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Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety
Part 6
IS&RDSS Application – Exemplary system operation cost analysis and maintenance optimization

Keywords

operation cost, complex systems, corrective maintenance, preventive maintenance

Abstract

The way of operation cost analysis of the complex technical system at the variable operation conditions and its application to the evaluation of the cost before and after the exemplary system operation process optimization is presented. The methods of corrective and preventive maintenance policy maximizing availability and minimizing renovation cost of the complex technical systems in variable operation conditions are presented and applied to the exemplary system.

12. The exemplary system operation cost analysis

12.1. The exemplary system operation cost analysis beforeand after its operation process optimization

In Section 3 [2], it is fixed that the exemplary system is composed of $n = 14$ components and that the numbers of the system components operating in various operation states z_b, $b = 1,2,3,4,$ are different. Namely, there are operating 6 system components at the operation states z_1, 8 system components at the operation states z_2 and 14 system components at the operation states z_3 and z_4. According to the arbitrary assumption, the approximate mean operation cost of the single basic component of the considered exemplary system that is used during the operation time θ = 1 year, independently of the operation state z_b, $b = 1,2,3,4,$ amounts

$$c_i\,(1,b) = 100\,\text{PLN},\; b = 1,2,3,4,\; i = 1,2,...,14,$$

whereas, the cost of each system singular basic component that is not used is equal to 0.
In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, we assume that the approximate cost of the system singular renovation is $c_{ig} = 1000\,\text{PLN}$. Similarly, in the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is not ignored, we assume that the approximate cost of the system singular renovation is $c_{nig} = 1500\,\text{PLN}$.

First, we will analyze the system operation cost before its operation process optimization.

Thus, under the assumptions, the total operation cost of the non-failed exemplary system during the operation time θ = 1 year, according to (13.2) from [1] and after considering (17) [3], is given by

$$C(1) \cong 0.214 \cdot 100 \cdot 6 + 0.038 \cdot 100 \cdot 8$$

$$+ 0.293 \cdot 100 \cdot 14 + 0.455 \cdot 100 \cdot 14$$

$$= 1206 \text{ PLN.} \tag{279}$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, from (76) [3], we know that the mean value of the number of exceeding by the system the critical reliability state during the operation time θ = 1 year is

$$H(1,2) = \frac{1}{357.68} \cong 0.0028.$$

Thus, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time θ = 1 year, according to (13.3) from [1], amounts

$$C_{ig}(1) \cong 1206 + 1000 \cdot 0.0028$$

$$= 1206 + 2.8 = 1208.8 \text{ PLN.} \tag{280}$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is non-ignored, from (77) [3], we know that the mean value of the number of the system renovations after exceeding the critical reliability state during the operation time θ = 1 year is given by

$$\overline{\overline{H}}(1,2) \cong \frac{1}{357.68 + 10} = \frac{1}{367.68} = 0.0027.$$

Thus, the total operation cost of the renewed exemplary system with ignored its renovation time during the operation time θ = 1 year, according to (13.5) from [1], amounts

$$C_{nig}(1) \cong 1206 + 1500 \cdot 0.0027$$

$$= 1206 + 4.05 = 1210.05 \text{ PLN.} \tag{281}$$

Now, we will analyze the system operation cost after its operation process optimization.

Proceeding similarly as before, the total operation cost of the non-failed exemplary system during the operation time θ = 1 year, according to (13.7) from [1] and after considering (90) [4] , is given by

$$\dot{C}(1) \cong 0.341 \cdot 100 \cdot 6 + 0.105 \cdot 100 \cdot 8$$

$$+ 0.245 \cdot 100 \cdot 14 + 0.309 \cdot 100 \cdot 14$$

$$= 1064.2 \text{ PLN.} \tag{282}$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, from (108) [4], we know that the mean value of the number of exceeding by the system the critical reliability state during the operation time θ = 1 year is

$$\dot{H}(1,2) = \frac{1}{410,20} \cong 0.0024.$$

Thus, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time θ = 1 year, according to (13.8) from [1], amounts

$$\dot{C}_{ig}(1) \cong 1064.2 + 1000 \cdot 0.0024$$

$$= 1064.2 + 2.4 = 1066.6 \text{ PLN.} \tag{283}$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is not ignored, from (109) [4], we know that the mean value of the number of the system renovations after exceeding the critical reliability state during the operation time θ = 1 year are is given by

$$\dot{\overline{\overline{H}}}(1,2) \cong \frac{1}{410.20 + 10} = \frac{1}{420.20} = 0.0024.$$

Thus, the total operation cost of the renewed exemplary system with ignored its renovation time during the operation time θ = 1 year, according to (13.10) from [1], amounts

$$\dot{C}_{nig}(\theta) \cong 1064.2 + 1500 \cdot 0.0024$$

$$= 1064.2 + 3.6 = 1067.8 \text{ PLN.} \tag{284}$$

The comparison of the results (279)-(281) with the results (282)-(284) justifies the sensibility of the system operation process optimization.

