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# Reliability prediction and optimization of complex technical systems with application in port transport

**Keywords**

complex system, reliability, operation process, prediction, optimization

**Abstract**

There are presented the methods of the reliability prediction and optimization of complex technical systems related to their operation processes. The general model of the reliability of complex technical systems operating at variable operation conditions linking the semi-Markov modeling of their operation processes with the multi-state approach to their reliability analysis and the linear programming are applied in maritime industry to the reliability and risk prediction and optimization of the container gantry crane.

## 1. Introduction

Most real technical systems are very complex. Large numbers of components and subsystems and their operating complexity cause that the identification, prediction and optimization of their reliability are complicated. The complexity of the systems' operation processes and their influence on changing in time the systems' structures and their components' reliability parameters is often very difficult to fix and to analyze and simultaneously frequently met in real practice.

The convenient tools for analyzing these problems are semi-Markov modelling of the system operation processes [2], [9] linked with multistate approach for the system reliability analysis [8], [10] and a linear programming for the system reliability optimization [4], [6]. An application of the proposed approach to reliability analysis and optimization of a container gantry crane is presented in this paper.

## 2. Complex technical system reliability analysis

### 2.1. Modelling complex technical systems operation processes

In analyzing the operation process of the complex technical system with the distinguished operation states $z_1, z_2, \ldots, z_\nu$, the semi-Markov process may be used to construct its general probabilistic model [7], [10]. To build this model the following parameters are defined:

- the vector of probabilities $[p_b(0)]_{1\times\nu}$ of the system operation process initials operation states;
- the matrix of probabilities $[p_{bl}]_{\nu\times\nu}$ of the system operation process transitions between the operation states;
- the matrix of conditional distribution functions $[H_{bl}(t)]_{\nu\times\nu}$ of the system operation process conditional sojourn times $\theta_{bl}$ at the operation states.

To describe the system operation process conditional sojourn times at the particular operation states the uniform distribution, the triangle distribution, the double trapezium distribution, the quasi-trapezium distribution, the exponential distribution, the Weibull distribution, the normal distribution and the chimney distribution are suggested as suitable [7].

Under these definitions and assumptions, the following main characteristics of the system operation process can be predicted:

- the vector $[H_b(t)]_{1x\nu}$, of the unconditional distribution functions of the sojourn times $\theta_b$ of the system operation process at the operation states

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t),\ b = 1,2,...,v; \quad (1)$$

- the vector $[M_b]_{1xv}$, of the mean values of the unconditional sojourn times $\theta_b$

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}\ \ b = 1,2,...,v, \quad (2)$$

where $M_{bl}$ are defined by the formula

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t),\ \ b,l = 1,2,...,v,\ \ b \neq l;$$

- the vector $[p_b]_{1xv}$ of the limit values of the transient probabilities $p_b(t)$ at the particular operation states

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l},\ b = 1,2,...,v, \quad (3)$$

where $M_b$, $b = 1,2,...,v,$ are given by (2), while the steady probabilities $\pi_b$ of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1; \end{cases}$$

- the vector $[\hat{M}_b]_{1xv}$ of the mean values of the total sojourn times $\hat{\theta}_b$ at the particular operation states for sufficiently large operation time $\theta$

$$\hat{M}_b = E[\hat{\theta}_b] = p_b \theta,\ b = 1,2,...,v, \quad (4)$$

where $p_b$ are given by (3).

## 2.2. Modelling reliability of multistate systems with ageing components

In the systems' reliability analysis it is practically reasonable to expand their two-state models to the multi-state models [1], [3], [5], [11]. Introducing the multi-state approach to reliability analysis of systems with ageing components we have to accept certain assumptions [5] that are as follows:

- $n$ is the number of the system components,
- $E_i$, $i$ = 1,2,...,$n$, are components of a system,
- all components and a system under consideration have the reliability state set {0,1,...,$z$}, $z \geq 1$,
- the reliability states are ordered, the reliability state 0 is the worst and the reliability state $z$ is the best,
- $T_i(u)$, $i$ = 1,2,...,$n$, are independent random variables representing the lifetimes of components $E_i$ in the reliability state subset {$u$,$u$+1,...,$z$}, while they were in the reliability state $z$ at the moment $t$ = 0,
- $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset {$u$,$u$+1,...,$z$} while it was in the reliability state $z$ at the moment $t$ = 0,
- the system states degrades with time $t$,
- $E_i(t)$ is a component $E_i$ reliability state at the moment $t$, $t \in < 0, \infty)$, given that it was in the reliability state $z$ at the moment $t$ = 0,
- $S(t)$ is a system $S$ reliability state at the moment $t$, $t \in < 0, \infty)$, given that it was in the reliability state $z$ at the moment $t$ = 0.

The above assumptions mean that the reliability states of the system with ageing components may be changed in time only from better to worse. The way in which the components and the system reliability states change is illustrated in *Figure 1*.

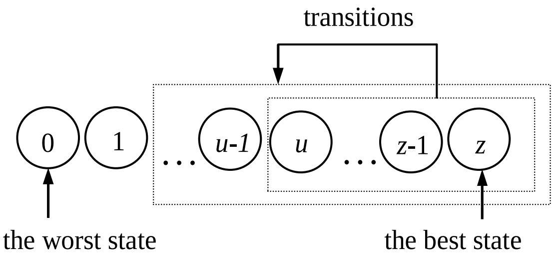


*Figure 1.* Illustration of a system and components reliability states changing

Under these assumptions, the following multi-state system reliability characteristics may be introduced and determined:

- the component multi-state reliability function

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1),...,R_i(t,z)],\ t \in < 0, \infty),$$

where $R_i(t,u)$, $u$ = 0,1,...,$z$, $i$ = 1,2,...,$n$, is the probability that the component $E_i$ is in the reliability state subset $\{u, u+1,..., z\}$ at the moment $t$, $t \in < 0, \infty)$, while it was in the reliability state $z$ at the moment $t$ = 0,

- the system multi-state reliability function

$$\boldsymbol{R}(t,\cdot) = [\boldsymbol{R}(t,0), \boldsymbol{R}(t,1),...,\boldsymbol{R}(t,z)],\ t \in < 0, \infty),$$

where $\boldsymbol{R}(t,u)$, $u$ = 0,1,...,$z$, is the probability that the system is in the reliability state subset $\{u, u+1,..., z\}$

at the moment $t$, $t \in <0, \infty)$, while it was in the reliability state $z$ at the moment $t = 0$,

- the system risk function $\boldsymbol{r}(t)$ which is the probability that the system is in the subset of reliability states worse than the critical reliability state $r$ while it was in the reliability state $z$ at the moment $t = 0$.

## 2.3. Complex technical systems reliability prediction

Designing the general reliability analytical models of complex multi-state technical systems related to their operation processes, linking their reliability model and their operation processes models and considering variable at different operation states their reliability structures and their components reliability parameters is practically very well justified [10]. Thus, we assume that the changes of the operation process states have an influence on the system multi-state components reliability and the system reliability structure, denoting the conditional reliability function of the system multi-state component $E_i$, $i = 1,2,...,n$, while the system is at the operation state $z_b$ by

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, ..., [R_i(t,z)]^{(b)}],$$

$$t \in <0, \infty), \ b = 1,2,...,v.$$

To predict the complex technical system reliability and risk we determine the following characteristics:

- the conditional reliability functions of the system while the system is at the operational states $z_b$

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1, [\boldsymbol{R}(t,1)]^{(b)}, ..., [\boldsymbol{R}(t,z)]^{(b)},$$

$$t \in <0, \infty), \ b = 1,2,...,v;$$

- the unconditional reliability function of the system

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), ..., \boldsymbol{R}(t,z)], \ t \in <0, \infty), \quad (5)$$

where

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)}, t \geq 0, u = 1,2,...,z; \quad (6)$$

- the mean values of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$

$$\mu(u) \cong \sum_{b=1}^{v} p_b \mu_b(u), \ u = 1,2,...,z, \quad (7)$$

where $\mu_b(u)$, $u = 1,2,...,z$, are the mean values of the system conditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$ while the system is at the operation state $z_b$, $b = 1,2,...,v$, defined by the formula

$$\mu_b(u) = \int_0^\infty [\boldsymbol{R}(t,u)]^{(b)} dt, \ u = 1,2,...,z, \quad (8)$$

and $p_b$ are given by (3);

- the standard deviations $\sigma(u)$, $b = 1,2,...,v$, of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, defined by the formula

$$\sigma^2(u) = 2\int_0^\infty t \ \boldsymbol{R}(t,u) dt - [\mu(u)]^2, \ u = 1,2,...,z; \quad (9)$$

- the mean values of the system unconditional lifetimes in the particular reliability states

$$\bar{\mu}(u) = \mu(u) - \mu(u+1), \ u = 1,2,...,z-1,$$

$$\bar{\mu}(z) = \mu(z), \quad (10)$$

- the system risk function

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,r), \ t \in <0, \infty), \quad (11)$$

- the moment when the risk exceeds a permitted level $\delta$

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (12)$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}(t)$.

