**Kuligowska Ewa**
*Maritime University, Gdynia, Poland*

# Preliminary Monte Carlo approach to complex technical system reliability analysis

**Keywords**

reliability, operation process, complex system, Monte Carlo simulation, piping oil transportation system

**Abstract**

The paper explores the mathematical and computer modeling of complex technical systems related to their operation processes. The complex technical system with the reliability structure and components' reliability parameters changing at the various operation states is defined. The selected system operation process parameters and the hypothetical distribution functions of the conditional sojourn times at the operation states are defined and identified. The reliability functions of the multistate system and components are introduced and their parameters are identified. The Monte Carlo simulation method is proposed to the complex system reliability evaluation. Moreover, the proposed theoretical and simulation tools are supported with the practical application to the multi-state port oil piping transportation system reliability analysis.

## 1. Introduction

The main issue of today's system reliability analysis is evaluation of more than two reliability states systems with changing their reliability structures and their components reliability parameters at the varying in time the system operation states [2]-[16]. Examples of such complex technical systems in real world for instance are energy generation and transmission systems, telecommunication systems, piping transportation systems of various substances and maritime transportation systems. Using the traditional analytical techniques is sometimes difficult to implement in the reliability analysis, modeling, prediction and optimization of those complex technical systems.

The Monte Carlo simulation method [18] applied to those problems can provide their approximate solutions in a relatively small amount of time. It allows examining the reliability of complex technical systems sampled in a number of random configurations in scientific computing. Taking into account the importance of reliability of complex multistate [17] technical systems in practice, the analysis is supported with a direct application to a port oil transportation system operating at one of the Baltic oil terminals.

## 2. System operation process

The operation processes of most real technical systems are very complex because of the large numbers of their operation states and the random transitions among them and the random sojourn lifetimes at them. To solve this complexity, the models of systems' operation processes can be constructed using semi-Markov processes [1] proposed in this section.

### 2.1. System operation process modeling and identification

We consider a multistate system operation process $Z(t)$, $t \in \langle 0,+\infty)$ with $v$, $v \in N$, distinguished discrete operation states from the set

$$Z = \{z_1, z_2, ..., z_v\}$$

with the conditional sojourn times $\theta_{bl}$ at the operation states $z_b$ when its next operation state is $z_l$, $b,l = 1,2,...,v$, $b \neq l$. We assume that $Z(t)$, is a semi-Markov process [1] and therefore, the sojourn times at the operation states may have arbitrary probability distributions [11].

Let $\Theta$ be the duration time of the experiment. Furthermore, we denote by $n(0)$ the realisation of the total number of the system operation process stay at the particular operation states at the initial moment $t=0$ and by $[n_b(0)]_{1\times v}$, $b=1,2,...,v$, the vector of realisations of the numbers of the system operation process transitions in the particular operation states $z_b$ at the initial moment $t=0$. In addition, we denote by $n_{bl}$ the realization of the numbers of the system operation process transitions from the state $z_b$ into the state $z_l$, $b,l=1,2,...,v$, $b\neq l$ and the realisation of the total numbers of the system operation process transitions from the operation state $z_b$ as $n_b$, $b=1,2,...,v$.

Consequently, the semi-Markov model can be described and identified using the following parameters and their evaluations:

- the vector $[p_b(0)]_{1\times v}$, of the initial probabilities of the system operation process $Z(t)$ at the moment $t=0$,

$$[p_b(0)]_{1\times v}=[p_1(0),p_2(0),...,p_v(0)],$$

where

$$p_b(0)=P(Z(0)=z_b)=\frac{n_b(0)}{n(0)},\ b=1,2,...,v;$$

- the matrix $[p_{bl}]_{v\times v}$ of the probabilities of the system operation process $Z(t)$ transitions between the operation states $z_b$ and $z_l$, $b,l=1,2,...,v$, $b\neq l$

$$[p_{bl}]_{v\times v}=\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \vdots & \vdots & \ddots & \vdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix},$$

where

$$p_{bl}=\frac{n_{bl}}{n_b} \text{ and } p_{bb}=0 \text{ for } b=1,2,...,v;$$

- the matrix of the conditional distribution functions

$$H_{bl}(t)=P(\theta_{bl}<t),\ \ b,l=1,2,...,v,\ b\neq l$$

of the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ at the operation states

$$[H_{bl}(t)]_{v\times v}=\begin{bmatrix} H_{11}(t) & H_{12}(t) & \cdots & H_{1v}(t) \\ H_{21}(t) & H_{22}(t) & \cdots & H_{2v}(t) \\ \vdots & \vdots & \ddots & \vdots \\ H_{v1}(t) & H_{v2}(t) & \cdots & H_{vv}(t) \end{bmatrix},$$

where $H_{bb}(t)=0$ for $b=1,2,...,v$, and the remaining ones can be estimated using the suggested suitable distributions and statistical methods given in [11].

Since, very often, we do not have numerous times of realisation then we assume that the suitable distributions describing the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ are the chimney distributions with the density functions of the form [11]:

$$h_{bl}(t)=\begin{cases} 0, & t<x_{bl} \\ \dfrac{A_{bl}}{z_{bl}^1-x_{bl}}, & x_{bl}\le t\le z_{bl}^1 \\ \dfrac{C_{bl}}{z_{bl}^2-z_{bl}^1}, & z_{bl}^1\le t\le z_{bl}^2 \\ \dfrac{D_{bl}}{y_{bl}-z_{bl}^2}, & z_{bl}^2\le t\le y_{bl} \\ 0, & t>y_{bl}, \end{cases}$$

where

$$0\le x_{bl}\le z_{bl}^1\le z_{bl}^2\le y_{bl}<+\infty,\ A_{bl},C_{bl},D_{bl}\ge 0.$$

The corresponding distribution function $H_{bl}(t)$ of the conditional sojourn time $\theta_{bl}$ takes the following form

$$H_{bl}(t)=\int_0^t h_{bl}(s)ds.$$

## 2.2. Parameters identification of port oil piping transportation system operation process

The oil piping transportation system under consideration is designated for reception, sending and storage the oil products such us petrol and oil. The terminal is composed of three parts (*A*, *B*, *C*) linked by three subsystems $S_1$, $S_2$ and $S_3$. The first and second are the series-parallel subsystems, each containing two pipelines, and the last one is "2 out of 3" subsystem [11]. The system scheme is shown in *Figure 1*.