12.2. Operation cost analysis of the impoved exemplary system

As in Section 12.1 we fixed that the exemplary system is composed of $n = 14$ components and that the numbers of the system components operating in various operation states z_b, $b = 1,2,3,4,$ are different. Namely, there are operating 6 system components at the operation states z_1, 8 system components at the operation states z_2 and 14 system components at the operation states z_3 and z_4. According to the arbitrary assumption, the approximate mean operation cost of the single basic, reserve and improved components of the considered exemplary system that are used during the operation time θ = 1000 days, independently of the operation state z_b, $b = 1,2,3,4,$ amount:

- the operation costs of basic components of the system with non-improved components in the operation state z_b, $b = 1,2,.3,4,$ during the system operation time θ

$$c_i^{(0)}(1000, b) = 1000 \text{ PLN}, \ i = 1,2,...,14,$$

- the operation costs of basic and reserve components of the system with a hot single reservation of components in the operation state z_b, $b = 1,2,...,v,$ during the system operation time θ

$$c_i^{(1)}(1000, b) = 1000 \text{ PLN}, \ i = 1,2,...,14,$$

- the operation costs of basic components of the system with a cold single reservation of components in the operation state z_b, $b = 1,2,.3,4,$ during the system operation time θ

$$c_i^{(2)}(1000, b) = 1000 \text{ PLN}, \ i = 1,2,...,14,$$

and the operation costs of reserve components of this system in the operation state z_b, $b = 1,2,.3,4,$ during the system operation time θ

$$\bar{c}_i^{(2)}(1000, b) = 500 \text{ PLN}, \ i = 1,2,...,14,$$

- the operation costs of basic system components of the system with improved components by reduction their rates of departures from the reliability state subsets in the operation state z_b, $b = 1,2,.3,4,$ during the system operation time θ

$$c_i^{(3)}(1000, \rho(2), b) = 1500 \text{ PLN}, \ i = 1,2,...,14,$$

whereas, the cost of each system singular basic component that is not used is equal to 0.

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, we assume that the approximate cost of the system singular renovation are:

- the cost of the singular renovation of the repairable system with ignored renovation time with non-improved components is

$$c_{ig}^{(0)} = 50 \text{ PLN},$$

- the cost of the singular renovation of the repairable system with ignored renovation time with a hot single reservation of components is

$$c_{ig}^{(1)} = 100 \text{ PLN},$$

- the cost of the singular renovation of the repairable system with ignored renovation time with a cold single reservation of components is

$$c_{ig}^{(2)} = 75 \text{ PLN},$$

- the cost of the singular renovation of the repairable system with ignored renovation time with improved components by reduction the rates of departures from the reliability state subsets

$$c_{ig}^{(3)} = 150 \text{ PLN}.$$

Similarly, in the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is not ignored, we assume that the approximate cost of the system singular renovation are:

- the cost of the singular renovation of the repairable system with non-ignored renovation time with non-improved components is

$$c_{nig}^{(0)} = 500 \text{ PLN},$$

- the cost of the singular renovation of the repairable system with non-ignored renovation time with a hot single reservation of components is

$$c_{nig}^{(1)} = 1000 \text{ PLN},$$

- the cost of the singular renovation of the repairable system with non-ignored renovation time with a cold single reservation of components is

$$c_{nig}^{(2)} = 750 \text{ PLN},$$

- the cost of the singular renovation of the repairable system with non-ignored renovation time with improved components by reduction the rates of departures from the reliability state subsets

$$c_{nig}^{(3)} = 150 \text{ PLN}.$$

First, we will analyze the system operation cost before its operation process optimization.
Thus, under the assumptions, the total operation cost of the non-failed exemplary system during the operation time θ = 1000 days, according to (13.17), (13.18), (13.19) or (13.20) from [1], respectively amount:
- the total operation cost of the non-repairable system with non-improved components

