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Towards the development of a topological model to assess networks performance: connectivity, robustness and reliability

Keywords

network, robustness, connectivity, reliability, failure, critical states

Abstract

Networks are the widest spreading system which Mankind has ever developed, today. New needs, services, and use are created each day where network systems represent the most critical aspect. Additionally, the interconnection between networks of different types is growing as never. This acceleration of interconnection of all sorts brings to the front scene the issue of performance measure in networks design and operation. For that purpose, engineers are orienting their major efforts towards the development of methods, models and codes to assess and measure networks performance in terms of probability of being connected. From this stand point of view, the engineers' attention is mainly focused on defining the network "Connectivity" and measuring it, in different manner, using probabilities. Subsequently, we are observing an accelerating course towards quantitative probabilistic models to describe and assess networks' Connectivity, since the sixties. Modelling realistic networks is still far from being satisfactorily achieved using quantitative probabilistic models.

On the other hand, little room has been lift to exploring the potential of graphical and topological models to develop qualitative and semi- quantitative models in order to assess networks performance. In this paper, we have tried to explore the potential of the topological modelling and are proposing an original one. The proposed model is still in its earliest phase of development. But, it sounds very promoting at that stage of maturation.

According to this model, the term "performance" may be extended, beyond the "connectivity probabilistic concept", to "robustness" and to a "deepest insight" of the "connectivity concept" itself. The impact is immediate on the way the "reliability concept" would be extended to cover network systems.

1. Introduction

Networks are the widest spreading system which Mankind has ever developed. New needs, services, and use are created each day where network systems represent the most critical aspect.

Network systems intervene in the tiniest of our daily-life activities: energy transmission (in all its forms), man and goods transportation, communications, distant inspection and control, detection and diagnosis, ...

The interconnection between networks of different types is growing as never. This acceleration of interconnection of all sorts creates higher pressure on network performances. It brings to the front scene the issue of performance measure in networks design and operation. Subsequently, the major effort of engineers today is oriented towards the development of methods, models and codes to assess and measure networks performance in terms of probability of being connected.

But, what "connected" is?

From this stand point of view, the engineers' attention is mainly focused on defining the network "Connectivity" and measuring it using probabilities. Subsequently, we are observing an

accelerating course towards quantitative probabilistic models to describe and assess networks' Connectivity, since the sixties. While the use of probabilities is a seducing target, the concept of "connectivity" itself still needs deeper understanding.
Presently, the situation is that a little room has been lift to exploring the potential of graphical and topological models to develop qualitative and semi-quantitative models in order to better understand and describe the "connectivity", and later, assess networks performance.
In this paper, we have tried to explore the potential of the topological modelling and are proposing an original one. It describes the connectivity in terms of tensors of different orders. In the paper, we focused on the applicability of the model rather than on its mathematical nature and bases.
The proposed model is still in its earliest phase of development. But, it sounds very promoting at that stage of maturation.
Subsequently, the term "performance" may be extended, beyond the probabilistic "connectivity concept", to "robustness" and to a "deepest insight" of the "connectivity concept" itself. The impact is immediate on the way the "reliability concept" would be extended to cover realistic network systems.

2. Description

Some researchers call it "terminal-pair reliability – TPR" problem. In communication engineering filed one may distinguish two different types of network topologies: Point-to-Multipoint (PMP) and mesh type networks. Mesh networks permit to largely extend the covered zone. However it introduces additional effects of interference which may decrease the quality of the received signals.
Reliability models for the PMP are given in [2], [11], [16], [19], [20] while other models treating the Mesh Network are given in [1], [8].
The most common manner to represent networks is to use graphs. A graph $G=(V,E)$ is a well-defined set of vertices (nodes), V, and edges (links), E.
Each node and each link is defined by a failure probability or a failure rate. These failure figures are functions of the used materials, technology, operational conditions and network's topology.
In mesh networks, three links-failure modes are generally identified such as: path loss, shadowing and signal fading [6]. These are the failure modes recognized by the IEEE 802.16 WG for communication networks.
Exponential models (Poisson's stochastic process) are often proposed [6], to describe failures occurrence.
Often, Reliability and availability concepts are confused [10].
Mandiratta, [14], recalls "Network reliability refers to the reliability of the overall network to provide communication in the event of a failure of a component in the network, and it depends on the sustainability of both hardware and software."
I, myself, would call that aptitude "Network Availability"!
Most of the researchers are working on the modeling of the *Connectivity* and the *Performance*, [14]. The *Connectivity* is thus defined as: the availability of a path from a source node to a destination node. So, it is not a network OVERALL measure, it is node-to-node local measure! The *Performance* is rather defined as: the ability of the network, in the presence of failures, to preserve existing connections (no dropped calls) and to initiate new connections (no blocked calls).
The same definition can be used for all other kinds of networks after minor syntax adaptations.