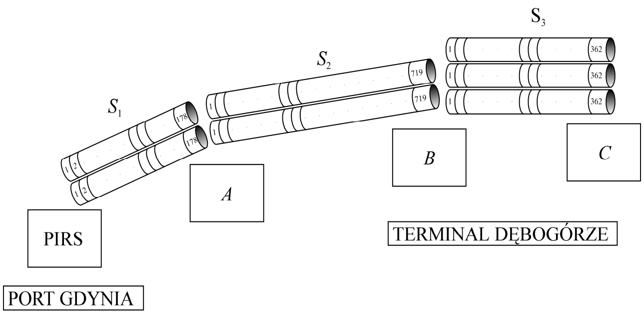



*Figure 1.* The scheme of a port oil transportation system.

The subsystem $S_1$ consists of $k = 2$ identical pipelines, each composed of $l = 178$ components:

- 176 pipe segments,
- 2 valves.

The subsystem $S_2$ consists of $k = 2$ identical pipelines, each composed of $l = 719$ components:

- 717 pipe segments,
- 2 valves.

The subsystem $S_3$ consists of $k = 3$, pipelines, two pipelines of the first type and one of the second type, each of them composed of $l = 362$ components:

- 360 pipe segments,
- 2 valves.

Taking into account the expert opinion, there are distinguished $v = 7$ operation states shown in the table below.

*Table 1.* List of operation states.

| State | Medium | Activity | Pipelines | Subsystem |
|---|---|---|---|---|
| $z_1$ | 1 kind | $\mathrm{B} \to \mathrm{C}$ | 2 out of 3 | $S_3$ |
| $z_2$ | 1 kind | $\mathrm{C} \to \mathrm{B}$ | 1 out of 3 | $S_3$ |
| $z_3$ | 1 kind | $\mathrm{B} \xrightarrow{A} \mathrm{Pier}$ | 1 out of 2 | $S_1$ |
| | | | 1 out of 2 | $S_2$ |
| $z_4$ | 1 kind | $\mathrm{Pier} \xrightarrow{A,B} \mathrm{C}$ | 1 out of 2 | $S_1$ |
| | | | 1 out of 2 | $S_2$ |
| | | | 2 out of 3 | $S_3$ |
| $z_5$ | 1 kind | $\mathrm{Pier} \xrightarrow{A} \mathrm{B}$ | 1 out of 2 | $S_1$ |
| | | | 1 out of 2 | $S_2$ |
| $z_6$ | 1 kind | $\mathrm{B} \to \mathrm{C}$ | 2 out of 3 | $S_3$ |
| | 1 kind | $\mathrm{Pier} \xrightarrow{A} \mathrm{B}$ | 1 out of 2 | $S_1$ |
| | | | 1 out of 2 | $S_2$ |
| $z_7$ | 1 kind | $\mathrm{B} \to \mathrm{C}$ | 1 out of 3 | $S_3$ |
| | 1 kind | $\mathrm{C} \to \mathrm{B}$ | 1 out of 3 | $S_3$ |

Using the procedure and formulas given in section 2.1., we determine the empirical parameters of the conditional sojourn times $\theta_{bl}$ at the operation states $z_b$. On the basis of statistical data coming from experts, during the experiment time $\Theta = 329$ days which is 0.901 year, the unknown parameters are evaluated as follows [11]:

- the number of the pipeline system operation process realizations

  $$n(0) = 41;$$

- the realizations

  $$n_1(0) = 14,\ n_2(0) = 2,\ n_3(0) = 0,\ n_4(0) = 0,$$
  $$n_5(0) = 9,\ n_6(0) = 8,\ n_7(0) = 8,$$

  of the numbers of staying of the system operation process respectively at the operation states $z_1, z_2, \ldots, z_7$ at the initial moments $t = 0$;

- the vector of realisations

  $$[p_b(0)]_{1\times v} = [0.34, 0.05, 0, 0, 0.23, 0.19, 0.19], \quad (1)$$

  of the initial probabilities $p_b(0),\ b = 1,2,...,7$, of the pipeline system operation process stay at the particular states $z_b$ at the time $t = 0$;

- the matrix

  $$[p_{bl}] = \begin{bmatrix} 0 & 0.022 & 0.022 & 0 & 0.534 & 0.111 & 0.311 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.488 & 0.023 & 0 & 0.023 & 0 & 0.233 & 0.233 \\ 0.095 & 0 & 0 & 0 & 0.667 & 0 & 0.238 \\ 0.531 & 0.062 & 0 & 0 & 0.219 & 0.188 & 0 \end{bmatrix} \quad (2)$$

  of the probabilities of the system operation process transitions between the various operation states;

- the empirical distribution functions of the system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$, $b,l = 1,2,...,7$, measured in hours, at the operation states $z_1, z_2, \ldots, z_7$ are as follows

$$H_{12}(t)=\begin{cases}0, & t<0\\ 0.00104\,t, & 0\geq t>960\\ 1, & t\geq 960\end{cases}$$

$$H_{13}(t)=\begin{cases}0, & t<0\\ 0.00417\,t, & 0\geq t>240\\ 1, & t\geq 240\end{cases}$$

$$H_{15}(t)=\begin{cases}0, & t<0\\ 0.00029t, & 0\leq t<2582\\ 0.000065t+0.583333, & 2582\leq t<6455\\ 1, & t\geq 6455\end{cases}$$

$$H_{16}(t)=\begin{cases}0, & t<0\\ 0.00036\,t, & 0\geq t>2780\\ 1, & t\geq 2780\end{cases}$$

$$H_{17}(t)=\begin{cases}0, & t<0\\ 0.00055t, & 0\leq t<1438\\ 0.00005t+0.785519, & 1438\leq t<5752\\ 1, & t\geq 5752\end{cases}$$

$$H_{21}(t)=\begin{cases}0, & t<0\\ 0.0002t, & 0\geq t>4980\\ 1, & t\geq 4980\end{cases}$$

$$H_{27}(t)=\begin{cases}0, & t<630\\ 0.00278t-1.75, & 630\geq t>990\\ 1, & t\geq 990\end{cases}$$

$$H_{31}(t)=\begin{cases}0, & t<0\\ 0.00265t, & 0\geq t>378\\ 1, & t\geq 378\end{cases}$$

$$H_{47}(t)=\begin{cases}0, & t<0\\ 0.00526t, & 0\geq t>190\\ 1, & t\geq 190\end{cases}$$