$$C^{(0)}(1000) \cong 0.214 \cdot 1000 \cdot 6$$

$$+ 0.038 \cdot 1000 \cdot 8 + 0.293 \cdot 1000 \cdot 14$$

$$+ 0.455 \cdot 1000 \cdot 14 = 12\,060 \text{ PLN}, \qquad (285)$$

- the total operation cost of the non-repairable system with a hot single reservation of components is

$$C^{(1)}(1000) \cong 2[\, 0.214 \cdot 1000 \cdot 6$$

$$+ 0.038 \cdot 1000 \cdot 8 + 0.293 \cdot 1000 \cdot 14$$

$$+ 0.455 \cdot 1000 \cdot 14\,] = 24\,120 \text{ PLN}, \qquad (286)$$

- the total operation cost of the non-repairable system with a cold single reservation of components is

$$C^{(2)}(1000) \cong 0.214 \cdot (1000 + 500) \cdot 6$$

$$+ 0.038 \cdot (1000 + 500) \cdot 8$$

$$+ 0.293 \cdot (1000 + 500) \cdot 14$$

$$+ 0.455 \cdot (1000 + 500) \cdot 14 = 18\,090 \text{ PLN}, \quad (287)$$

- the total operation cost of the non-repairable system with improved components by reduction their rates of departures from the reliability state subsets is

$$C^{(3)}(1000) \cong 0.214 \cdot 1500 \cdot 6$$

$$+ 0.038 \cdot 1500 \cdot 8 + 0.293 \cdot 1500 \cdot 14$$

$$+ 0.455 \cdot 1500 \cdot 14 = 18\,090 \text{ PLN}. \qquad (287)$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time θ = 1000 days, according to (13.27), (13.29), (13.31) or (13.33) from [1], respectively amount:
- the total operation cost of the repairable system with ignored renovation time with non-improved components

$$C_{ig}^{(0)}(1000) \cong 12\,060 + 50 \cdot 2.80$$

$$= 12\,060 + 140 = 12\,200 \text{ PLN} \qquad (288)$$

as according to (76) [3]

$$H^{(0)}(1000,2) = \frac{1000}{357.68} \cong 2.80,$$

- the total operation cost of the repairable system with ignored renovation time with a hot single reservation of components is

$$C_{ig}^{(1)}(1000) \cong 24\,120 + 100 \cdot 1.40$$

$$= 24\,120 + 140 = 24\,260 \text{ PLN} \qquad (289)$$

as according to (273) [5]

$$H^{(1)}(1000,2) = \frac{1000}{712.01} \cong 1.40,$$

- the total operation cost of the repairable system with ignored renovation time with a cold single reservation of components is

$$C_{ig}^{(2)}(1000) \cong 18\,090 + 75 \cdot 1.02$$

$$= 18\,090 + 76.5 = 18\,166.5 \text{ PLN} \quad (290)$$

as according to (274) [5]

$$H^{(2)}(1000,2) = \frac{1000}{977.87} \cong 1.02,$$

- the total operation cost of the repairable system with ignored renovation time with components improved by reduction their rates of departures is

$$C_{ig}^{(3)}(1000) \cong 18\,090 + 150 \cdot 2.47$$

$$= 18\,090 + 370.5 = 18\,460.5 \text{ PLN} \quad (291)$$

as according to (275) [5]

$$H^{(3)}(1000,2) = \frac{1000}{404.89} \cong 2.47.$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is non-ignored and has the distribution function with the mean value $\mu_0(2) = 10$, the total operation cost of the renewed exemplary system with non-ignored its renovation time during the operation time θ = 1000 days, according to (13.42), (13.44), (13.46) or (13.48) from [1], respectively amount:

- the total operation cost of the repairable system with non-ignored renovation time with non-improved components

$$C_{nig}^{(0)}(1000) \cong 12\,060 + 500 \cdot 2.72$$

$$= 12\,060 + 1360 = 13\,420 \text{ PLN} \quad (292)$$

as according to (77) [3]

$$\overline{\overline{H}}^{(0)}(1000,2) = \frac{1000}{357.68 + 10} \cong 2.72,$$

- the total operation cost of the repairable system with non-ignored renovation time with a hot single reservation of components

$$C_{nig}^{(1)}(\theta) \cong 24\,120 + 1000 \cdot 1.39$$

$$= 24\,120 + 1390 = 25\,510 \text{ PLN}, \quad (293)$$

as according to (276) [5]