3. Connectivity modelling

As in [14], we opt for the Network *Connectivity* metric, $C(t)$, such as:

$$C(t) = \frac{N_{con}(t)}{N_{tot}},$$

where,

$N_{con}(t)$: is the number of connected node-pairs at time t,
N_{tot}: is the total number of node-pairs in the network
Very often distinction is done between three modes of network *Connectivity* measure, *such as:*

<u>*Two-terminal connectivity*</u>
It measures the ability of the network to satisfy the communication needs of a specific pair of nodes. *Two-terminal* availability defines the probability that there is at least one available path in the network connecting a specified pair of nodes.

k-terminal connectivity
It measures the ability of the network to satisfy the communication needs of a subset *k* of specified nodes. The *k-terminal* availability is defined as the probability that for *k* specified target nodes there is at least one connecting path between each pair of the *k* nodes, in the network.

All-terminal connectivity
It measures the ability of the network to satisfy the communication needs of all nodes in the network. *All-terminal availability* determines the probability that there is at least one path connecting each pair of nodes in the network

Mendiratta, [14], proposed a model to determine the network *connectivity*, based on a given minimal *Connectivity* condition. In that model, n is the minimal acceptable number of connecting nodes in a network containing N nodes. The network *connectivity*, under this minimal connectivity condition, will be measured such that:

$$P(n:N;C)=\sum_{j=n}^{N}\binom{N}{j}p^{j}(1-p)^{N-j}$$

where,

$P(n:N;C)$: The probability that the network is available (/connecting) (at least n nodes out of N are available).

$\binom{N}{j}$: The number of possible combinations. The number of possible sets containing j nodes out of N.

p: The probability that a given node is available (connected)

Many works have been carried on in order to develop algorithms based on the previous model, [3]. Besides, most of the researchers, working on the determination of the network availability, treat the problem as consecutive k-out-of-N failure (K:N;F) problem.

Very often, the following assumptions are considered:

- Nodes are completely reliable; only links fail. (failure rates of nodes are by so far smaller than the link failure ones).
- Link failures are independently random events, [5], [17].
- Sometimes link failures are supposed equally probable. This assumption is often made because no detailed information about link failures is available, whereas information about the average failure is available, [9].

Most of R&D efforts are devoted to the development of numerical algorithms in order to determine the probability that a given network may have a determined level of *connectivity*, [2], [4], [7], [11], [12], [14], [16], [20].

Most of the researchers called this probability “Reliability” of consecutive K-out-of-N system. Recently, this has been evolved to a k-within consecutive-$r\times s$-out-of-$m\times n:F$ system, [13]. This is a generalisation of the problem of consecutive k-ou-of-N failures.

The network availability is generally determined either analytically or numerically. Analytical schemes are limited by the size and the topological complexity of the network. For large and complex network numerical methods such as: Mont-Carlo simulation, neural-network models [5], or genetic algorithms are developed.

Some interesting applications are given in [18], [15].

Another major task in network design is to optimize network reliability (connectivity, performance, resilience, …) versus cost.

Three classic meta-heuristic procedures are often recognized for solving large and realistic designs: steepest descent, simulated annealing and genetic algorithms.

These procedures are clearly described and compared in [4], with an interesting list of corresponding references. The final result of the optimization procedure depends on which measure of the connectivity one will consider.

The 1st set of difficulties is related to failure data availability. The second difficulty arises from the combinatorial aspect of the problem. The 3rd difficulty arises from the interdependence between different failure paths (cut-sets).

4. A topological model

Networks may be represented using graphs. A graph $G=(V,E)$ is a well-defined set of vertices (nodes), V, and edges (links), E.