## 2.4. Complex technical systems reliability optimization

### 2.4.1. Optimal transient probabilities of complex technical system operation process at operation states

Considering the equation (6), it is natural to assume that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equation (7) for the mean values of the system unconditional lifetimes in the reliability state subsets.

From the linear equation (7), we can see that the mean value of the system unconditional lifetime $\mu(u)$, $u = 1,2,..., z,$ is determined by the limit values of transient probabilities $p_b$, $b = 1,2,...,\nu$, of the system operation process at the operation states given by (3) and the mean values $\mu_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,..., z,$ of the system conditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ given by (8). Therefore, the system lifetime optimization approach based on the linear programming [4], [8]-[9], can be proposed. Namely, we may look for the corresponding optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities $p_b$, $b = 1,2,...,\nu$, of the system operation process at the operation states to maximize the mean value $\mu(u)$ of the unconditional system lifetimes in the reliability state subsets $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ under the assumption that the mean values $\mu_b(u)$, $b = 1,2,...,\nu$, $u = 1,2,..., z,$ of the system conditional lifetimes in the reliability state subsets are fixed. As a special and practically important case of the above formulated system lifetime optimization problem, if $r$, $r = 1,2,..., z,$ is a system critical reliability state, we may look for the optimal values $\dot{p}_b$, $b = 1,2,...,\nu$, of the transient probabilities $p_b$, $b = 1,2,...,\nu$, of the system operation process at the system operation states to maximize the mean value $\mu(r)$ of the unconditional system lifetime in the reliability state subset $\{r, r+1, ,..., z\}$, $r = 1,2,..., z,$ under the assumption that the mean values $\mu_b(r)$, $b = 1,2,...,\nu$, $r = 1,2,..., z,$ of the system conditional lifetimes in this reliability state subset are fixed. More exactly, we may formulate the optimization problem as a linear programming model with the objective function of the following form

$$\mu(r) = \sum_{b=1}^{\nu} p_b \mu_b(r) \qquad (13)$$

for a fixed $r \in \{1,2,..., z\}$ and with the following bound constraints

$$\breve{p}_b \le p_b \le \hat{p}_b, \; b = 1,2,...,\nu, \qquad (14)$$

$$\sum_{b=1}^{\nu} p_b = 1, \qquad (15)$$

where

$$\mu_b(r), \; \mu_b(r) \ge 0, \; b = 1,2,...,\nu, \qquad (16)$$

are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,..., z\}$ and

$$\breve{p}_b, \; 0 \le \breve{p}_b \le 1 \text{ and}$$

$$\hat{p}_b, \; 0 \le \hat{p}_b \le 1, \; \breve{p}_b \le \hat{p}_b, \; b = 1,2,...,\nu, \qquad (17)$$

are lower and upper bounds of the unknown transient probabilities $p_b$, $b = 1,2,...,\nu$, respectively.

Now, we can obtain the optimal solution of the formulated by (13)-(15) the linear programming problem, i.e. we can find the optimal values $\dot{p}_b$ of the transient probabilities $p_b$, $b = 1,2,...,\nu$, that maximize the objective function given by (13).

First, we arrange the system conditional lifetime mean values $\mu_b(r)$, $b = 1,2,...,\nu$, in non-increasing order

$$\mu_{b_1}(r) \ge \mu_{b_2}(r) \ge \ldots \ge \mu_{b_\nu}(r),$$

where $b_i \in \{1,2,...,\nu\}$ for $i = 1,2,...,\nu$.

Next, we substitute

$$x_i = p_{b_i}, \; \breve{x}_i = \breve{p}_{b_i}, \; \hat{x}_i = \hat{p}_{b_i} \text{ for } i = 1,2,...,\nu \qquad (18)$$

and we maximize with respect to $x_i$, $i = 1,2,...,\nu$, the linear form (13) that after this transformation takes the form

$$\mu(r) = \sum_{i=1}^{\nu} x_i \mu_{b_i}(r) \qquad (19)$$

for a fixed $r \in \{1,2,..., z\}$ with the following bound constraints

$$\breve{x}_i \le x_i \le \hat{x}_i, \; i = 1,2,...,\nu, \qquad (20)$$

$$\sum_{i=1}^{\nu} x_i = 1, \qquad (21)$$

where

$$\mu_{b_i}(r), \; \mu_{b_i}(r) \ge 0, \; i = 1,2,...,\nu,$$

are fixed mean values of the system conditional lifetimes in the reliability state subset $\{r, r+1,..., z\}$ arranged in non-increasing order and

$$\breve{x}_i,\ \ 0 \le \breve{x}_i \le 1 \ \text{ and}$$

$$\hat{x}_i,\ \ 0 \le \hat{x}_i \le 1,\ \ \breve{x}_i \le \hat{x}_i,\ \ i = 1,2,...,\nu, \tag{22}$$

are lower and upper bounds of the unknown probabilities $x_i$, $i = 1,2,...,\nu$, respectively.
To find the optimal values of $x_i$, $i = 1,2,...,\nu$, we define

$$\breve{x} = \sum_{i=1}^{\nu} \breve{x}_i,\ \ \hat{y} = 1 - \breve{x} \tag{23}$$

and

$$\breve{x}^0 = 0,\ \ \hat{x}^0 = 0 \ \text{ and}$$

$$\breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i \ \text{ for } I = 1,2,...,\nu. \tag{24}$$

Next, we find the largest value $I \in \{0,1,...,\nu\}$ such that

$$\hat{x}^I - \breve{x}^I < \hat{y} \tag{25}$$

and we fix the optimal solution that maximize (19) in the following way:
i) if $I = 0$, the optimal solution is

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \ \text{ and } \ \dot{x}_i = \breve{x}_i \ \text{ for } i = 2,3,...,\nu; \tag{26}$$

ii) if $0 < I < \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \ \text{ for } i = 1,2,...,I,\ \dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}$$

$$\text{and } \dot{x}_i = \breve{x}_i \ \text{ for } \ i = I+2, I+3,...,\nu; \tag{27}$$

iii) if $I = \nu$, the optimal solution is

$$\dot{x}_i = \hat{x}_i \ \text{ for } i = 1,2,...,\nu. \tag{28}$$

Finally, after making the inverse to (18) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \ \text{ for } \ i = 1,2,...,\nu, \tag{29}$$

that maximize the system mean lifetime in the reliability state subset $\{r, r+1,..., z\}$, defined by the linear form (13), giving its maximum value in the following form

$$\dot{\mu}(r) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(r) \tag{30}$$

for a fixed $r \in \{1,2,..., z\}$.