$$\overline{\overline{H}}^{(1)}(1000,2) = \frac{1000}{712.01 + 10} \cong 1.39,$$

- the total operation cost of the repairable system with non-ignored renovation time with a cold single reservation of components

$$C_{nig}^{(2)}(\theta) \cong 18\,090 + 750 \cdot 1.01$$

$$= 18\,090 + 757.5 = 18\,847.50 \text{ PLN} \quad (12.17)$$

as according to (11.167) [5]

$$\overline{\overline{H}}^{(2)}(1000,2) = \frac{1000}{977.87 + 10} \cong 1.01,$$

- the total operation cost of the repairable system with non-ignored renovation time with improved components by reduction the rates of departures in the reliability state subsets

$$C_{nig}^{(3)}(1000, \rho(2)) \cong 18\,090 + 1500 \cdot 2.41$$

$$= 18\,090 + 3615 = 21\,705 \text{ PLN} \quad (296)$$

as according to (278) [5]

$$\overline{\overline{H}}^{(3)}(1000,2) = \frac{1000}{404.89 + 10} \cong 2.41.$$

Now, we will analyze the improved exemplary system operation cost after its operation process optimization.

Proceeding similarly as before, the total operation cost of the non-failed exemplary system during the operation time θ = 1000 days, according to (13.52), (13.53), (13.54) or (13.55) from [1], respectively amount:

- the total operation cost of the non-repairable system with non-improved components

$$\dot{C}^{(0)}(1000) \cong 0.341 \cdot 1000 \cdot 6$$

$$+ 0.105 \cdot 1000 \cdot 8 + 0.245 \cdot 1000 \cdot 14$$

$$+ 0.309 \cdot 1000 \cdot 14 = 10\,642 \text{ PLN}, \quad (297)$$

- the total operation cost of the non-repairable system with a hot single reservation of components is

$$\dot{C}^{(1)}(1000) \cong 2[\,0.341 \cdot 1000 \cdot 6$$

$$+ 0.105 \cdot 1000 \cdot 8 + 0.245 \cdot 1000 \cdot 14$$

$$+ 0.309 \cdot 1000 \cdot 14] = 21\,284 \text{ PLN}, \quad (298)$$

- the total operation cost of the non-repairable system with a cold single reservation of components is

$$\dot{C}^{(2)}(1000) \cong 0.341 \cdot (1000 + 500) \cdot 6$$

$$+ 0.105 \cdot (1000 + 500) \cdot 8$$

$$+ 0.245 \cdot (1000 + 500) \cdot 14$$

$$+ 0.309 \cdot (1000 + 500) \cdot 14 = 15\,963 \text{ PLN}, \quad (299)$$

- the total operation cost of the non-repairable system with improved components by reduction their rates of departures from the reliability state subsets is

$$\dot{C}^{(3)}(1000) \cong 0.341 \cdot 1500 \cdot 6 + 0.105 \cdot 1500 \cdot 8$$

$$+ 0.245 \cdot 1500 \cdot 14 + 0.309 \cdot 1500 \cdot 14$$

$$= 15\,963 \text{ PLN}. \quad (300)$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time θ = 1000 days, according to (13.58), (13.60), (13.63) or (13.65) from [1], respectively amount:
- the total operation cost of the repairable system with ignored renovation time with non-improved components

$$\dot{C}_{ig}^{(0)}(1000) \cong 10\,642 + 50 \cdot 2.44$$

$$= 10\,642 + 122 = 10\,764 \text{ PLN} \quad (301)$$

as according to (108) [4]

$$\dot{H}^{(0)}(1000,2) = \frac{1000}{410.20} \cong 2.44,$$

- the total operation cost of the repairable system with ignored renovation time with a hot single reservation of components is

$$\dot{C}_{ig}^{(1)}(1000) \cong 21\,284 + 100 \cdot 1.28$$

$$= 21\,284 + 128 = 21\,412 \text{ PLN} \quad (302)$$

as according to (108) [4]

$$\dot{H}^{(1)}(1000,2) = \frac{1000}{781.33} \cong 1.28,$$

- the total operation cost of the repairable system with ignored renovation time with a cold single reservation of components is

$$\dot{C}_{ig}^{(2)}(1000) \cong 15\,963 + 75 \cdot 0.93$$

$$= 15\,963 + 69.75 = 16\,032.75 \text{ PLN} \quad (303)$$

as according to (108) [4]