A network is a graph containing at least 3 nodes, $N\geq 3$.

We are looking for constructing a connectivity measure dependant on the number of links necessary to link two nodes in a given network. The proposed connectivity measure C_{ij}^{n} is a (binary) scalar (tensor) of order n describes the connecting state between two independent nodes i and j in a given network, such that: it takes the value 1 if the two nodes are connected, otherwise it takes the value 0. The order n refers to the minimal necessary number of links to connect the two nodes, i.e. the order of the minimal cut-set to connect.

The C_{il}^{1} is called the network identity tensor. It describes the 1st order connecting state between all the nodes, i.e., it determines the couples of nodes that are directly connected. In a certain way, it describes the topology of the network and contains all the information we need to know about the connectivity state of the network.

Connectivity

We will define a connectivity measure C_{ij}^{n+1} exclusively based on the topological characteristics of the network. This connectivity measure, C_{ij}^{n+1}, is determined by the following recursive relation:

$$T_{ij}^{n+1} = C_{il}^{1} \,.\, C_{lj}^{n} \quad i, l, j \in [1, N]$$

$$C_{ij}^{n+1}(T_{ij}^{n+1}) = 1, \forall T_{ij}^{n+1} > 0$$

otherwise,

$$C_{ij}^{n+1}(T_{ij}^{n+1}) = 0$$

where,

C_{il}^{1}: is the network identity tensor.

T_{ij}^{n+1}: is the total number of cuts of order less than or equal to $n+1$, connecting two nodes (i, j).

N : is the total number of nodes in the network.

For each node, one would determine a connectivity indicator, I_i^n, and a connectivity ratio, R_i^n, of a given order, such that:

$$I_i^n = \left(\sum_{j=1}^{N} C_{ij}^{n}\right) - 1,$$

and

$$R_i^n = \frac{I_i^n}{(N-1)}$$

Based on all nodes' connectivity indicator, one can define a network overall connectivity indicator, $I_{overall}^{n}$, and connectivity ratio, $R_{overall}^{n}$, of a given order, such that:

$$I_{overall}^{n} = \sum_{j=1}^{N} I_i^n ,$$

and

$$R_{overall}^{n} = \frac{\sum_{i=1}^{N} R_i^n}{N}$$

At last, one may define the network highest order, n_{∞}, such that, this is the minimum necessary order of cut-sets (links) so that all nodes may become mutually connected. That is can be described as following:

$$n_{\infty} = Min.\left(n; C_{ij}^{n} = 1, \forall i, j \in [1, N]\right)$$

Robustness

Robustness is a measure of a network tolerance to failure. One possible manner to qualitatively measure network's robustness is to use the network overall 1st order connectivity indicator, $I_{overall}^{1}$, or the network 1st order overall connectivity ratio, $R_{overall}^{1}$. Higher is $I_{overall}^{1}$ (or $R_{overall}^{1}$), higher is the network Robustness.

The most robust network (for a given technology, and given operating conditions) will be for

$$I_{overall}^{1} = N(N-1) \text{ (or } R_{overall}^{1} = 100\%).$$

The worst robust network (for a given technology, and given operating conditions) will be for

$$I_{overall}^{1} = 2N \text{ (or } R_{overall}^{1} = \frac{2}{(N-1)}\%).$$

Where N is the total number of nodes in the nodes in the network.

Reliability

Reliability is a measure of a network probability to fail. One possible manner to qualitatively measure the network's reliability is to use the network highest order n_{∞} indicator. Lower is n_{∞}, higher is the network Reliability.

The most reliable network (for a given technology, and given operating conditions) will be at n_{∞} = 1.

Where N is the total number of nodes in the network.
The worst reliable network (for a given technology, and given operating conditions) will be for $n_\infty \le \frac{N-2}{2}+1$. Where N is the total number of nodes in the network.

<u>*Author's Remarks*</u>
This way of describing the network seems to us promoting and would open a wide field to more advanced models describing networks connectivity, robustness, and reliability.

In fact, the author sees the connectivity as the most fundamental measure in describing the performance of a network. Still, it is not the most explored compared to the reliability measure, as we have already detailed above.
We are going to demonstrate in the following that the proposed model may be used to assess the network connectivity in a deepest and simplest manner. It produces pertinent measures although it does not directly produce probabilities.