### 2.4.2. Optimal reliability and safety characteristics of complex technical system

From the expression (30) for the maximum mean value $\dot{\mu}(r)$ of the system unconditional lifetime in the reliability state subset $\{r, r+1,..., z\}$, replacing in it the critical reliability state $r$ by the reliability state $u$, $u = 1,2,..., z$, we obtain the corresponding optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subsets $\{u, u+1,..., z\}$ of the form

$$\dot{\mu}(u) = \sum_{b=1}^{\nu} \dot{p}_b \mu_b(u) \ \text{ for } \ u = 1,2,..., z. \tag{31}$$

Further, according to (5)-(6), the corresponding optimal unconditional multistate reliability function of the system is the vector

$$\dot{\boldsymbol{R}}(t,\cdot) = [1,\ \dot{\boldsymbol{R}}(t,1),\ ...,\ \dot{\boldsymbol{R}}(t,z)\ ], \tag{32}$$

with the coordinates given by

$$\dot{\boldsymbol{R}}(t,u) \cong \sum_{b=1}^{\nu} \dot{p}_b [\boldsymbol{R}(t,u)]^{(b)} \ \text{ for } t \ge 0, \tag{33}$$

$$u = 1,2,..., z.$$

By applying (9), the corresponding optimal values of the variances of the system unconditional lifetimes in the system reliability state subsets are

$$\dot{\sigma}^2(u) = 2\int_0^\infty t\ \dot{\boldsymbol{R}}(t,u)dt - [\dot{\mu}(u)]^2,\ \ u = 1,2,..., z, \tag{34}$$

where $\dot{\mu}(u)$ is given by (31) and $\dot{\boldsymbol{R}}(t,u)$ is given by (33).
And, by (10), the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are

$$\dot{\bar{\mu}}(u) = \dot{\mu}(u) - \dot{\mu}(u+1),\ \ u =,1,..., z-1,$$

$$\dot{\bar{\mu}}(z) = \dot{\mu}(z). \tag{35}$$

Moreover, considering (11) and (12), the corresponding optimal system risk function and the optimal moment when the risk exceeds a permitted level $\delta$, respectively are given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,r),\ \ t \ge 0, \tag{36}$$

and

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta), \tag{37}$$

where $\dot{\boldsymbol{R}}(t,r)$ is given by (33) for $u = r$ and $\dot{\boldsymbol{r}}^{-1}(t)$, if it exists, is the inverse function of the optimal risk function $\dot{\boldsymbol{r}}(t)$.

### 2.4.3. Optimal sojourn times of complex technical system operation process at operation states and operation strategy

Replacing in (3) the limit transient probabilities $p_b$ of the system operation process at the operation states by their optimal values $\dot{p}_b$ found in Section 2.4.1 and the mean values $M_b$ of the unconditional sojourn times at the operation states by their corresponding unknown optimal values $\dot{M}_b$ maximizing the mean value of the system lifetime in the reliability states subset $\{r, r+1, ..., z\}$ defined by (13), we get the system of equations

$$\dot{p}_b = \frac{\pi_b \dot{M}_b}{\sum_{l=1}^{v} \pi_l \dot{M}_l}, \ \ b = 1,2,...,v. \tag{38}$$

After simple transformations the above system takes the form

$$
\begin{aligned}
&(\dot{p}_1 - 1)\pi_1 \dot{M}_1 + \dot{p}_1 \pi_2 \dot{M}_2 + ... + \dot{p}_1 \pi_v \dot{M}_v = 0 \\
&\dot{p}_2 \pi_1 \dot{M}_1 + (\dot{p}_2 - 1)\pi_2 \dot{M}_2 + ... + \dot{p}_2 \pi_v \dot{M}_v = 0 \\
&\cdot \\
&\cdot \\
&\cdot \\
&\dot{p}_v \pi_1 \dot{M}_1 + \dot{p}_v \pi_2 \dot{M}_2 + ... + (\dot{p}_v - 1)\pi_v \dot{M}_v = 0,
\end{aligned}
\tag{39}
$$

where $\dot{M}_b$ are unknown variables we want to find, $\dot{p}_b$ are optimal transient probabilities determined by (29) and $\pi_b$ are steady probabilities defined in Section 2.1.

Since the system of equations (39) is homogeneous and it can be proved that the determinant of its main matrix is equal to zero, then it has nonzero solutions and moreover, these solutions are ambiguous. Thus, if we fix some of the optimal values $\dot{M}_b$ of the mean values $M_b$ of the unconditional sojourn times at the operation states, for instance by arbitrary fixing one or a few of them, we may find the values of the remaining once and this way to get the solution of this equation.

Having this solution, it is also possible to look for the optimal values $\dot{M}_{bl}$ of the mean values $M_{bl}$ of the conditional sojourn times at the operation states using the following system of equations

$$\sum_{l=1}^{v} p_{bl} \dot{M}_{bl} = \dot{M}_b, \ \ b = 1,2,...,v, \tag{40}$$

obtained from (2) by replacing $M_b$ by $\dot{M}_b$ and $M_{bl}$ by $\dot{M}_{bl}$, were $p_{bl}$ are known probabilities of the system operation process transitions between the operation states $z_b$ i $z_l$, $b, l = 1,2,...,v,$ $b \neq l,$ defined in Section 2.1.

Another very useful and much easier to be applied in practice tool that can help in planning the operation processes of the complex technical systems are the system operation process optimal mean values of the total system operation process sojourn times $\hat{\theta}_b$ at the particular operation states $z_b$, $b = 1,2,...,v,$ during the fixed system operation time $\theta,$ that can be obtain by the replacing in the formula (4) the transient probabilities $p_b$ at the operation states $z_b$ by their optimal values $\dot{p}_b$ and resulting in the following expession

$$\dot{\hat{M}}_b = \dot{E}[\hat{\theta}_b] = \dot{p}_b \theta, \ \ b = 1,2,...,v. \tag{41}$$

The knowledge of the optimal values $\dot{M}_b$ of the mean values of the unconditional sojourn times and the optimal values $\dot{M}_{bl}$ of the mean values of the conditional sojourn times at the operation states and the optimal mean values $\dot{E}[\hat{\theta}_b]$ of the total sojourn times at the particular operation states during the fixed system operation time may by the basis for changing the complex technical systems operation processes in order to ensure these systems operation more reliable and safer.

## 3. Container gantry crane reliability prediction and optimization

### 3.1. Container gantry crane description

We analyze the reliability of the container gantry crane that is operating at the container terminal placed at the seashore [8]. The considered container terminal is engaged in trans-shipment of containers. The loading of containers is carried out by using the gantry cranes called Ship-To-Shore (STS).

We consider the STS container gantry crane that is composed of 5 basic subsystems $S_1$, $S_2$, $S_3$, $S_4$

and $S_5$ having an essential influence on its reliability. Those subsystems are as follows:

$S_1$ - the power supply subsystem,

$S_2$ - the control and monitoring subsystem,

$S_3$ - the arm getting up and getting down subsystem,

$S_4$ - the transferring subsystem,

$S_5$ - the loading and unloading subsystem.

The gantry crane power supply subsystem $S_1$ consists of:

- a high voltage cable delivering the energy from the substation to the gantry crane $E_1^{(1)}$,
- a drum allowing the cable unreeling during the crane transferring $E_2^{(1)}$,
- an inner crane power supply cable $E_3^{(1)}$ ,
- a device transmitting the energy from the high voltage cable to the inner crane cable $E_4^{(1)}$,
- main and supporting voltage transformers $E_5^{(1)}$,
- a low voltage power supply cable $E_6^{(1)}$,
- relaying and protective electrical components $E_7^{(1)}$.

The gantry crane control and monitoring subsystem $S_2$ consists of:

- a crane software controller precisely analyzing the situation and takes suitable actions in order to assure correct work of the crane $E_1^{(2)}$,
- a measuring and diagnostic device sending signals about the crane state to the software controller $E_2^{(2)}$,
- a transmitter of signals from the controller to elements executing the set of commands $E_3^{(2)}$,
- devices carrying out the controller's orders (a permission to work, a blockade of work, etc.) $E_4^{(2)}$,
- control panels (an engine room, an operator's cabin, a crane arm cabin) $E_5^{(2)}$,
- control and steering cables' connections $E_6^{(2)}$.

The gantry crane arm getting up and getting down subsystem $S_3$ consists of:

- a propulsion unit (an engine, a rope drum, a transmission gear, a clutch, breaks, a rope) $E_1^{(3)}$,
- a set of rollers and multi-wheels $E_2^{(3)}$,
- a crane arm (joints, hooks fastening the arm) $E_3^{(3)}$.