$$\dot{H}^{(2)}(1000,2) = \frac{1000}{1070.01} \cong 0.93,$$

- the total operation cost of the repairable system with ignored renovation time with components improved by reduction their rates of departures is

$$\dot{C}_{ig}^{(3)}(1000) \cong 15\,963 + 150 \cdot 2.15$$

$$= 15\,963 + 322.5 = 16\,285.5 \text{ PLN} \quad (304)$$

as according to (108) [4]

$$\dot{H}^{(3)}(1000,2) = \frac{1000}{464.47} \cong 2.15.$$

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is non-ignored and has the distribution function with the mean value $\mu_0(2) = 10$, the total operation cost of the renewed exemplary system with non-ignored its renovation time during the operation time θ = 1000 days, according to (13.68), (13.70), (13.72) or (13.74) from [1], respectively amount:
- the total operation cost of the repairable system with non-ignored renovation time with non-improved components

$$\dot{C}_{nig}^{(0)}(1000) \cong 10\,642 + 500 \cdot 2.38$$

$$= 10\,642 + 1190 = 11\,832 \text{ PLN} \quad (305)$$

as according to (109) [4]

$$\dot{\bar{\bar{H}}}^{(0)}(1000,2) = \frac{1000}{410.20+10} \cong 2.38,$$

- the total operation cost of the repairable system with non-ignored renovation time with a hot single reservation of components

$$\dot{C}^{(1)}_{nig}(1000) \cong 21\ 284 + 1000 \cdot 1.26$$

$$= 21\ 284 + 1260 = 22\ 544 \text{ PLN}, \qquad (306)$$

as according to (109) [4]

$$\dot{\bar{\bar{H}}}^{(1)}(1000,2) = \frac{1000}{781.33+10} \cong 1.26,$$

- the total operation cost of the repairable system with non-ignored renovation time with a cold single reservation of components

$$\dot{C}^{(2)}_{nig}(1000) \cong 15\ 963 + 750 \cdot 0.93$$

$$= 15\ 963 + 697.5 = 16\ 660.50 \text{ PLN} \qquad (307)$$

as according to (109) [4]

$$\dot{\bar{\bar{H}}}^{(2)}(1000,2) = \frac{1000}{1070.01+10} \cong 0.93,$$

- the total operation cost of the repairable system with non-ignored renovation time with improved components by reduction the rates of departures in the reliability state subsets

$$\dot{C}^{(3)}_{nig}(1000, \rho(2)) \cong 15963 + 1500 \cdot 2.11$$

$$= 15\ 963 + 3165 = 19\ 123 \text{ PLN} \qquad (308)$$

as according to (109) [4]

$$\dot{\bar{\bar{H}}}^{(3)}(1000,2) = \frac{1000}{464.47+10} \cong 2.11.$$

The comparison of the results (285)-(296) with the results (297)-(308) justifies the sensibility of the system operation process optimization.

13. The exemplary system corrective and preventive maintenance policy optimization

13.1. Maintenance policy maximizing system availability

To optimize the exemplary system corrective and preventive maintenance policy maximizing its availability, we use its following reliability and renewal parameters:

- the number of the system and components reliability states 4 (z = 3),
- the system and components critical reliability state r = 2,
- the 2-nd coordinate of the system unconditional reliability function $\boldsymbol{R}(t,\cdot)$

$$\boldsymbol{R}(t,2) = 0.214 \cdot [\boldsymbol{R}(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}(t,2)]^{(2)}$$

$$+ 0.293 \cdot [\boldsymbol{R}(t,2)]^{(3)} + 0.455 \cdot [\boldsymbol{R}(t,2)]^{(4)} \qquad (309)$$

for $t \geq 0$,

where $[\boldsymbol{R}(t,2)]^{(b)}$, $b = 1,2,3,4$, are respectively given by (21), (28), (43), (58) [3].

- the derivative of the 2-nd coordinate of the system unconditional reliability function $\boldsymbol{R}(t,\cdot)$

$$\boldsymbol{R}'(t,2) = 0.214 \cdot [\boldsymbol{R}'(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}'(t,2)]^{(2)}$$

$$+ 0.293 \cdot [\boldsymbol{R}'(t,2)]^{(3)} + 0.455 \cdot [\boldsymbol{R}'(t,2)]^{(4)} \text{ for } t \geq 0,$$

- the mean value of the system corrective maintenance (renovation) time $\mu_0(2) = 10$,
- the mean value of the system preventive maintenance (renovation) time $\mu_1(2) = 5$.