5. Case description

The studied network is schematically presented in *Figure 1*.

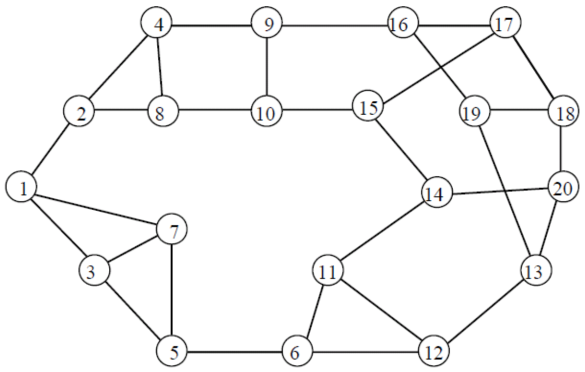

Figure1. Schematic presentation of NET-I, [5]

It describes a network composed of 20 nodes and 30 links, exported from [5]. Each node is connected to 3 other nodes. It belongs to PMP type of networks with connectivity of the order 2 according to the conventional terminology. That is one of the most widely used PMP type of connectivity in networks design [2], [11], [16], [19], [20].
The term connectivity at that level is local and only describes the connectivity between each node and the others. It does, by no means, describe the overall network connectivity.

One may use a metric like the one proposed in[14] to measure the overall network connectivity, which is based on the quotient between the number of connected node-pairs and the total number of node-pairs in the network. But, we can argue that it may not be rigorous enough.
The 1st step in applying the proposed model is to construct the network identity tensor C_{ij}^{1}.
Examining the presentation given in *Figure 1*, one may deduce the identity matrix given in *Table 1* or expressed in the following manner:

$$C_{ij}^{1} = 1,$$

For the following (i, j):
$(1,2)$, $(1,3)$, $(1,7)$, $(2,1)$, $(2,4)$, $(2,8)$, $(3,1)$, $(3,5)$, $(3,7)$, $(4,2)$, $(4,8)$, $(4,9)$, $(5,3)$, $(5,6)$, $(5,7)$, $(6,5)$, $(6,11)$, $(6,12)$, $(7,1)$, $(7,3)$, $(7,5)$, $(8,2)$, $(8,4)$, $(8,10)$, $(9,4)$, $(9,10)$, $(9,16)$, $(10,8)$, $(10,9)$, $(10,15)$, $(11,6)$, $(11,12)$, $(11,14)$, $(12,6)$, $(12,11)$, $(12,13)$, $(13,12)$, $(13,19)$, $(13,20)$, $(14,11)$, $(14,15)$, $(14,20)$, $(15,10)$, $(15,14)$, $(15,17)$, $(16,9)$, $(16,17)$, $(16,19)$, $(17,15)$, $(17,16)$, $(17,18)$, $(18,17)$, $(18,19)$, $(18,20)$, $(19,13)$, $(19,16)$, $(19,13)$, $(19,18)$, $(20,13)$, $(20,14)$, $(20,18)$, (30 connected node-pairs).

All other node-pairs (links) are not directly connected and their corresponding matrix elements are, then, equal to zero.

6. Network connectivity

In a network with N nodes, the potential number of the node-pairs M is given by:

$$M = \frac{N(N-1)}{2}$$

For N =20, the potential number of connected node-pairs M is 190.
If one uses the metric proposed in [14] to measure the overall network connectivity $C_{overall}^{1}$, that will give:

$$C_{overall}^{1} = \frac{30}{190} \approx 15.8\%$$

It is worth recalling that $C_{overall}^{1}$ is the network overall 1st order connectivity, i.e. it determines

connections with cut-sets of 1st orders: only one egg between each connected node-pair.
According to the proposed model, higher orders of connectivity can be determined, as well.
Let's determine the second order connectivity tensor, C_{ij}^2. According to the proposed model, C_{ij}^2 is determined as following:

$$T_{ij}^2 = C_{il}^1 \, . \, C_{lj}^1 \, ,$$

then

$$C_{ij}^2 = 1,$$

if

$$T_{ij}^2 > 0$$

$$C_{ij}^2 = 0., \text{ otherwise}$$

Applying the model on our case study, the progress of the network overall connectivity with the order of the connectivity can be analysed in tables 1 to 6.
In the tables, we have determined both the PMP connectivity of different orders for each node in the network Net-I.
In order to illustrate the interest of the model, we modified the original network design (Net-I) by adding two links (7,8) and (7,11). The modified network (Net-II) is schematically presented in *Figure 2*. We are hopefully expecting to improve the network connectivity & robustness.
In *Table 7*, we are comparing between the PMP connectivity of different order.
A comparison between the network overall connectivity at different orders, between Net-I and Net-II is given in *Figure 3,* as well.
One may conclude that adding the two mentioned links has significantly improved the performance of the original network, Net-I.
One interesting remark is that after some given order (n_∞), the PNP connectivity comes to be equal to 1 between all possible node-pairs in the network:

$$C_{il}^{n_\infty} = \left(\frac{N(N-1)}{2}\right)$$

That means each node communicates with all the others in the network. Consequently, the network

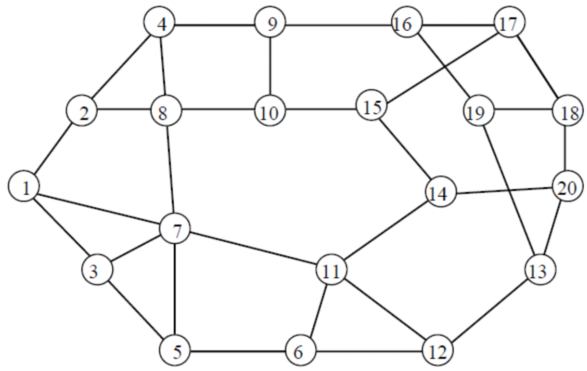

Figure 2. Schematic presentation of NET-II

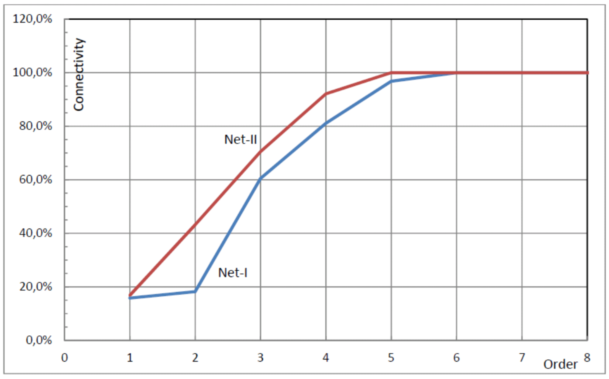

Figure 3. Connectivity v-s order for Net-I and Net-II

100%. The highest order, at which the network average connectivity reaches 100%, will be referred to as the "network maximum order", n_∞. In our study case, n_∞ = 6 and 5, for Net-I and Net-5, respectively.

__Robustness__
Robustness could be considered as the aptitude of the net work to tolerate failures. The proposed model considers the network average connectivity of the 1st order as a measure of the network connectivity. That may be explained in that way: higher is the network connectivity (of the 1st order) level, higher is its tolerance to failures.
For the purpose of robustness measuring, the network average connectivity of the 1st order would be a pertinent measure.

__Reliability__
The model considers the network maximum order n_∞ as a direct measure of the network unreliability. Higher is the network maximum order n_∞, lower is its reliability level.

To explain that, let's consider the PMP Link of the node-pair (12,19), $L_{12,19}$ which describes the success of the connection between this node-pair, and is given by:

$L_{12,19} = l_{12,13}.l_{13,19} + l_{12,11}.l_{11,14}.l_{14,20}.l_{20,18}.l_{18,19}$ + cut-sets of higher order

This above expression is a logical one in the Boolean sense, where the operators (Dot) and (Plus) replace to the operators (AND) and (OR), respectively.

If one assumes that all links are almost identical and their failure rates are of the same order of magnitude, then, the probability of losing the path $l_{12,11}.l_{11,14}.l_{14,20}.l_{20,18}.l_{18,19}$ is higher than the probability of losing the shorter path $l_{12,13}.l_{13,19}$.

Longer are the minimal cut-sets describing the success, lower is the tolerance to failures.

7. Conclusion

An overview of the state-of-the-art in modelling networks reliability is given in the paper. It leads to the conclusion that network connectivity is the basic concept in describing networks reliability. However, a deeper description of networks connectivity can't be achieved only through probabilistic models. It requires the use of topological models.