The gantry crane transferring subsystem $S_4$ consists of:

- a driving unit (an engine, a clutch, breaks, a transmission gear, gantry crane wheels) $E_1^{(4)}$.

The gantry crane loading and unloading subsystem $S_5$ consists of the winch unit $E_1^{(5)}$ composed of:

- a propulsion unit (an engine, a clutch, breaks, a transmission gear, ropes),
- a winch head (which a container grab is connected to),
- a container's grab,
- a container's grab stabilizing unit

and the cart unit $E_2^{(5)}$ composed of:

- a propulsion unit (an engine, a clutch, breaks, a transmission gear, cart wheels, ropes),
- rails which cart is moving on during the operation,
- a crane cart.

## 3.2. Container gantry crane operation process identification

We assume that the container gantry crane reliability structure and its subsystems $S_\upsilon$, $\upsilon = 1,2,3,4,5,$ and components reliability depend on its changing in time operation states [8]. Taking into account expert opinions about the operations process $Z(t)$ of the considered container gantry crane we distinguish the following as its six operation states:

- an operation state $z_1$ – the crane standby with the power supply on and the control system off,
- an operation state $z_2$ – the crane prepared either to starting or finishing the work with the crane arm angle position of 90°,
- an operation state $z_3$ – the crane prepared either to starting or finishing the work with the crane arm angle position of 0°,
- an operation state $z_4$ – the crane transferring either to or from the loading and unloading area with the crane arm angle position of 90°,
- an operation state $z_5$ – the crane transferring either to or from the loading and unloading area with the crane arm angle position of 0°,
- an operation state $z_6$ – the containers' loading and unloading with the crane arm angle position of 0°.

Moreover, we fix that there are possible the transitions between all system operation states.

Hence and on the basis of statistical data coming from experts [8], the probabilities $p_{bl}$, $b,l = 1,2,...,6,$ of the container gantry crane operation process transitions from the operation state $z_b$ into the operation state $z_l$ were fixed and they amount

$$[p_{bl}] =$$

$$\begin{bmatrix} 0 & 0.648 & 0.336 & 0.008 & 0 & 0.008 \\ 0.525 & 0 & 0.373 & 0.093 & 0 & 0.009 \\ 0.105 & 0.111 & 0 & 0 & 0.118 & 0.666 \\ 0.417 & 0.583 & 0 & 0 & 0 & 0 \\ 0.005 & 0 & 0.220 & 0 & 0 & 0.775 \\ 0.012 & 0 & 0.628 & 0 & 0.360 & 0 \end{bmatrix}.$$

The matrix $[h_{bl}(t)]_{6x6}$ of conditional density functions of the container gantry crane operation process conditional sojourn times $\theta_{bl}$ at the operation state $z_b$ while the next transition is into the operation state $z_l$, $b,l = 1,2,...,6,$ was evaluated and they are given in [8]. Knowing these distributions, the conditional mean sojourn times of the container gantry crane at the particular operation states can be evaluated and they are as follows:

$$M_{12} = 456.978,\ M_{13} = 36.860,\ \ M_{14} = 50,\ M_{16} = 3,$$

$$M_{21} = 7.887,\ M_{23} = 9.121,\ M_{24} = 1.545,\ M_{26} = 16,$$

$$M_{31} = 5.5,\ M_{32} = 4.343,\ M_{35} = 6.822,\ M_{36} = 7.857,$$

$$M_{41} = 2,\ M_{42} = 2.143,$$

$$M_{51} = 10,\ M_{53} = 2.899,\ M_{56} = 24.681,$$

$$M_{61} = 22.6,\ M_{63} = 23.117,\ M_{65} = 20.512. \qquad (42)$$

Next, applying (7), the unconditional mean sojourn times of the container gantry crane operation process at the particular operation states can be evaluated and they are as follows:

$$M_1 \cong 308.93,\ M_2 \cong 7.83,\ M_3 \cong 7.09,$$

$$M_4 \cong 2.08, M_5 \cong 19.82,\ M_6 \cong 22.17. \qquad (43)$$

Using those results, according to (3), the limit values of the transient probabilities $p_b(t)$ of the gantry crane operation process at the operation states $z_b$ are

$$p_1 = 0.6874,\ p_2 = 0.0187,\ \ p_3 = 0.0515,$$

$$p_4 = 0.0005,\ p_5 = 0.0717,\ \ p_6 = 0.1702. \qquad (44)$$

The system operation process optimal mean values of the total sojourn times at the particular operation states during the fixed system operation time $\theta = 1$ year = 365 days, after aplying (4), are

$$E[\hat{\theta}_1] = 251 \text{ days},\ E[\hat{\theta}_2] = 7 \text{ days},$$

$$E[\hat{\theta}_3] = 19 \text{ day}, E[\hat{\theta}_4] = 0.2 \text{ day},$$

$$E[\hat{\theta}_5] = 26 \text{ days},\ E[\hat{\theta}_6] = 62 \text{ days}. \qquad (45)$$

## 3.3. Container gantry crane reliability prediction

We assume that subsystems $S_\upsilon$, $\upsilon = 1,2,3,4,5$ are composed of four-state components, i.e. $z$ = 3 and their reliability states are 0, 1, 2 and 3 with the multi-state reliability functions given by the vectors

$$[R_i^{(\upsilon)}(t,\cdot)]^{(b)}$$

$$= [1,\ [R_i^{(\upsilon)}(t,1)]^{(b)}, [R_i^{(\upsilon)}(t,2)]^{(b)}, [R_i^{(\upsilon)}(t,3)]^{(b)}\ ],$$

$$b = 1,2,...,6,$$

where

$$R_i^{(\upsilon)}(t,u)]^{(b)} = P([T_i^{(\upsilon)}(u)]^{(b)} > t | Z(t) = z_b)$$

$$\text{for } t \in < 0,\infty),\ \ u = 1,2,3,\ \ b = 1,2,...,6,$$

is the conditional reliability function standing the probability that the conditional lifetime $[T_i^{(\upsilon)}(u)]^{(b)}$ of the container gantry crane component in the reliability states subset $\{u, u+1,...,3\}$ is greater than $t$, while the system operation process $Z(t)$ is at the operation state $z_b$, $b = 1,2,...,6$.

In [8], on the basis of expert opinions, the reliability functions of the container gantry crane components in different operation states were approximately determined. Further, they were used in [8] to this system reliability analysis and evaluation.

Assuming that the container gantry crane is in the reliability state subset $\{u,u+1,...,3\ \}$ if all its subsystems are in this subset of reliability states, we conclude that the gantry crane is a four-state series system [10] of subsystems $S_1$, $S_2$, $S_3$, $S_4$, $S_5$ with the scheme presented in *Figure 2*.

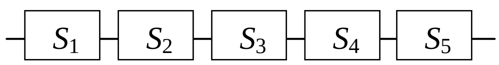


*Figure 2.* General scheme of container gantry crane reliability structure

Next, we assume that changes of the container gantry crane operation states have an influence on the subsystems $S_\upsilon$, $\upsilon = 1,2,3,4,5,$ components reliability and on the gantry crane reliability structures as well. The container gantry crane operation process influence on the system reliability structure is expressed below.

At the system operation state $z_1$, the container gantry crane is composed of the subsystem $S_1$ which is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$ (subsystems) with the structure showed in *Figure 3*.

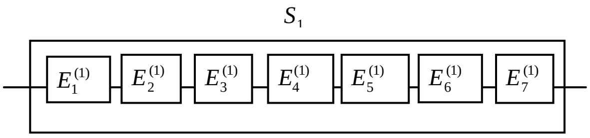

*Figure 3*. The scheme of the container gantry crane at operation state $z_1$.

At the system operation states $z_2$ and $z_3$, the container gantry crane is composed of the subsystems $S_1$, $S_2$ and $S_3$ forming a series structure shown in *Figure 4*. The subsystem $S_1$ is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem $S_2$ is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, and the subsystem $S_3$ is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$.