Moreover, to apply the algorithm proposed in [1], we fix:

- the measure of the method of secants accuracy $\varepsilon = 0.001$,
- the number of the values of the system preventive maintenance period η for which we find the values of the availability coefficient of the exemplary system in the cases when there is no its optimal value $\kappa = 20$,
- the values of the system preventive maintenance period η for which we find the values of the availability coefficient of the system in the cases when there is no its optimal value

$$\eta_i = (i-1)0.2\mu(2) = (i-1)0.2 \cdot 358, \qquad (310)$$
$$i = 1,2,...,20.$$

where $\mu(2)$, is given by (69) [3],

Since

$$\mu_0(2) = 10 > \mu_1(2) = 5$$

we are looking for the optimal value $\dot{\eta}$ of the preventive maintenance period η that maximizes the availability coefficient of the system $A(\eta, r)$ given by (14.6) from [1] by determining, if it exists, its approximate value from the equation (14.12) from [1] by applying the method of secants in the interval $< a, b >$ as follows:

- we define, obtained after the transformation of the equation (14.12) from [1], the function

$$f(\eta) = \lambda(\eta,2) \int_0^\eta \boldsymbol{R}(t,2)dt + \boldsymbol{R}(\eta,2) - \frac{\mu_0(2)}{\mu_0(2) - \mu_1(2)}$$

$$= \lambda(\eta,2) \int_0^\eta \boldsymbol{R}(t,2)dt + \boldsymbol{R}(\eta,2) - 2 \quad \text{for } \eta \geq 0,$$

where $\boldsymbol{R}(t, 2)$ is given by (309) and

$$\lambda(\eta, r) = -\frac{\boldsymbol{R}'(\eta,2)}{\boldsymbol{R}(\eta,2)},$$

- we define the interval $< a, b >$ assuming $a = 0$ and finding b such that

$$f(b) > 0,$$

- we use the recurrent formula

$$\eta_0 = a,$$

$$\eta_{k+1} = \eta_k - \frac{f(\eta_k)}{f(b) - f(\eta_k)}(b - \eta_k) \quad (311)$$

for $k = 0,1,..., K$,

where K is such that

$$f(\eta_{K+1}) < \varepsilon$$

and $\varepsilon = 0.001$ is the measure of the method of secants accuracy,

- we fix the optimal value $\dot{\eta}$ of the preventive maintenance period η assuming

$$\dot{\eta} = \eta_{K+1}.$$

As a result of the computer calculations, we recognize that there is no optimal value $\dot{\eta}$ of the exemplary system preventive maintenance period η that maximize the value of its availability coefficient. The values of the system preventive maintenance period η defined by (13.2) and the values of the availability coefficient of the exemplary system are given in *Table 4.*

Table 4. The values of the availability coefficient of the exemplary system

η	$A(\eta,2)$	η	$A(\eta,2)$
0.0	0.0	716.0	0.97202
71.6	0.93036	787.6	0.97220
143.2	0.95742	859.2	0.97233
214.8	0.96501	930.8	0.97243
286.4	0.96809	1002.4	0.97250
358.0	0.96961	1074.0	0.97256
429.6	0.97054	1145.6	0.97262
501.2	0.97110	1217.2	0.97265
572.8	0.97152	1288.8	0.97268
644.4	0.97180	1360.4	0.97271

13.2. Maintenance policy minimizing system renowation cost

To optimize the exemplary system corrective and preventive maintenance policy minimizing its cost of renovation, we use its following reliability and operation cost parameters:
- the system and components critical reliability state $r = 2$,
- the 2-nd coordinate of the system unconditional reliability function $\boldsymbol{R}(t,\cdot)$

$$\boldsymbol{R}(t,2) = 0.214 \cdot [\boldsymbol{R}(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}(t,2)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}(t,2)]^{(3)} + 0.455 \cdot [\boldsymbol{R}(t,2)]^{(4)} \quad (312)$$

for $t \geq 0$,

where $[\boldsymbol{R}(t,2)]^{(b)}$, $b = 1,2,3,4$, are respectively given by (21), (28), (43), (58) [3].
- the derivative of the 2-nd coordinate of the system unconditional reliability function $\boldsymbol{R}(t,\cdot)$

$$\boldsymbol{R}'(t,2) = 0.214 \cdot [\boldsymbol{R}'(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}'(t,2)]^{(2)}$$

$$+ 0.293 \cdot [\boldsymbol{R}'(t,2)]^{(3)} + 0.455 \cdot [\boldsymbol{R}'(t,2)]^{(4)} \text{ for } t \geq 0,$$