$$H_{15}(t)=\begin{cases}0, & t<0\\ 0.0005t, & 0\leq t<1906\\ 0.00002t+0.95229, & 1906\leq t<4765\\ 1, & t\geq 4765\end{cases}$$

$$H_{52}(t)=\begin{cases}0, & t<0\\ 0.00417t, & 0\geq t>240\\ 1, & t\geq 240\end{cases}$$

$$H_{54}(t)=\begin{cases}0, & t<0\\ 0.00667t, & 0\geq t>150\\ 1, & t\geq 150\end{cases}$$

$$H_{56}(t)=\begin{cases}0, & t<0\\ 0.00154t, & 0\leq t<389\\ 0.00051t+0.4, & 389\leq t<1167\\ 1, & t\geq 1167\end{cases}$$

$$H_{57}(t)=\begin{cases}0, & t<0\\ 0.00031t, & 0\leq t<2869\\ 0.00002t+0.89986, & 2869\leq t<8607\\ 1, & t\geq 8607\end{cases}$$

$$H_{61}(t)=\begin{cases}0, & t<0\\ 0.00323t, & 0\geq t>310\\ 1, & t\geq 310\end{cases}$$

$$H_{65}(t)=\begin{cases}0, & t<300\\ 0.00131t, & 300\leq t<900\\ 0.00048t-0.00117, & 900\leq t<1200\\ 1, & t\geq 1200\end{cases}$$

$$H_{67}(t)=\begin{cases}0, & t<0\\ 0.0013t, & 0\leq t<615\\ 0.00033t-0.00055, & 615\leq t<1230\\ 1, & t\geq 1230\end{cases}$$

$$H_{71}(t)=\begin{cases}0, & t<62.5\\ 0.00064t, & 62.5\leq t<1432.5\\ 0.00009t-0.00034, & 1432.5\leq t<2802.5\\ 1, & t\geq 2802.5\end{cases}$$

$$H_{72}(t)=\begin{cases}0, & t<0\\ 0.00333t, & 0\geq t>300\\ 1, & t\geq 300\end{cases}$$

$$H_{75}(t) = \begin{cases} 0, & t < 0 \\ 0.00018t, & 0 \le t < 4850 \\ 0.00001t - 0.0002, & 4850 \le t < 14550 \\ 1, & t \ge 14550 \end{cases}$$

$$H_{76}(t) = \begin{cases} 0, & t < 0 \\ 0.00022t, & 0 \le t < 3850 \\ 0.00004t - 0.00046, & 3850 \le t < 7700 \\ 1, & t \ge 7700 \end{cases}$$

The remaining distribution functions, besides of those for which $b = l$, could not be evaluated because of the lack of data.

## 3. Complex technical system reliability

Taking into account the importance of the safety and operating process effectiveness of real technical systems it seems reasonable to expand the two-state approach to multi-state approach [2]-[17] in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time [11] gives the possibility for more precise analysis of their reliability and operation processes effectiveness.
To be able to apply practically the general joint models linking the multistate systems reliability models with the models of their operation processes to the evaluation the reliability of real complex technical systems it is necessary to use the statistical methods concerned with determining unknown parameters of the these models [11]. Particularly, the unknown parameters of the conditional multistate reliability functions of the system components at the various operation states should be identified. It is also necessary to have the methods of testing the hypotheses concerned with the conditional multistate reliability functions of the system components at the system various operation states.

### 3.1. System and its components reliability modeling and identification

In multistate reliability analysis, to define the system with degrading components we assume that its reliability states may be changed in time only from better to worse [11]. Then, the multistate reliability function of a component $E_i$, $i = 1,2,...,n,$ can be defined by the vector [11]

$$R_i(t,\cdot) = [R_i(t,0), R_i(t,1),..., R_i(t,z)],$$

where the coordinates

$$R_i(t,u) = P(E_i(t) \ge u \mid E_i(0) = z) = P(T_i(u) > t),$$

for $t \in \langle 0,+\infty)$, $i = 1,2,...,n,$, $u = 1,2,...,z,$

are the reliability functions defined as the probability that the component $E_i$ is in the reliability state subset $\{u, u+1,..., z\}$ at the moment $t$, $t \in \langle 0,+\infty)$, while it was in the reliability state $z$ at the moment $t = 0$ and $T_i(u)$ is the component $E_i$ lifetime in this subset of reliability states.
Further, we assume that the system components at the system operation states $z_b$, $b = 1,2,...,v,$ have the exponential reliability functions, i.e.

$$[R_i(t,u)]^{(b)} = \exp[-[\lambda_i(u)]^{(b)}]$$

for $t \in \langle 0,+\infty)$, $u = 1,2,...,z.$, $b = 1,2,...,v$.

The approximate data on system reliability components estimating the unknown parameters on the basis of expert opinion are used in case as we do not have the statistical data. Mainly, the mean values

$$[\mu(u)]^{(b)} = E[T^{(b)}(u)],\ u = 1,2,...,z,\ \ b = 1,2,...,v,$$

of the system component lifetimes $T^{(b)}(u)$, $u = 1,2,...,z$, $b = 1,2,...,v,$ in the reliability state subset $\{u, u+1,..., z\}$ while the system is at the operation state $z_b$ are estimated by experts. Further, we estimate the values $[\hat{\lambda}(u)]^{(b)}$ of the components unknown intensities of departure from the reliability state subsets $\{u, u+1,..., z\}$ using the following formula

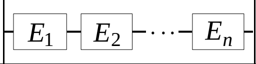
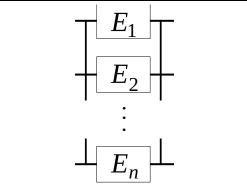
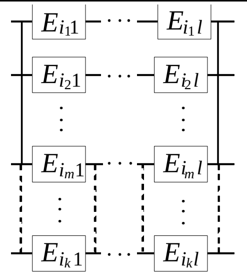
$$[\lambda(u)]^{(b)} \cong [\hat{\lambda}(u)]^{(b)} = \frac{1}{[\hat{\mu}(u)]^{(b)}},$$

(3)

for $u = 1,2,...,z,$ $b = 1,2,...,v.$

The selected for further considerations, the exemplary multistate system reliability structures in the reliability state subset $\{u, u+1,..., z\}$ are given in *Table 2*.
The numbers $m, k, l$ are called the system structure shape parameters.