At the system operation states $z_4$ and $z_5$, the container gantry crane is composed of the subsystems $S_1$, $S_2$, $S_3$ and $S_4$ forming a series structure shown in *Figure 5*. The subsystem $S_1$ is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem $S_2$ is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, the subsystem $S_3$ is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$, and the subsystem $S_4$ consists of a component $E_1^{(4)}$.

At the system operation state $z_6$, the container gantry crane is composed of the subsystems $S_1$, $S_2$, $S_3$ and $S_5$ forming a series structure shown in *Figure 6*. The subsystem $S_1$ is a series system composed of $n = 7$ components $E_i^{(1)}$, $i = 1,2,..,7$, the subsystem $S_2$ is a series system composed of $n = 6$ components $E_i^{(2)}$, $i = 1,2,..,6$, the subsystem $S_3$ is a series system composed of $n = 3$ components $E_i^{(3)}$, $i = 1,2,3$ and the subsystem $S_5$ is a series system composed of $n = 2$ components $E_i^{(5)}$, $i = 1,2.$

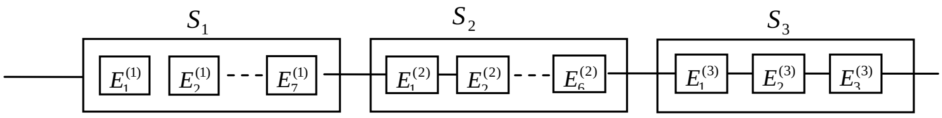

*Figure 4.* The scheme of the container gantry crane at operation states $z_2$ and $z_3$.

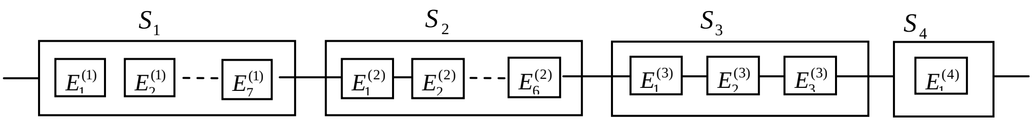

*Figure 5*. The scheme of the container gantry crane at operation states $z_4$ and $z_5$.

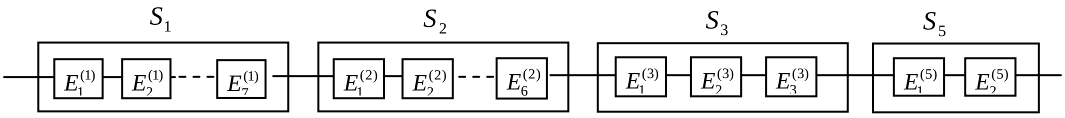

*Figure 6.* The scheme of the container gantry crane at operation state $z_6$.

On the basis of expert opinions and statistical data given in [8], the container gantry crane reliability structures and their components reliability functions and the container gantry crane conditional reliability functions at different operation states can be determined. Further, in the case when the gantry crane operation time is large enough, using the system conditional reliability functions at particular operation states, we may conclude the unconditional reliability function of the container gantry crane is given by the vector

$$\boldsymbol{R}(t,\ \cdot) = [1,\ \boldsymbol{R}(t,1),\ \boldsymbol{R}(t,2),\ \boldsymbol{R}(t,3)],\ t \geq 0, \qquad (46)$$

where, according to (5)-(6), after considering the values of $p_b$, $b = 1,2,...,6$, given by (44), its co-ordinates are

$$\boldsymbol{R}(t,u) = 0.6874 \cdot [\boldsymbol{R}(t,u)]^{(1)} + 0.0187 \cdot [\boldsymbol{R}(t,u)]^{(2)}$$

$$+ 0.0515 \cdot [\boldsymbol{R}(t,u)]^{(3)} + 0.0005 \cdot [\boldsymbol{R}(t,u)]^{(4)}$$

$$+ 0.0717 \cdot [\boldsymbol{R}(t,u)]^{(5)} + 0.1702 \cdot [\boldsymbol{R}(t,u)]^{(6)}, \qquad (47)$$

for $t \geq 0$, $u = 1,2,3,$ where $[\boldsymbol{R}(t,u)]^{(b)}$ for $b = 1,2,...,6,$ are the container gantry crane conditional reliability functions at particular operation states $z_b$, $b = 1,2,...,6,$ given in [8].

Next, according to (7), considering the results from [8], it follows that the mean values of the container gantry crane unconditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ respectively are:

$$\mu(1) \cong 4.14 \text{ years},$$

$$\mu(2) = 0.6874 \cdot 3.79 + 0.0187 \cdot 1.53 + 0.0515 \cdot 1.52$$

$$+ 0.0005 \cdot 1.43 + 0.0717 \cdot 1.46 + 0.1702 \cdot 1.28$$

$$\cong 3.04 \text{ years}, \qquad (48)$$

$$\mu(3) \cong 2.22 \text{ years}.$$

From the above, according to (10), the mean values of the system lifetimes in the particular reliability states 1, 2, 3 are:

$$\bar{\mu}(1) = 1.10,\ \bar{\mu}(2) = 0.82,\ \bar{\mu}(3) = 2.22 \text{ years}. \qquad (49)$$

Since the critical reliability state is $r = 2$, then the system risk function, according to (11), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,2), \qquad (50)$$

where $\boldsymbol{R}(t,2)$ is given by (47) for $u = 2$.

Hence, by (12), the moment when the system risk function exceeds a permitted level [10], for instance $\delta$ = 0.05, obtained from (50), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.126 \text{ year}. \qquad (51)$$

### 3.4. Container gantry crane operation process optimization

Considering the equations (46)-(47), it is natural to assume that the container gantry crane operation process has a significant influence on the system reliability. This influence is also clearly expressed in the formula (48) for the mean value of the container gantry crane unconditional lifetime in the reliability state subsets that can be used for this system operation process optimization performed in the accordance with the procedure proposed in Section 2.4.1.

In this case, as the container gantry crane critical state is $r = 2,$ then considering the expression (48) the objective function, defined in the equation (13), takes the form

$$\mu(2) = p_1 \cdot 3.79 + p_2 \cdot 1.53 + p_3 \cdot 1.52 + p_4 \cdot 1.43$$

$$+ p_5 \cdot 1.46 + p_6 \cdot 1.28 \qquad (52)$$

and we assume, the following coming from experts bound constraints

$$0.50 \leq p_1 \leq 0.90,\ 0.01 \leq p_2 \leq 0.03,$$

$$0.03 \leq p_3 \leq 0.07,\ 0.0004 \leq p_4 \leq 0.0007,$$

$$0.05 \leq p_5 \leq 0.09,\ 0.09 \leq p_6 \leq 0.30,$$

$$\sum_{b=1}^{6} p_b = 1. \qquad (53)$$

Now, in order to find the optimal values $\dot{p}_b$ of the transient probabilities $p_b$, $b = 1,2,...,6,$ that maximize the objective function (52), according to the procedure given in Section 2.4.1, we arrange the system conditional lifetimes mean values $\mu_b(2)$, $b = 1,2,...,6,$ in non-increasing order

$$\mu_1(2) \geq \mu_2(2) \geq \mu_3(2) \geq \mu_5(2) \geq \mu_4(2) \geq \mu_6(2).$$

Next, according to (18), we substitute

$$x_1 = p_1,\ x_2 = p_2,\ x_3 = p_3,$$

$$x_4 = p_5,\ \ x_5 = p_4,\ \ x_6 = p_6, \qquad (54)$$

and we maximize with respect to $x_i$, $i = 1,2,...,6$, the linear form (52) that after considering the substitution (54) takes the form

$$\mu(2) = x_1 \cdot 3.79 + x_2 \cdot 1.53 + x_3 \cdot 1.52$$

$$+ x_4 \cdot 1.46 + x_5 \cdot 1.43 + x_6 \cdot 1.28, \qquad (55)$$

with suitable bound constraints resulting from (54). Further, according to the procedure given in Section 2.4.1, we calculate

$$\breve{x} = \sum_{i=1}^{6} \breve{x}_i = 0.6804,$$
$$\hat{y} = 1 - \breve{x} = 1 - 0.6804 = 0.3196, \qquad (56)$$

and we find

$$\breve{x}^0 = 0,\ \ \hat{x}^0 = 0,\ \ \hat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.50,\ \ \hat{x}^1 = 0.90,\ \ \hat{x}^1 - \breve{x}^1 = 0.40,$$

$$\breve{x}^2 = 0.51,\ \ \hat{x}^2 = 0.93,\ \hat{x}^2 - \breve{x}^2 = 0.42,$$

$$\breve{x}^3 = 0.54,\ \ \hat{x}^3 = 1.00,\ \hat{x}^3 - \breve{x}^3 = 0.46,$$
$$\ldots$$

$$\breve{x}^6 = 0.6804,\ \hat{x}^6 = 1.3907,\ \ \hat{x}^6 - \breve{x}^6 = 0.7103. \qquad (57)$$

