value (of $\mu_{fc}(f_{cm})$ membership function), which should be established for determined classes of concrete on the ground the fuzzy-statistical experiment.

2.1. Fuzzy membership of compressive strength

The three-phase method (fuzzy-statistical) has been used to determine membership functions of test characteristics [6, 7]. Random variables ξ and η were defined. Every experiment determines a pair of numbers ξ and η, where ξ is demarcation point for considered and lower class of concrete and η is demarcation point for considered and higher class concrete. Variable (ξ, η) can be assumed as a two-dimensional random variable. Then trough sampling it is possible to obtain $p_\xi(f_{cm})$ and $p_\eta(f_{cm})$ as marginal probability distributions. In general, ξ and η follow a normal distribution $N(m_\xi,\sigma_\xi)$ and $N(m_\eta,\sigma_\eta)$.

The membership function for i-class of concrete can be described below:

$$\mu_{Ci}(f_{cm}) = \int_{-\infty}^{f_{cm}} p_\eta(f_{cm})df_{cm} = F\left(\frac{f_{cm} - m_\eta}{\sigma_\eta}\right) \tag{6}$$

The membership function for considered and less i-class of concrete can be described by the following formula:

$$\mu_{Ci-1}(f_{cm}) = \int_{f_{cm}}^{+\infty} p_\xi(f_{cm})df_{cm} = 1 - F\left(\frac{f_{cm} - m_\xi}{\sigma_\xi}\right) \tag{7}$$

Contrast, the fuzzy membership functions considered and higher i- class of concrete has follow form:

$$\mu_{Ci+1}(f_{cm}) = 1 - \int_{f_{cm}}^{+\infty} p_\xi(f_{cm})df_{cm} - \int_{-\infty}^{f_{cm}} p_\eta f(f_{cm}) \tag{8}$$

$$\mu_{Ci+1}(f_{cm}) = 1 - \left[1 - F\left(\frac{f_{cm} - m_\xi}{\sigma_\xi}\right)\right] - F\left(\frac{f_{cm} - m_\eta}{\sigma_\eta}\right) \tag{9}$$

The final form of the formula is:

$$\mu_{Ci+1}(f_{cm}) = F\left(\frac{f_{cm} - m_\xi}{\sigma_\xi}\right) - F\left(\frac{f_{cm} - m_\eta}{\sigma_\eta}\right) \tag{10}$$

Where:

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp(-0{,}5z^2)dz \tag{11}$$

With the membership functions for different classes of concrete, and the mean value of compressive strength for neighboring classes estimated based on random simulation, we can calculate the degree of membership values of the considered batch of concrete to different classes. Depending on the value of $\mu_{f,Ci}(f_{cm})$, we can decide on the inclusion of lots of concrete to the appropriate class of concrete. This decision may be more or less conservative, depending on its impact on the qualitative assessment of the produced concrete and the impact on the requirements of safety, quality and economics.

3. Numerical example

Compliance criteria given in the formulas (1) were performed by the computations method: generating 100 000 random groups of size *n=3* in accordance with normal distribution, generating class of concrete (concrete classes of the three neighboring classes of concrete C_{i-1}, C_i, C_{i+1} with 1/3 probability), generating standard deviation and defective fraction. To generate random numbers with standard normal distribution, the method Box and Muller [5] was used. The table of probability distribution of random vector (ξ, η) and the histogram of marginal probability distributions, for the considered and lower class of concrete and one for the considered and higher class of concrete, were built. Graphs of the density function of boundary probability distributions $p_\xi(f_{cm})$ and $p_\eta(f_{cm})$ are the basis for the designation of membership function for each class of concrete.

On the basis of simulations for the concrete class C16/25, generating 100 000 random groups of size n = 3 in accordance with normal distribution, marginal density functions of distributions and fuzzy membership functions were estimated for each class of concrete.

The marginal probability distributions $p_{\xi}(x)$, $p_{\eta}(x)$ and the plot of original and modified membership function of concrete class C16/20 and neighboring classes (C12/15, C20/25) are presented by the *Figure 1*.