- the mean value of the cost of the exemplary system corrective maintenance (renovation) $c_0(1) = 1000$ PLN,
- the mean value of the cost of the exemplary system preventive maintenance (renovation) $c_1(1) = 800$ PLN,
- the measure of the method of secants accuracy $\varepsilon = 0.001$,
- the number of the values of the system age ς for which we find the values of the system renovation cost in the cases when there is no its optimal value $\kappa = 20$,
- the values of the system age ς for which we find the values of the system renovation cost in the cases when there is no optimal value

$$\varsigma_i = i \cdot 0.2\mu(2) = i \cdot 0.2 \cdot 358,\ i = 1,2,...,20. \quad (313)$$

where $\mu(2)$, is given by (69) [3],

After fixing the above system reliability and operation cost input parameters, we use the procedure described in Section 14.2.2 of [1].
Since

$$c_0(2) = 1000 > c_1(2) = 800,$$

we are looking for the optimal value $\dot{\varsigma}$ of the system age ς at which the system preventive renovation is performed that minimizes the system renovation cost per unit time $C(\varsigma,2)$ given by (14.20) from [1] by determining, if it exists, its approximate value from the equation (14.23) from [1] by applying the method of secants in the interval $< a, b >$ as follows:
- we define, obtained after the transformation of the equation (14.23) from [1], the function

$$\psi(\varsigma) = \boldsymbol{R}^2(\varsigma,2) - \varsigma\boldsymbol{R}'(\varsigma,2)\ \boldsymbol{R}(\varsigma,2)$$

$$-\frac{c_0(2)\boldsymbol{R}(\varsigma,2) - c_1(2)\boldsymbol{R}'(\varsigma,2)L(\varsigma,2)}{c_0(2) - c_1(2)}$$

for $\varsigma \geq 0$,

where $\boldsymbol{R}(t,2)$ is given by (312) and

$$L(\varsigma,2) = \int_0^{\varsigma} t\frac{\boldsymbol{f}(t,2)}{1-\boldsymbol{R}(\varsigma,2)}dt = -\int_0^{\varsigma} t\frac{\boldsymbol{R}'(t,2)}{1-\boldsymbol{R}(\varsigma,2)}dt$$

$$= \frac{-\varsigma\boldsymbol{R}(\varsigma,2)] + \int_0^{\varsigma} \boldsymbol{R}(\varsigma,2)dt}{1-\boldsymbol{R}(\varsigma,2)},$$

- we define the interval $< a, b >$ assuming $a = 0$ and finding b such that

$$\psi(b) > 0,$$

- we use the recurrent formula

$$\varsigma_0 = a,$$

$$\varsigma_{k+1} = \varsigma_k - \frac{\psi(\eta_k)}{\psi(b) - \psi(\eta_k)}(b - \varsigma_k)$$

for $k = 0,1,...,K$,

where K is such that

$$\psi(\varsigma_{K+1}) < \varepsilon$$

and ε is the measure of the method of secants accuracy,
- we fix the optimal value $\dot{\varsigma}$ of the system age ς assuming

$$\dot{\varsigma} = \varsigma_{K+1},$$

As a result of the computer calculations, we recognize that there is no optimal value $\dot{\varsigma}$ of the exemplary system age ς at which the system preventive renovation is performed that minimize the system renovation cost. The exemplary values of the system age ς at which the system preventive renovation is performed defined by (313) and the values of the renovation cost of the exemplary system are given in *Table 5.*

Table 5. The values of the renovation cost of the exemplary system

ς	$C(\varsigma,1)$	ς	$C(\varsigma,1)$
71.6	11.53	787.6	2.92
143.2	6.27	859.2	2.90
214.8	4.68	930.8	2.87
286.4	3.96	1002.4	2.86
358.0	3.58	1074.0	2.84
429.6	3.34	1145.6	2.83
501.2	3.19	1217.2	2.83

572.8	3.09	1288.8	2.82
644.4	3.02	1360.4	2.81
716.0	2.96	1432.0	2.81

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