From the above, as according to (56) after considering the inequality

$$\hat{x}^I - \breve{x}^I < 0.3196, \qquad (58)$$

it follows that the largest value $I \in \{0,1,...,6\}$ such that the inequality (58) is satisfied, is $I = 0$.
Therefore, we fix the optimal solution that maximize linear function (55) according to the rule given in Section 2.4.1 and we get

$$\dot{x}_1 = \hat{y} + \breve{x}_1 = 0.3196 + 0.5 = 0.8196,$$

$$\dot{x}_2 = \breve{x}_2 = 0.01,\ \ \dot{x}_3 = \breve{x}_3 = 0.03,\ \ \dot{x}_4 = \breve{x}_4 = 0.05,$$

$$\dot{x}_5 = \breve{x}_5 = 0.0004,\ \ \dot{x}_6 = \breve{x}_6 = 0.09.$$

Finally, after making the substitution inverse to (54), we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_1 = 0.8196,\ \ \dot{p}_2 = \dot{x}_1 = 0.01,$$

$$\dot{p}_3 = \dot{x}_3 = 0.03,\ \dot{p}_4 = \dot{x}_5 = 0.0004,$$

$$\dot{p}_5 = \dot{x}_4 = 0.05,\ \dot{p}_6 = \dot{x}_6 = 0.09, \qquad (59)$$

that maximize the system mean lifetime in the reliability state subset $\{2,3\}$ expressed by the linear form (52) giving, according (59), its optimal value

$$\dot{\mu}(2) = \dot{p}_1 \cdot 3.79 + \dot{p}_2 \cdot 1.53 + \dot{p}_3 \cdot 1.52 + \dot{p}_4 \cdot 1.43$$

$$+ \dot{p}_5 \cdot 1.46 + \dot{p}_6 \cdot 1.28 \cong 3.36. \qquad (60)$$

### 3.5. Container gantry crane reliability optimization

To make the optimization of the reliability of the container gantry crane we need the optimal values $\dot{p}_b$, $b = 1,2,...,6$, of the transient probabilities $p_b$, $b = 1,2,...,6$, in particular operation states determined by (59). Using the optimal solution (59), we obtain the optimal mean values of the container gantry crane unconditional lifetimes in the reliability state subset $\{1,2,3\}$ and $\{3\}$ that respectively are

$$\dot{\mu}(1) \cong 4.61,\ \dot{\mu}(3) \cong 2.45 \qquad (61)$$

and the optimal solutions for the mean values of the container gantry crane unconditional lifetimes in the particular reliability states 1, 2 and 3 are

$$\dot{\bar{\mu}}(1) = 1.25,\ \dot{\bar{\mu}}(2) = 0.91,\ \dot{\bar{\mu}}(3) = 2.45. \qquad (62)$$

Moreover, according to (32)-(33), the corresponding optimal unconditional multistate reliability function of the container gantry crane is given by the vector

$$\dot{\boldsymbol{R}}(t,\cdot) = [1,\ \dot{\boldsymbol{R}}(t,1),\ \dot{\boldsymbol{R}}(t,2),\ \dot{\boldsymbol{R}}(t,3)\,], \qquad (63)$$

where after considering the values of $\dot{p}_b$ given by (59), its coordinates are as follows

$$\dot{\boldsymbol{R}}(t,u) = 0.8196 \cdot [\boldsymbol{R}(t,u)]^{(1)} + 0.01 \cdot [\boldsymbol{R}(t,u)]^{(2)}$$

$$+ 0.03 \cdot [\boldsymbol{R}(t,u)]^{(3)} + 0.0004 \cdot [\boldsymbol{R}(t,u)]^{(4)}$$

$$+ 0.05 \cdot [\boldsymbol{R}(t,u)]^{(5)} + 0.09 \cdot [\boldsymbol{R}(t,u)]^{(6)}, \qquad (64)$$

for $t \geq 0$, $u = 1,2,3$, where $[\boldsymbol{R}(t,u)]^{(b)}$ for $b = 1,2,...,6$, are the container gantry crane conditional reliability functions at particular operation states $z_b$, $b = 1,2,...,6$, given in [8].

Since the critical reliability state is $r = 2$, then according to (36) the optimal system risk function is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,2) \text{ for } t \geq 0, \quad (65)$$

where $\dot{\boldsymbol{R}}(t,2)$ is given by (64) for $u = 2$.
Hence an from (37), the moment when the optimal system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.149 \text{ year.} \quad (66)$$

The comparison of the values of the container gantry crane reliability characteristics before the system operation process optimization given by (46)-(51) with their corresponding values after the system operation process optimization respectively given by (60)-(66) justifies the sensibility of the performed system operation process optimization.

### 3.6. Optimal sojourn times of container gantry crane operation process at operation states

Having the values of the optimal transient probabilities determined by (59), it is possible to find the optimal conditional and unconditional mean values of the sojourn times of the container gantry crane operation process at the operation states and the optimal mean values of the total unconditional sojourn times of the container gantry crane operation process at the operation states during the fixed operation time as well.
Substituting the optimal transient probabilities at operation states determined in (59) and the steady probabilities

$$\pi_1 \cong 0.0951, \ \pi_2 \cong 0.1020, \ \pi_3 \cong 0.3100,$$

$$\pi_4 \cong 0.0102, \ \pi_5 \cong 0.1547, \ \pi_6 \cong 0.3280.$$

determined from the system of equation given in Section 2.1 into (39), we get the following system of equations

$$-0.017156\dot{M}_1 + 0.0835992\dot{M}_2$$
$$+0.254076\dot{M}_3 + 0.0083599\dot{M}_4$$
$$+0.1267921\dot{M}_5 + 0.2688288\dot{M}_6 = 0$$

$$0.000951\dot{M}_1 - 0.10098\dot{M}_2$$
$$+0.0031\dot{M}_3 + 0.000102\dot{M}_4$$
$$+0.001547\dot{M}_5 + 0.00328\dot{M}_6 = 0$$

$$0.002853\dot{M}_1 + 0.00306\dot{M}_2$$
$$-0.3007\dot{M}_3 + 0.000306\dot{M}_4$$
$$+0.004641\dot{M}_5 + 0.00984\dot{M}_6 = 0$$

$$0.000038\dot{M}_1 + 0.0000408\dot{M}_2$$
$$+0.000124\dot{M}_3 - 0.0101959\dot{M}_4$$
$$+0.0000618\dot{M}_5 + 0.0001312\dot{M}_6 = 0$$

$$0.004755\dot{M}_1 + 0.0051\dot{M}_2$$
$$+0.0155\dot{M}_3 + 0.00051M_4$$
$$-0.146965\dot{M}_5 + 0.0164\dot{M}_6 = 0$$

$$0.008559\dot{M}_1 + 0.00918\dot{M}_2$$
$$+0.0279\dot{M}_3 + 0.000918\dot{M}_4$$
$$+0.013923\dot{M}_5 - 0.29848\dot{M}_6 = 0 \quad (67)$$

with the unknown optimal mean values $\dot{M}_b$ of the system unconditional sojourn times in the operation states we are looking for.
Since the determinant of the main matrix of the homogeneous system of equations (67) is equal to 0, then its rank is less than 6 and there are non-zero solutions of this system of equations that are ambiguous and dependent on one or more parameters. Thus, we may fix some of them and determine the remaining ones. In our case, according to (43), after considering experts opinion, we conclude that it is sensible to assume

$$\dot{M}_4 \cong 2. \quad (68)$$

Consequently, from (67), we get the system of equations

$$-0.017156\dot{M}_1 + 0.0835992\dot{M}_2$$
$$+0.254076\dot{M}_3 + 0.1267921\dot{M}_5$$