*Table 2.* Selected reliability structures

| Structure | Scheme | Lifetime |
|---|---|---|
| Series |  | $T(u) = \min_{1 \le i \le n}\{T_i(u)\}$ |
| Parallel |  | $T(u) = \max_{1 \le i \le n}\{T_i(u)\}$ |
| Series – *m out of k* |  | $T_i(u) = \min_{1 \le j \le l}\{T_{ij}(u)\}$ $T(u) = T_{(k-m+1)}(u)$ |

### 3.2. System reliability at variable operation conditions

In reliability analysis of complex systems at the variable operation conditions we assume that the changes of the system operation process $Z(t)$ states have an impact on the system's components and its structure.

We denote the conditional multistate reliability function $[R_i(t,\cdot)]^{(b)}$ of the system component $E_i$, $i = 1,2,...,n,$ while the system is at the operation state $z_b$, $b = 1,2,...,v,$ by a vector

$$[R_i(t,\cdot)]^{(b)} = [1,[R_i(t,1)]^{(b)},...,[R_i(t,z)]^{(b)}],$$

where

$$[R_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t \mid Z(t) = z_b),$$

for $t \in \langle 0,+\infty)$, $u = 1,2,...,z$, $b = 1,2,...,v,$ and $T_i^{(b)}(u)$ is the component $E_i$ conditional lifetime in the subset of reliability states $\{u, u+1,..., z\}$ while the system is at the operation state $z_b$, $b = 1,2,...,v.$

Similarly, the conditional reliability function of the system at the operational state $z_b$, $b = 1,2,...,v,$ is defined by a vector

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [1,[\boldsymbol{R}(t,1)]^{(b)},...,[\boldsymbol{R}(t,z)]^{(b)}],$$

where $[\boldsymbol{R}_n(t,u)]^{(b)} = P(T^{(b)}(u) > t \mid Z(t) = z_b),$ for $t \in \langle 0,+\infty)$, $u = 1,2,...,z,$ $b = 1,2,...,v,$ $n \in N$ and $T^{(b)}(u)$ is the system conditional lifetime in the subset of reliability states $\{u, u+1,..., z\}$ while the system is at the operation state $z_b$, $b = 1,2,...,v.$

Under the above definitions, the unconditional reliability function of the system is given by

$$\boldsymbol{R}(t,\cdot) = [1,[\boldsymbol{R}(t,1)],...,[\boldsymbol{R}(t,z)]],$$

where $\boldsymbol{R}(t,u) = P(T(u) > t),$ for $t \in \langle 0,+\infty)$, $u = 1,2,...,z,$ and $T(u)$ is the unconditional lifetime of the system in the reliability state subset $\{u, u+1,..., z\}$.

In the case when the system operation time $\theta$ is large enough, the coordinates of the unconditional reliability function of the system are given by [11]

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)}, \qquad (4)$$

for $t \in \langle 0,+\infty)$, $u = 1,2,...,z,$, where $p_b$, $b = 1,2,...,v,$ are the system operation process limit transient probabilities [11].

Further, for $u = r,$ if $r$ is the system critical reliability state, then the system risk function is given by [11]

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,r), \qquad (5)$$

for $t \in \langle 0,+\infty)$ and if $\tau$ is the moment when the system risk function exceeds a permitted level $\delta$, then if $\boldsymbol{r}^{-1}(t)$ exists we have

$$\tau = \boldsymbol{r}^{-1}(\delta), \qquad (6)$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}(t)$.

### 3.3. Port oil transportation system components reliability identification

Based on expert opinion, there are distinguished three ($z = 2$) reliability states [11]:

- a reliability state 2 – piping operation is fully safe,
- a reliability state 1 – piping operation is less safe and more dangerous because of the possibility of environment pollution,
- a reliability state 0 – piping is destroyed.

The components of subsystems $S_v$, $v = 1,2,3,$ have reliability functions

$$[R_{ij}^{(v)}(t,\cdot)] = [1,[R_{ij}^{(v)}(t,1)]^{(b)},[R_{ij}^{(v)}(t,2)]],$$

with the exponential coordinates of the form

$$[R_{ij}^{(v)}(t,1)] = \exp[-\lambda_{ij}^{(v)}(1)t],$$
$$[R_{ij}^{(v)}(t,2)] = \exp[-\lambda_{ij}^{(v)}(2)t].$$

The approximate evaluation of the unknown intensities of departure of components results from formula (3).
The reliability parameters of each pipeline components of the port oil transportation system based on the data coming from experts are given in *Table 3* [11].

*Table 3.* Reliability parameters

| Subsystem | Coordinate | Components | Intensity |
|---|---|---|---|
| $S_1$ | $[R_{ij}^{(1)}(t,1)]$ | $i=1,2$<br>$j=1,2,...,176$ | 0.0062 |
| | | $i=1,2$<br>$j=177,178$ | 0.0167 |
| | $[R_{ij}^{(1)}(t,2)]$ | $i=1,2$<br>$j=1,2,...,176$ | 0.0088 |
| | | $i=1,2$<br>$j=177,178$ | 0.0182 |
| $S_2$ | $[R_{ij}^{(2)}(t,1)]$ | $i=1,2$<br>$j=1,2,...,717$ | 0.0062 |
| | | $i=1,2$<br>$j=718,719$ | 0.0166 |
| | $[R_{ij}^{(2)}(t,2)]$ | $i=1,2$<br>$j=1,2,...,717$ | 0.0088 |
| | | $i=1,2$<br>$j=718,719$ | 0.0181 |
| $S_3$ pipeline of the 1st type | $[R_{ij}^{(3)}(t,1)]$ | $i=1,2$<br>$j=1,2,...,360$ | 0.0059 |
| | | $i=1,2$<br>$j=361,362$ | 0.0166 |
| | $[R_{ij}^{(3)}(t,2)]$ | $i=1,2$<br>$j=1,2,...,360$ | 0.0074 |
| | | $i=1,2$<br>$j=361,362$ | 0.0181 |
| $S_3$ pipeline of the 2nd type | $[R_{ij}^{(3)}(t,1)]$ | $i=3$<br>$j=1,2,...,360$ | 0.0071 |
| | | $i=3$<br>$j=361,362$ | 0.0166 |
| | $[R_{ij}^{(3)}(t,2)]$ | $i=3$<br>$j=1,2,\ldots,360$ | 0.0079 |
| | | $i=3$<br>$j=361,362$ | 0.0181 |

## 4. Monte Carlo approach to system reliability evaluation

The Monte Carlo simulation uses randomly generated numbers to calculate approximate solutions of a given problem. In this article the native C# method *NextDouble()* was used for all calculations and each generated sequence of numbers from 0 to 1 was converted into time. Moreover, the lifetime of a complex system was determined and the failures were identified.