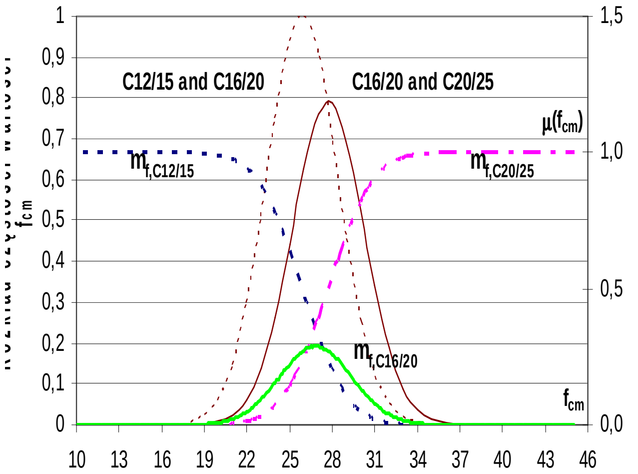

Figure 1. The membership function of f_{cm} concrete classes for concrete class C12/15, C16/20 and C20/25

The plot for membership function of concrete class does not accept value 1,0 [*Figure 1*]. This suggests that concrete class division is too numerous.

The analysis was conducted for the second-class concrete getting the results of the confirmatory foundation about too numerous classes of concrete.

Class of concrete C16/20 (sample size *n=3*, $\mu_{,Ci}(f_{cm})$) were considered to estimate the membership function of the fuzzy value of concrete classes. Assuming the normal distributions of demarcation points $\xi \rightarrow N(m_{\xi}, \sigma_{\xi})$ and $\eta \rightarrow N(m_{\eta,}, \sigma_{\eta})$ of concrete class C8/10 and C16/20, and C16/20 and C25/30, respectively, mean values m_{ξ} = 10,19 MPa, m_{ξ} = 21,72 MPa and standard deviation σ_{ξ}=3,29 MPa and σ_{ξ}=2,18 MPa were estimated. Then the membership function of value of test coefficient for concrete class C16/20 was calculated according to formulas 6, 7, 10 and 11 which is presented on the Fig. 2.

We can accept batch of concrete, with values of:

- f_{cm}=15,8 MPa compressive strength with the confidence level 0,82 to the concrete class C16/20
- f_{cm} as values from intervals (10; 15,8) MPa with the confidence level from interval (0,5; 0,82) to the concrete class C16/20 or to C8/10,
- f_{cm} as values from intervals (15,8; 22) MPa with the confidence level from interval (0,08; 0,5) to the concrete class C25/30.

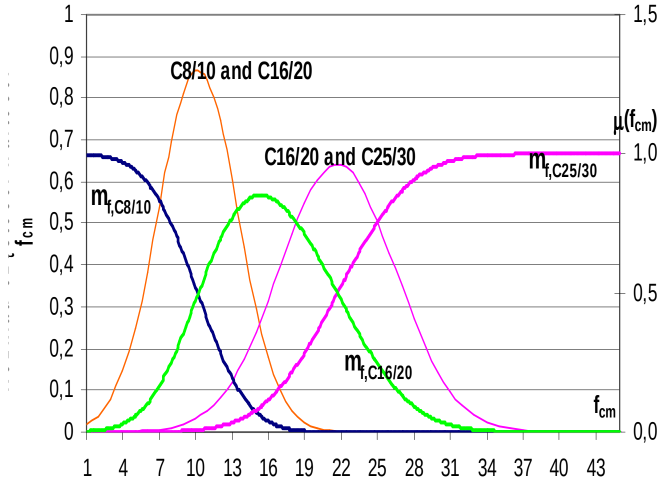

Figure 2. The membership function of f_{cm} concrete classes for concrete class C8/10, C16/20 and C25/30 (the analysis for the second-class concrete)

3. Conclusions

Uncertainties related to the assessment and classification of concrete strength and attempts to apply a fuzzy measures of safety in the designing and analysis of building structures are the reasons of the formulation of new fuzzy-statistical classification procedures for produced concrete. Defects and deficiencies of the statistical criteria of conformity evaluation gives additional reason for this solution.

Fuzzy-statistical and fuzzy methods allow taking into account the opposing requirements of safety, quality, economy. The inclusion of these requirements is possible by determination of test characteristic value with degree of membership less than unity.

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