The paper presents a topological way of describing networks connectivity and extends the model to describe networks robustness and reliability using topological indicators. Three measures are developed: nodes-to-nodes connectivity (C_{ij}^{n}), node connectivity indicator (I_{i}^{n}), and network overall connectivity indicator ($I_{overall}^{n}$), of order n.

The ongoing work focuses on the development of a methodology to determine the cut-sets and their interdependencies.

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Appendix

Table 1. The identity matrix C_{ij}^{n} of the network NET-I, described in *Figure1*

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	I_i^1	R_i^1	
1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	15,8%	
2	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	3	15,8%	
3	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	15,8%	
4	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	3	15,8%	
5	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	15,8%	
6	0	0	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	3	15,8%	
7	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	15,8%	
8	0	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	3	15,8%	
9	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	3	15,8%	
10	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0	3	15,8%	PMP
11	0	0	0	0	0	1	0	0	0	0	1	1	0	1	0	0	0	0	0	0	3	15,8%	Connectivity
12	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	3	15,8%	
13	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	3	15,8%	
14	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	1	3	15,8%	
15	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	0	0	0	3	15,8%	
16	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	1	0	3	15,8%	
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	3	15,8%	
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	3	15,8%	
19	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	1	0	3	15,8%	
20	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	1	3	15,8%	
																					30		Total N° of Links
$I_{overall}^1 / R_{overall}^1$																					3	15,8%	Average/node

Table 2. The 2nd order PMP-connectivity of the network NET-I, described in *Figure1*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	I_i^2	R_i^2	
1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	6	31,6%	
2	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	7	36,8%	
3	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	5	26,3%	
4	1	1	0	1	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	6	31,6%	
5	1	0	1	0	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	6	31,6%	
6	0	0	1	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	7	36,8%	
7	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	5	26,3%	
8	1	1	0	1	0	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0	6	31,6%	
9	0	1	0	1	0	0	0	1	1	1	0	0	0	0	1	1	1	0	1	0	8	42,1%	
10	0	1	0	1	0	0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	8	42,1%	PMP
11	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	0	0	0	0	1	7	36,8%	Connectivity
12	0	0	0	0	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	7	36,8%	
13	0	0	0	0	0	1	0	0	0	0	1	1	1	1	0	1	0	1	1	1	8	42,1%	
14	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	0	1	1	0	1	9	47,4%	
15	0	0	0	0	0	0	0	1	1	1	1	0	0	1	1	1	1	1	0	1	9	47,4%	
16	0	0	0	1	0	0	0	0	1	1	0	0	1	0	1	1	1	1	1	0	8	42,1%	
17	0	0	0	0	0	0	0	0	1	1	0	0	0	1	1	1	1	1	1	1	8	42,1%	
18	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	7	36,8%	
19	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1	1	1	1	1	7	36,8%	
20	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	1	8	42,1%	
$I^2_{overall} / R^2_{overall}$																					7,1	37,4%	Average/node

Table 3. The 3rd order PMP-connectivity of the network NET-I, described in *Figure1*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	I_i^3	R_i^3	
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	9	47,4%	
2	1	1	1	1	1	0	1	1	1	1	0	0	0	0	1	1	0	0	0	0	10	52,6%	
3	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	9	47,4%	
4	1	1	1	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	1	0	11	57,9%	
5	1	1	1	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	9	47,4%	
6	1	0	1	0	1	1	1	0	0	0	1	1	1	1	1	0	0	0	1	1	11	57,9%	
7	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	0	0	9	47,4%	
8	1	1	1	1	0	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0	11	57,9%	
9	1	1	0	1	0	0	0	1	1	1	0	0	1	1	1	1	1	1	1	0	12	63,2%	
10	1	1	0	1	0	0	0	1	1	1	1	0	0	1	1	1	1	1	1	1	13	68,4%	PMP
11	0	0	1	0	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1	1	13	68,4%	Connectivity
12	0	0	1	0	1	1	1	0	0	0	1	1	1	1	1	1	0	1	1	1	12	63,2%	
13	0	0	0	0	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	12	63,2%	
14	0	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	14	73,7%	
15	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	15	78,9%	
16	0	1	0	1	0	0	0	1	1	1	0	1	1	1	1	1	1	1	1	1	13	68,4%	
17	0	0	0	1	0	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	12	63,2%	
18	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	11	57,9%	
19	0	0	0	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	13	68,4%	
20	0	0	0	0	0	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	11	57,9%	
$I_{overall}^3 / R_{overall}^3$																					11,5	60,5%	Average/node