### 4.1. Monte Carlo simulation application to port oil transportation system reliability evaluation

The algorithm of Monte Carlo simulation for the reliability evaluation of the port oil piping transportation system is presented in *Figure 2*.

The first step is to define the initial operation state $z_b(g)$, $b = 1,2,5,6,7,$ using the formula

$$z_b(g) = \begin{cases} z_1, & 0 \le g < 0.34 \\ z_2, & 0.34 \le g < 0.39 \\ z_5, & 0.39 \le g < 0.62 \\ z_6, & 0.62 \le g < 0.81 \\ z_7, & 0.81 \le g \le 1 \end{cases}$$

where $g$ is a randomly generated number between 0 and 1. We can observe that according to (1), the port oil transportation system does not occupy the operation states $z_3$ and $z_4$ at the initial moment $t = 0,$ as the probabilities of staying in these operational states are equal to 0.
The next operation state $z_l$, $l = 1,2,...,7,$ is generated, according to (2), from $z_{bl}(g)$, $b = 1,2,5,6,7,$ defined as

$$z_{1l}(g) = \begin{cases} z_2, & 0 \le g < 0.022 \\ z_3, & 0.022 \le g < 0.044 \\ z_5, & 0.044 \le g < 0.578 \\ z_6, & 0.578 \le g < 0.689 \\ z_7, & 0.689 \le g \le 1, \end{cases}$$

$$z_{2l}(g) = \begin{cases} z_1, & 0 \le g < 0.2 \\ z_7, & 0.2 \le g \le 1, \end{cases}$$

$$z_{3l}(g) = z_1, \ 0 \le g \le 1,$$

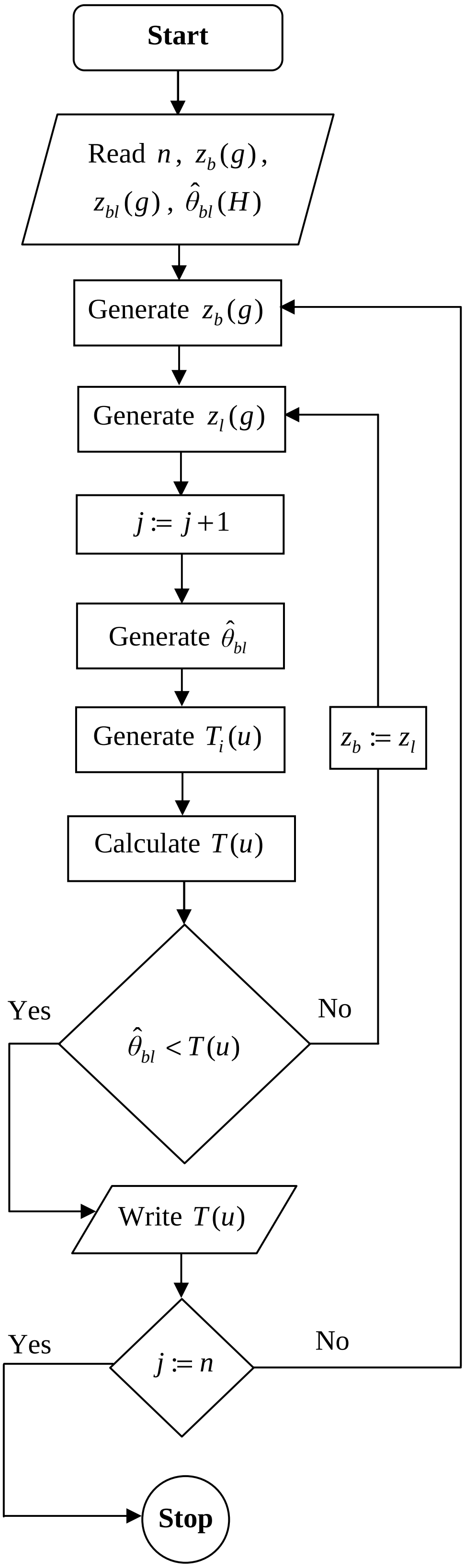



*Figure 2.* Monte Carlo algorithm for piping system reliability

$$z_{4l}(g) = z_7,\ 0 \le g \le 1,$$

$$z_{5l}(g) = \begin{cases} z_1, & 0 \le g < 0.488 \\ z_2, & 0.488 \le g < 0.511 \\ z_4, & 0.511 \le g < 0.534 \\ z_6, & 0.534 \le g < 0.767 \\ z_7, & 0.767 \le g \le 1, \end{cases}$$

$$z_{6l}(g) = \begin{cases} z_1, & 0 \le g < 0.095 \\ z_5, & 0.095 \le g < 0.762 \\ z_7, & 0.762 \le g \le 1, \end{cases}$$

$$z_{7l}(g) = \begin{cases} z_1, & 0 \le g < 0.531 \\ z_2, & 0.531 \le g < 0.593 \\ z_5, & 0.593 \le g < 0.812 \\ z_6, & 0.812 \le g \le 1. \end{cases}$$

For instance, if $z_b(g) = z_1$, then the next operation state would be $z_2$, $z_3$, $z_5$, $z_6$ or $z_7$ generated from $z_{1l}(g)$.

To apply the Monte Carlo method we assume that the particular conditional sojourn times at the operation state $z_b$, $b = 1,2,...,7$, are randomly generated using the inverse functions $\hat{\theta}_{bl}(H)$ of the empirical distribution functions $H_{bl}(t)$ defined in subsection 2.2. This way, the empirical conditional sojourn times measured in hours are as follows

$$\hat{\theta}_{12}(H) = 960H,$$

$$\hat{\theta}_{13}(H) = 240H,$$

$$\hat{\theta}_{15}(H) = \begin{cases} 3442.67H, & 0 \le H < 0.75 \\ 15492 \times (H - 0.58), & 0.75 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{16}(H) = 2780H,$$

$$\hat{\theta}_{17}(H) = \begin{cases} 1830.18H, & 0 \le H < 0.79 \\ 20132 \times (H - 0.71), & 0.79 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{21}(H) = 4980H,$$

$$\hat{\theta}_{27}(H) = 360 \times (H + 1.75),$$

$$\hat{\theta}_{31}(H) = 378H,$$

$$\hat{\theta}_{47}(H) = 190H,$$

$$\hat{\theta}_{51}(H) = \begin{cases} 2001.3H, & 0 \le H < 0.95 \\ 60039 \times (H - 0.92), & 0.95 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{52}(H) = 240H,$$