$+0.2688288\dot{M}_6 = -0.0167198$

$0.000951\dot{M}_1 - 0.10098\dot{M}_2$

$+0.0031\dot{M}_3 + 0.001547\dot{M}_5$

$+0.00328\dot{M}_6 = -0.000204$

$0.002853\dot{M}_1 + 0.00306\dot{M}_2$

$-0.3007\dot{M}_3 + 0.004641\dot{M}_5$

$+0.00984\dot{M}_6 = -0.000612$

$0.000038\dot{M}_1 + 0.0000408\dot{M}_2$

$+0.000124\dot{M}_3 + 0.0000618\dot{M}_5$

$+0.0001312\dot{M}_6 = 0.0203918$

$0.004755\dot{M}_1 + 0.0051\dot{M}_2$

$+0.0155\dot{M}_3 - 0.146965\dot{M}_5$

$+0.0164\dot{M}_6 = -0.00102$

$0.008559\dot{M}_1 + 0.00918\dot{M}_2$

$+0.0279\dot{M}_3 + 0.013923\dot{M}_5$

$-0.29848\dot{M}_6 = -0.001836$

and we solve it with respect to $\dot{M}_1$, $\dot{M}_2$, $\dot{M}_3$, $\dot{M}_5$ and $\dot{M}_6$.
This way obtained the solutions of the system of equations (67), are

$$\dot{M}_1 \cong 439.9400,\ \dot{M}_2 \cong 5.0046,\ \dot{M}_3 \cong 4.9401,$$

$$\dot{M}_4 \cong 2,\ \dot{M}_5 \cong 16.4988,\ \dot{M}_6 \cong 14.0069. \qquad (69)$$

It can be seen that these solutions differ much from the values $M_1$, $M_2$, $M_3$, $M_4$, $M_5$ and $M_6$ estimated by (43).
Having these solutions, it is also possible to look for the optimal values $\dot{M}_{bl}$ of the mean values $M_{bl}$ of the conditional sojourn times at the operation states. Namely, substituting the probabilities of the system operation process transitions between the operation states given in Section 3.2 and the optimal mean values $\dot{M}_b$ given by (69) into (40), we get the following system of equations

$0.648\dot{M}_{12} + 0.336\dot{M}_{13} + 0.008\dot{M}_{14}$

$+ 0.008\dot{M}_{16} = 439.9400$

$0.525\dot{M}_{21} + 0.373\dot{M}_{23} + 0.093\dot{M}_{24}$

$+ 0.009\dot{M}_{26} = 5.0046$

$0.105\dot{M}_{31} + 0.111\dot{M}_{32} + 0.118\dot{M}_{35}$

$+ 0.666\dot{M}_{36} = 4.9401$

$0.417\dot{M}_{41} + 0.583\dot{M}_{42} = 2$

$0.005\dot{M}_{51} + 0.220\dot{M}_{53} + 0.775\dot{M}_{56} + = 16.4988$

$0.012\dot{M}_{61} + 0.628\dot{M}_{63} + 0.360\dot{M}_{65} = 14.0069.$

with the unknown optimal values $\dot{M}_{bl}$ we want to find.
As the solutions of the above system of equations are ambiguous, then we arbitrarily fix some of them, for instance because of practically important reasons, and we find the remaining ones. In this case we proceed as follows:

- we fix in the first equation $\dot{M}_{13} = 40$, $\dot{M}_{14} = 50$, $\dot{M}_{16} = 4$ and we find $\dot{M}_{12} \cong 657.5$;
- we fix in the second equation $\dot{M}_{23} = 9$, $\dot{M}_{24} = 2$, $\dot{M}_{26} = 16$ and we find $\dot{M}_{21} \cong 2.51$;
- we fix in the third equation $\dot{M}_{31} = 6$, $\dot{M}_{32} = 4$, $\dot{M}_{35} = 7$ and we find $\dot{M}_{36} \cong 4.56$;
- we fix in the fourth equation $\dot{M}_{41} = 2$ and we find $\dot{M}_{42} \cong 2$;
- we fix in the fifth equation $\dot{M}_{51} = 10$, $\dot{M}_{53} = 3$ and we find $\dot{M}_{56} \cong 20.37$;
- we fix in the sixth equation $\dot{M}_{61} = 23$, $\dot{M}_{65} = 21$ and we find $\dot{M}_{63} \cong 9.83$. (70)

Other very useful and much easier to be applied in practice tool that can help in planning the operation process of the ferry technical system are the system operation process optimal mean values of the total sojourn times at the particular operation states during the fixed system operation time $\theta$.

Assuming as in Section 3.2, the system operation time $\theta = 1$ year = 365 days, after aplying (41), we get their values

$$\dot{E}[\hat{\theta}_1] = \dot{p}_1\theta = 0.8196 \cdot 365 \cong 299.15,$$

$$\dot{E}[\hat{\theta}_2] = \dot{p}_2\theta = 0.01 \cdot 365 = 3.65,$$

$$\dot{E}[\hat{\theta}_3] = \dot{p}_3\theta = 0.03 \cdot 365 = 10.95,$$

$$\dot{E}[\hat{\theta}_4] = \dot{p}_4\theta = 0.0004 \cdot 365 \cong 0.15,$$

$$\dot{E}[\hat{\theta}_5] = \dot{p}_5\theta = 0.05 \cdot 365 = 18.25,$$

$$\dot{E}[\hat{\theta}_6] = \dot{p}_6\theta = 0.09 \cdot 365 = 32.85. \quad (71)$$

that differ from the values of $E[\hat{\theta}_i]$, $i = 1,2,...,6$, determined by (45).
In practice, the knowledge of the optimal values of $\dot{M}_b$, $\dot{M}_{bl}$ and $\dot{E}[\hat{\theta}_b]$ given respectively by (69)-(71), can be very important and helpful for the container gantry crane operation process planning and improving in order to make the system operation more reliable and safer.

### 3.7. Parameters and characteristics of container gantry crane operation process before and after its optimization

From Section 3.2, we have the values of the following container gantry crane operation process parameters before its optimization:

- the conditional mean sojourn times of the container gantry crane at the particular operation states

$$M_{12} = 456.978,\ M_{13} = 36.860,\ M_{14} = 50,\ M_{16} = 3,$$

$$M_{21} = 7.887,\ M_{23} = 9.121,\ M_{24} = 1.545,\ M_{26} = 16,$$

$$M_{31} = 5.5,\ M_{32} = 4.343,\ M_{35} = 6.822,\ M_{36} = 7.857,$$

$$M_{41} = 2,\ M_{42} = 2.143,$$

$$M_{51} = 10,\ M_{53} = 2.899,\ M_{56} = 24.681,$$

$$M_{61} = 22.6,\ M_{63} = 23.117,\ M_{65} = 20.512. \quad (72)$$

- the unconditional mean sojourn times of the container gantry crane at the particular operation states

$$M_1 = 308.93,\ M_2 = 7.83,\ M_3 = 7.09,$$

$$M_4 = 2.08,\ M_5 \cong 19.82,\ M_6 \cong 22.17; \quad (73)$$

- the transient probabilities of the container gantry crane operation process at the operational states

$$p_1 = 0.6874,\ p_2 = 0.0187,\ p_3 = 0.0515,$$

$$p_4 = 0.0005,\ p_5 = 0.0717,\ p_6 = 0.1702; \quad (74)$$

- the total sojourn times of the container gantry crane operation process in particular operation states during the operation time $\theta = 1$ year = 365 days

$$E[\hat{\theta}_1] = 251 \text{ days},\ E[\hat{\theta}_2] = 7 \text{ days},$$

$$E[\hat{\theta}_3] = 19 \text{ day},\ E[\hat{\theta}_4] = 0.2 \text{ day},$$

$$E[\hat{\theta}_5] = 26 \text{ days},\ E[\hat{\theta}_6] = 62 \text{ days}. \quad (75)$$