Table 4. the 4th order PMP-connectivity of the network NET-I, described in *Figure1*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	I_i^4	R_i^4	
1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	13	68,4%	
2	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	1	0	14	73,7%	
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	13	68,4%	
4	1	1	1	1	1	0	1	1	1	1	0	0	1	1	1	1	1	1	1	0	15	78,9%	
5	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0	1	1	14	73,7%	
6	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	16	84,2%	
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	13	68,4%	
8	1	1	1	1	1	0	1	1	1	1	1	0	0	1	1	1	1	1	1	1	16	84,2%	
9	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	17	89,5%	
10	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18	94,7%	PMP
11	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	17	89,5%	Connectivity
12	1	0	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	16	84,2%	
13	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	16	84,2%	
14	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18	94,7%	
15	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	17	89,5%	
16	1	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	16	84,2%	
17	0	1	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	15	78,9%	
18	0	0	0	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	14	73,7%	
19	0	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	16	84,2%	
20	0	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	14	73,7%	
$I^4_{overall} / R^4_{overall}$																					15,4	81,1%	Average/node

Table 5. the 5[th] order PMP-connectivity of the network NET-I, described in *Figure1*

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	I_i^5	R_i^5	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	17	89,5%	
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	17	89,5%	
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	17	89,5%	
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	PMP
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	Connectivity
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
17	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	17	89,5%	
18	0	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	16	84,2%	
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
20	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	18	94,7%	
$I_{overall}^5 / R_{overall}^5$																					18,4	96,8%	Average/node

Table 6. the 6th order PMP-connectivity of the network NET-I, described in *Figure1*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	I_i^6	R_i^6	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	PMP
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	Connectivity
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	19	100,0%	
$I_{overall}^6 / R_{overall}^6$																					19	100,0%	Average/node

Table 7. Comparison between the PMP connectivity and average Connectivity of NET-I and NET-II

	Identity PMP Connectivity		2nd Order PMP Connectivity		3rd Order PMP Connectivity		4th Order PMP Connectivity		5th Order PMP Connectivity		6th Order PMP Connectivity		
#	Net-I	Net-II	Net-I	Net-II	Net-I	Net-II	Net-I	Net-II	Net-I	Net-II	Net-I	Net-II	
1	3	3	6	7	9	12	13	16	17	19	19	19	
2	3	3	7	7	10	11	14	16	19	19	19	19	
3	3	3	5	7	9	11	13	15	17	19	19	19	
4	3	3	6	7	11	13	15	18	19	19	19	19	
5	3	3	6	7	9	12	14	16	19	19	19	19	
6	3	3	7	7	11	12	16	18	19	19	19	19	
7	3	5	5	11	9	15	13	19	17	19	19	19	
8	3	4	6	10	11	15	16	19	19	19	19	19	
9	3	3	8	8	12	13	17	18	19	19	19	19	
10	3	3	8	9	13	16	18	19	19	19	19	19	
11	3	4	7	11	13	17	17	19	19	19	19	19	
12	3	3	7	8	12	14	16	19	19	19	19	19	
13	3	3	8	8	12	13	16	18	19	19	19	19	
14	3	3	9	10	14	17	18	19	19	19	19	19	
15	3	3	9	9	15	16	17	19	19	19	19	19	
16	3	3	8	8	13	13	16	17	19	19	19	19	
17	3	3	8	8	12	12	15	16	17	19	19	19	
18	3	3	7	7	11	11	14	15	16	19	19	19	
19	3	3	7	7	13	13	16	17	19	19	19	19	
20	3	3	8	8	11	12	14	17	18	19	19	19	
Ave rage	3	3,2	7,1	8,2	11,5	13,4	15,4	17,5	18,4	19	19	19	
	15.8%	16.8%	18.2%	43.2%	60.5%	70.5%	81.1%	92.1%	96.8%	100%	100%	100%	