$$\hat{\theta}_{54}(H) = 150H,$$

$$\hat{\theta}_{56}(H) = \begin{cases} 648.33H, & 0 \le H < 0.6 \\ 1945 \times (H - 0.4), & 0.6 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{57}(H) = \begin{cases} 3187.78H, & 0 \le H < 0.9 \\ 57380 \times (H - 0.85), & 0.9 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{61}(H) = 310H,$$

$$\hat{\theta}_{65}(H) = \begin{cases} 4200H & 0 \le H < 0.07 \\ 763.64 \times (H + 0.32), & 0.07 \le H < 0.86 \\ 2100 \times (H - 0.43), & 0.86 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{67}(H) = \begin{cases} 768.75H, & 0 \le H < 0.8 \\ 3075 \times (H - 0.6), & 0.8 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{71}(H) = \begin{cases} 1552.67H, & 0 \le H < 0.92 \\ 11645 \times (H - 0.8), & 0.92 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{72}(H) = 300H,$$

$$\hat{\theta}_{75}(H) = \begin{cases} 5658.33H, & 0 \le H < 0.86 \\ 67900 \times (H - 0.79), & 0.86 \le H \le 1, \end{cases}$$

$$\hat{\theta}_{76}(H) = \begin{cases} 4620H, & 0 \le H < 0.83 \\ 23100 \times (H - 0.67), & 0.83 \le H \le 1, \end{cases}$$

where $H$ is a randomly generated number between 0 and 1.
The system conditional lifetimes in the reliability states subsets $\{u, u+1, ..., z\}$ are approximated using exponential sampling formula

$$T_{ij}(u) = -\frac{1}{\lambda_{ij}^{(v)}(u)} ln(1-g),$$

where $\lambda_{ij}^{(v)}(u)$, $i = 1,2,...,v$, is the intensity of the subsystem $S_v$, $v = 1,2,3$, given in *Table 3*, $g$ is a randomly generated number from 0 to 1. The lifetime of the system is counted according to the formula given in *Table 2*.
In this paper we will focus on the multistate approach in the reliability analysis by the assumption that the reliability state 1 is a critical one. More general approach will be discussed in the future papers. The simulation was made with 1000 runs. The results for the generated operational states are presented in *Table 4*. The initial operation state is $z_5$. The histogram of the pipeline system lifetime is presented in *Table 5* and illustrated in *Figure 3*.

*Table 4.* Comparison of tries and failures

| State | Nº of transitions | Nº of failures |
|---|---|---|
| 12 | 1098 | 0 |
| 13 | 1047 | 0 |
| 15 | 26923 | 10 |
| 16 | 5545 | 1 |
| 17 | 15481 | 3 |
| 21 | 924 | 0 |
| 27 | 3759 | 0 |
| 31 | 1047 | 3 |
| 47 | 1239 | 3 |
| 51 | 25468 | 275 |
| 52 | 1204 | 1 |
| 54 | 1239 | 0 |
| 56 | 12307 | 52 |
| 57 | 12089 | 241 |
| 61 | 2443 | 6 |
| 65 | 17263 | 111 |
| 67 | 6091 | 30 |
| 71 | 20569 | 73 |
| 72 | 2382 | 0 |
| 75 | 8367 | 126 |
| 76 | 7064 | 65 |

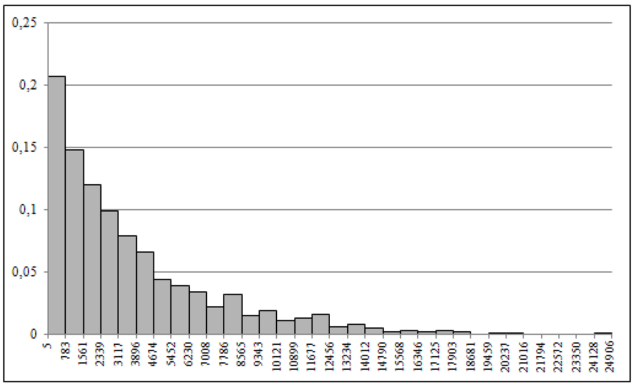



*Figure 3.* Graph of the histogram of the pipeline system lifetime

*Table 5.* Histogram of the pipeline system lifetime

| № | $x^j$ | $y^j$ | $n^j$ | $f(t,1)$ |
|---|---|---|---|---|
| 1 | 4.70 | 782.88 | 207 | 0.207 |
| 2 | 782.88 | 1561.06 | 148 | 0.148 |
| 3 | 1561.06 | 2339.24 | 120 | 0.120 |
| 4 | 2339.24 | 3117.41 | 99 | 0.099 |
| 5 | 3117.41 | 3895.59 | 79 | 0.079 |
| 6 | 3895.59 | 4673.77 | 66 | 0.066 |
| 7 | 4673.77 | 5451.95 | 44 | 0.044 |
| 8 | 5451.95 | 6230.13 | 39 | 0.039 |
| 9 | 6230.13 | 7008.30 | 34 | 0.034 |
| 10 | 7008.30 | 7786.48 | 22 | 0.022 |
| 11 | 7786.48 | 8564.66 | 32 | 0.032 |
| 12 | 8564.66 | 9342.84 | 15 | 0.015 |
| 13 | 9342.84 | 10121.02 | 19 | 0.019 |
| 14 | 10121.02 | 10899.20 | 11 | 0.011 |
| 15 | 10899.20 | 11677.37 | 13 | 0.013 |
| 16 | 11677.37 | 12455.55 | 16 | 0.016 |
| 17 | 12455.55 | 13233.73 | 6 | 0.006 |
| 18 | 13233.73 | 14011.91 | 8 | 0.008 |
| 19 | 14011.91 | 14790.09 | 5 | 0.005 |
| 20 | 14790.09 | 15568.26 | 2 | 0.002 |
| 21 | 15568.26 | 16346.44 | 3 | 0.003 |
| 22 | 16346.44 | 17124.62 | 2 | 0.002 |
| 23 | 17124.62 | 17902.80 | 3 | 0.003 |
| 24 | 17902.80 | 18680.98 | 2 | 0.002 |
| 25 | 18680.98 | 19459.15 | 0 | 0 |
| 26 | 19459.15 | 20237.33 | 1 | 0.001 |
| 27 | 20237.33 | 21015.51 | 1 | 0.001 |
| 28 | 21015.51 | 21793.69 | 0 | 0 |
| 29 | 21793.69 | 22571.87 | 0 | 0 |
| 30 | 22571.87 | 23350.04 | 0 | 0 |
| 31 | 23350.04 | 24128.22 | 0 | 0 |
| 32 | 24128.22 | 24906.40 | 1 | 0.001 |

After analyzing and comparing the histogram with the graph of exponential distribution function

$$f(t,1) = \begin{cases} 0, & t < 0 \\ \lambda(1)\exp[-\lambda(1)t], t \geq 0, \end{cases}$$

where $0 \leq \lambda(1) < +\infty$, we formulate the null hypothesis:

$H_0$: The pipeline transportation system has the exponential reliability function.