From Section 3.6, we have the values of the following container gantry crane operation process parameters after its optimization:

- the optimal conditional mean sojourn times of the container gantry crane at the particular operation states

$$\dot{M}_{12} \cong 657.5,\ \dot{M}_{13} = 40,\ \dot{M}_{14} = 50,\ \dot{M}_{16} = 4,$$

$$\dot{M}_{21} \cong 2.51,\ \dot{M}_{23} = 9,\ \dot{M}_{24} = 2,\ \dot{M}_{26} = 16,$$

$$\dot{M}_{31} = 6,\ \dot{M}_{32} = 4,\ \dot{M}_{35} = 7,\ \dot{M}_{36} \cong 4.56,$$

$$\dot{M}_{41} = 2,\ \dot{M}_{42} \cong 2,\ \dot{M}_{51} = 10,$$

$$\dot{M}_{53} = 3,\ \dot{M}_{56} \cong 20.37,$$

$$\dot{M}_{61} = 23,\ \dot{M}_{65} = 21,\ \dot{M}_{63} \cong 9.83. \quad (76)$$

- the optimal unconditional mean sojourn times of the container gantry crane in the particular operation states

$$\dot{M}_1 \cong 439.94,\ \dot{M}_2 \cong 5.00,\ \dot{M}_3 \cong 4.94,$$

$$\dot{M}_4 \cong 2,\ \dot{M}_5 \cong 16.50,\ \dot{M}_6 \cong 14.01; \quad (77)$$

- the optimal transient probabilities of the container gantry crane operation process at the operational states

$\dot{p}_1 = 0.8196, \ \dot{p}_2 = 0.01, \ \dot{p}_3 0.03,$
$\dot{p}_4 = 0.0004, \ \dot{p}_5 = 0.05, \ \dot{p}_6 = 0.09;$ (78)

- the optimal total sojourn times of the container gantry crane operation process in particular operation states during the operation time $\theta = 1$ year = 365 days

$\dot{E}[\hat{\theta}_1] = 299.15$ days, $\dot{E}[\hat{\theta}_2] = 3.65$ days,

$\dot{E}[\hat{\theta}_3] = 10.95$ days, $\dot{E}[\hat{\theta}_4] = 0.15$ days,

$\dot{E}[\hat{\theta}_5] = 18.25$ days, $\dot{E}[\hat{\theta}_6] = 32.85$ days. (79)

### 3.8. Characteristics of container gantry crane reliability before and after operation process optimization

From Section 3.3, we have the values of the following container gantry crane reliability characteristics before its operation process optimization:

- the expected values of the container gantry crane unconditional lifetimes respectively in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$

$\mu(1) = 4.14$ years, $\mu(2) = 3.04$ years,

$\mu(3) = 2.22$ years; (80)

- the mean values of the unconditional lifetimes respectively in the particular reliability states 1, 2, 3

$\bar{\mu}(1) = 1.10 \ \ \bar{\mu}(2) = 0.82, \ \bar{\mu}(3) = 2.22$ years; (81)

- the moment when the system risk function exceeds a permitted level

$\tau \cong 0.126$ year. (82)

From Sections 3.4 and 3.5, we have the values of the following container gantry crane reliability parameters and characteristics after its operation process optimization:

- the optimal expected values of the container gantry crane unconditional lifetimes respectively in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$

$\dot{\mu}(1) = 4.61$ years, $\dot{\mu}(2) = 3.36$ years,

$\dot{\mu}(3) = 2.45$ years; (83)

- the optimal mean values of the unconditional lifetimes respectively in the particular reliability states 1, 2, 3

$\dot{\bar{\mu}}(1) = 1.25, \ \dot{\bar{\mu}}(2) = 0.91, \ \dot{\bar{\mu}}(3) = 2.45$ years; (84)

- the optimal moment when the system risk function exceeds a permitted level

$\dot{\tau} \cong 0.149$ year. (85)

### 3.9. Suggestions on new strategy of container gantry crane operation process organizing

The comparison of the values of the selected in Section 3.8 container gantry crane reliability characteristics before the system operation process optimization given by (80)-(82) with their values after the system operation process optimization respectively given by (83)-(85) justifies the sensibility of the performed system operation process optimization.

From the performed in Section 3.7 analysis of the results of the container gantry crane operation process optimization it can be suggested to organize the system operation process in the way that causes the replacing (or the approaching/convergence to) the conditional mean sojourn times $M_{bl}$ of the system at the particular operation states before the optimization given by (72) by their optimal values $\dot{M}_{bl}$ after the optimization given by (76). The possibility of fulfilling this suggestion of the operation process parameters changing is not easy and has to be checked in practice.

It seems to be practically a bit easier way, changing the operation process characteristics that results in replacing (or the approaching/convergence to) the unconditional mean sojourn times $M_b$ of the container gantry crane at the particular operation states before the optimization given by (73) by their optimal values $\dot{M}_b$ after the optimization given by (77).

The easiest way of the system operation process reorganizing is that leading to the replacing (or the approaching/convergence to) the total sojourn times $E[\hat{\theta}_b]$ of the container gantry crane operation process at the particular operation states during the operation time $\theta = 1$ year before the optimization given by (75) by their optimal values $\dot{E}[\hat{\theta}_b]$ after the optimization given by (79).

## 4. Conclusion

The joint model of the reliability of complex technical systems at the variable operation conditions linking the semi-Markov modeling of the system operation processes with the multi-state approach to system reliability analysis was presented and applied to the evaluation of the container gantry crane reliability characteristics. Next, the final results obtained from this joint model and the linear programming were used to perform this complex technical system reliability optimization.

These tools practical application to reliability and risk evaluation and optimization of a container gantry crane operating at the variable operation conditions and the results achieved are interesting for reliability practitioners from maritime industry and from other industrial sectors as well. These tools can be useful in reliability and operation prediction and optimization of a very wide class of real technical systems operating in varying conditions that have an influence on changing their reliability structures and their components reliability characteristics.

## References

[1] Aven, T. (1985). Reliability evaluation of multi-state systems with multi-state components. *IEEE Trans on Reliab*, Vol. 34, 473-479.

[2] Grabski, F. (2002). *Semi-Markov Models of Systems Reliability and Operations*. Warsaw: Systems Research Institute, Polish Academy of Sciences.

[3] Hudson, J. & Kapur, K. (1985). Reliability bounds for multistate systems with multistate components. *Oper Res*, Vol. 33, 735-744.

[4] Klabjan, D. & Adelman, D. Existence of optimal policies for semi-Markov decision processes using duality for infinite linear programming. *Siam J. on Control and Optim*, Vol. 44(6), 2104-2122.

[5] Kołowrocki, K. (2004). *Reliability of Large Systems*. Elsevier.

[6] Kolowrocki, K. & Soszynska, J. Reliability, risk and availability based optimization of complex technical systems operation processes. Part 1. Theoretical backgrounds. *Electron. J of Reliab & Risk Anal: Theory & Appl* , vol. 2(4), 141-152.

[7] Kolowrocki, K. & Soszynska, J. (2010). Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems. *Electron J of Reliab & Risk Anal: Theory & Appl*, Vol. 1(2), 184-200.

[8] Kołowrocki, K. & Soszyńska-Budny, J. (2010). *The container gantry crane operation, reliability, risk, availability identification, prediction and optimization - Testing IS&RDSS*. Task 9.5 in WP9: Applications and Testing of Packages of Tools in Complex Maritime Transportation Systems and Processes Safety and Reliability Optimization. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0.Gdynia Maritime University.

[9] Limnios, N. & Oprisan, G. (2001). *Semi-Markov Processes and Reliability*. Birkhauser, Boston.

[10] Soszynska, J. (2010). Reliability and risk evaluation of a port oil pipeline transportation system in variable operation conditions. *Int J of Press Vessels and Pip*, Vol. 87(2-3), 81-87.

[11] Xue, J. (1985). On multi-state system analysis. *IEEE Trans on Reliab*, Vol. 34, 329-337.