Further, we estimate the unknown parameter $\lambda(u)$ of the density function of the hypothetical exponential distribution and obtain

$$\lambda(1) = \frac{1}{\overline{T}(1)} \cong \frac{1}{3719.56} \cong 0.00027, \qquad (7)$$

where $\overline{T}(1)$ is the empirical mean value of system conditional lifetimes in the reliability state subset $\{1,2\}$.

Hence, we get the following form of the reliability function coordinate

$$\boldsymbol{R}(t,1) = \begin{cases} 0, & t < 0 \\ \exp[-0.00027t], & t \geq 0. \end{cases}$$

To verify the hypothesis $H_0$ we join the intervals $I_j = \langle x^j, y^j)$ that have the number $n^j$ of realizations less than 4 into $\bar{r} = 22$ new intervals.
The new intervals and new realizations of the histogram are presented in *Table 6*.

*Table 6.* Joined intervals and new realizations of the histogram of the pipeline system lifetime

| № | $\bar{x}^j$ | $\bar{y}^j$ | $\bar{n}^j$ | $p_j$ |
|---|---|---|---|---|
| 1 | 4.70 | 782.88 | 207 | 0,19 |
| 2 | 782.88 | 1561.06 | 148 | 0,15 |
| 3 | 1561.06 | 2339.24 | 120 | 0,12 |
| 4 | 2339.24 | 3117.41 | 99 | 0,10 |
| 5 | 3117.41 | 3895.59 | 79 | 0,08 |
| 6 | 3895.59 | 4673.77 | 66 | 0,07 |
| 7 | 4673.77 | 5451.95 | 44 | 0,05 |
| 8 | 5451.95 | 6230.13 | 39 | 0,04 |
| 9 | 6230.13 | 7008.30 | 34 | 0,04 |
| 10 | 7008.30 | 7786.48 | 22 | 0,03 |
| 11 | 7786.48 | 8564.66 | 32 | 0,02 |
| 12 | 8564.66 | 9342.84 | 15 | 0,02 |
| 13 | 9342.84 | 10121.02 | 19 | 0,02 |
| 14 | 10121.02 | 10899.20 | 11 | 0,01 |
| 15 | 10899.20 | 11677.37 | 13 | 0,01 |
| 16 | 11677.37 | 12455.55 | 16 | 0,01 |
| 17 | 12455.55 | 13233.73 | 6 | 0,01 |
| 18 | 13233.73 | 14011.91 | 8 | 0,01 |
| 19 | 14011.91 | 14790.09 | 5 | 0,00 |
| 20 | 14790.09 | 16346.44 | 5 | 0,01 |
| 21 | 16346.44 | 17902.80 | 5 | 0,00 |
| 22 | 17902.80 | 24906.40 | 5 | 0,01 |

The hypothetical probabilities that the system lifetime $T(u)$ takes values from the new intervals are given according to the formula

$$p_j(u) = P(T(u) \in \bar{I}_j(u))$$
$$= \boldsymbol{R}(\bar{x}^j(u),u) - \boldsymbol{R}(\bar{y}^j(u),u),$$

for $j = 1,2,...,\bar{r}$, under the assumption that the hypothesis $H_0$ is true.

The next step is to calculate the realization of the $\chi^2$ (chi-square)-Pearson's statistics $u_n$, according to the formula given in [11], which amounts

$$u_n = \sum_{j=1}^{\bar{r}(u)} \frac{\left(\bar{n}^j(u) - \bar{n}p_j(u)\right)^2}{\bar{n}p_j(u)} \cong 21.89.$$

Assuming the significance level $\alpha = 0.05$ for $\bar{r}(1) - l - 1 = 22 - 1 - 1 = 20$ degrees of freedom, from the tables of the $\chi^2$-Pearson's distribution we find the value $u_\alpha = 31.41$. The critical domain and acceptance domain in the form of the intervals are presented in *Figure 4*.

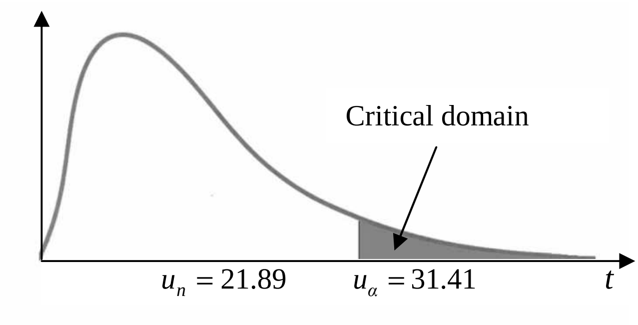



*Figure 4.* The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

The obtained value $u_n$ belongs to the acceptance domain

$$u_n = 21.89 \le u_\alpha = 31.41,$$

thus, at the significance level $\alpha = 0.05$ we do not reject the hypothesis $H_0$ stating that the pipeline system reliability function is exponential.

## 4.2. Comparison of the results

Based on the analytical formulas (4)-(6) and assuming $r = 1$, the following results were obtained [11]:

- the expected value of the system unconditional lifetimes in the reliability state subset $\{1,2\}$

$$\mu(1) = \int_0^{+\infty} \boldsymbol{R}(t,1)\,dt = 0.37 \text{ year;}$$

- the system risk function, when $r = 1$ is the system critical reliability state, is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,1),$$

for $t \in \langle 0,+\infty)$, where $\boldsymbol{R}(t,1)$ is given in [11] by

$$\begin{aligned}\boldsymbol{R}(t,1) = {} & 4\exp[-9.9176t] + 8\exp[-10.3496t] \\ & - 8\exp[-12.5078t] - 2\exp[-14.396t] \\ & - 4\exp[-14.8282t] + 4\exp[-16.9864t] \\ & - 2\exp[-11.0422t] - 4\exp[-11.4742t] \\ & + 4\exp[-13.6324t] + \exp[-15.5208t] \\ & + 2\exp[-15.9528t] - 2\exp[-18.111t];\end{aligned}$$

- the moment when the system risk function exceeds a permitted level $\delta = 0.05$

$$\tau = \boldsymbol{r}^{-1}(\delta) = 0.066 \text{ year,}$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the risk function $\boldsymbol{r}(t)$.

The values of those characteristics obtained by using Monte Carlo method according to (7) are presented below

$$\mu(1) = \int_0^{+\infty} \exp[-0.00027t]\,dt = 0.42,$$

$$\boldsymbol{r}(t) = 1 - \exp[-0.00027t],$$

$$\tau = \boldsymbol{r}^{-1}(0.05) = -3719.56\ln[0.95] \cong 0.022 \text{ year.}$$

The graph of the risk function $\boldsymbol{r}(t)$ of the piping transportation system is given in *Figure 5*.

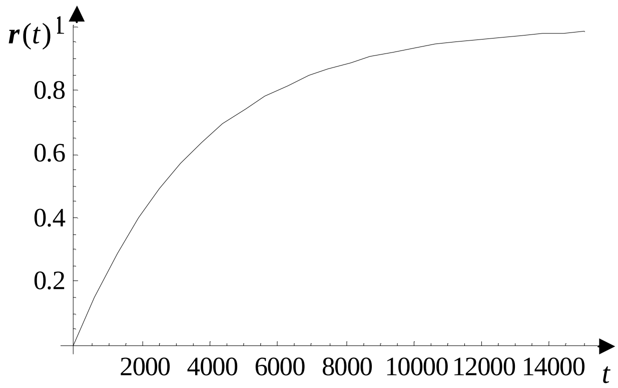



*Figure 5.* The graph of the piping transportation system risk function $\boldsymbol{r}(t)$

## 5. Conclusions

The Monte Carlo simulation method was used for complex technical system reliability evaluation. The obtained results were compared with the results of the analytical methods presented in [11]. The differences are enough large and therefore further analysis of these two methods is necessary and their accuracy have to be investigated and their convergence improved.

The first and natural idea of analysis is to observe whether the increasing the number of runs provides more accurate simulation results.

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## References

[1] Grabski, F. (2002). *Models of Systems Reliability and Operations Analysis* (*in Polish*). System Research Institute, Polish Academy of Science. 2002.

[2] Kołowrocki, K. & Soszyńska, J. (2009). Reliability, risk and availability based optimization of complex technical systems operation processes. Part 1. Theoretical backgrounds. *Electronic Journal Reliability & Risk Analysis: Theory & Application*, Vol. 2, No 4, 142-152.

[3] Kołowrocki, K. & Soszyńska, J. (2009). Reliability, risk and availability based optimization of complex technical systems operation processes. Part 2. Application in port transportation. *Electronic Journal Reliability & Risk Analysis: Theory & Application*, Vol. 2, No 4, 153-167.

[4] Kołowrocki, K. & Soszyńska, J. (2010). Reliability modeling of a port oil transportation system's operation processes. *International Journal of Performability Engineering*, Vol. 6, No 1, 77-87.

[5] Kolowrocki, K. & Soszynska, J. (2009). Safety and risk evaluation of Stena Baltica ferry in variable operation conditions. *Electronic Journal Reliability & Risk Analysis: Theory & Applications*, Vol.2, No 4, 168-180.

[6] Kolowrocki, K. & Soszynska, J. (2010). Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems (part1). *Electronic Journal Reliability: Theory and Applications*, Vol. 1, No 2, 184-200.

[7] Kolowrocki, K. & Soszynska, J. (2010). Methods and algorithms for evaluating unknown parameters of components reliability of complex technical systems (part2). *Electronic Journal Reliability: Theory and Applications*, Vol. 1, No 2, 201-210.

[8] Kolowrocki, K. & Soszynska, J. (2010). Reliability, availability and safety of complex technical systems: modelling – identification – prediction – optimization. *Summer Safety and Reliability Seminars – SSARS 2010, Journal of Polish Safety and Reliability Association*, Vol. 1, 133-158.

[9] Kolowrocki, K. & Soszynska, J. (2010). Safety and risk optimization of a ferry technical system. *Summer Safety and Reliability Seminars – SSARS 2010, Journal of Polish Safety and Reliability Association*, Vol. 1, 159-172.

[10] Kolowrocki, K. &. Soszynska, J. (2011). On safety analysis of complex technical maritime transportation system. *Journal of Risk and Reliability*, 225 (3), 345-354.

[11] Kołowrocki, K. & Soszyńska-Budny, J. (2011). *Reliability and Safety of Complex Technical Systems and Processes: Modeling – Identification – Prediction – Optimization*. Springer, ISBN 978-0-85729-693-1.

[12] Soszyńska, J. (2007). Systems reliability analysis in variable operation conditions. International *Journal of Reliability, Quality and Safety Engineering*. Special Issue: System Reliability and Safety, Vol. 14, No 6, 617-634.

[13] Soszyńska, J. (2009). Asymptotic approach to reliability evaluation of large "m out of l" – series system in variable operation conditions. *Electronic Journal Reliability & Risk Analysis: Theory & Application*, Vol. 2, No 2, 9-43.

[14] Soszyńska, J. (2010). Reliability and risk evaluation of a port oil pipeline transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping* Vol. 87, No 2-3, 81-87.

[15] Soszyńska-Budny, J. (2011). Reliability and risk evaluation of a container gantry crane at variable operation conditions. *Summer Safety and Reliability Seminars – SSARS 2011, Journal of Polish Safety and Reliability Association*, Vol. 2, 453-463.

[16] Sun, Z-L., Ming Ng, K., Soszyńska-Budny, J. & Habibullah, M.S. (2011). Application of the LP-ELM model on transportation system lifetime optimization. *IEEE Transactions on Intelligent Transportation Systems*, Vol.12, Issue 4, 1484-1494.

[17] Xue, J. &. Yang, K. (1995). Dynamic reliability analysis of coherent multi-state systems. *IEEE Transactions on Reliability* 4, Vol. 44, 683-688.
[18] Zio, E. & Marseguerra, M. (2002). *Basics of the Monte Carlo Method with Application to System Reliability*. LiLoLe, ISBN 3-934447-06-6.