Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety
Part 6
IS&RDSS Application – Exemplary system operation cost analysis and maintenance optimization

12. The exemplary system operation cost analysis

12.1. The exemplary system operation cost analysis before and after its operation process optimization

In Section 3 [2], it is fixed that the exemplary system is composed of \( n = 14 \) components and that the numbers of the system components operating in various operation states \( z_b \), \( b = 1,2,3,4 \), are different. Namely, there are operating 6 system components at the operation states \( z_1 \), 8 system components at the operation states \( z_2 \) and 14 system components at the operation states \( z_3 \) and \( z_4 \). According to the arbitrary assumption, the approximate mean operation cost of the single basic component of the considered exemplary system that is used during the operation time \( \theta = 1 \) year, independently of the operation state \( z_b \), \( b = 1,2,3,4 \), amounts

\[
c_i (1,b) = 100 \text{ PLN}, \quad b = 1,2,3,4, \quad i = 1,2,\ldots,14,
\]

whereas, the cost of each system singular basic component that is not used is equal to 0.

In the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is ignored, we assume that the approximate cost of the system singular renovation is \( c_{ig} = 1000 \text{ PLN} \). Similarly, in the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is not ignored, we assume that the approximate cost of the system singular renovation is \( c_{ig} = 1500 \text{ PLN} \).

First, we will analyze the system operation cost before its operation process optimization.
Thus, under the assumptions, the total operation cost of the non-failed exemplary system during the operation time $\theta = 1$ year, according to (13.2) from [1] and after considering (17) [3], is given by

$$C(1) \equiv 0.214 \cdot 100 \cdot 6 + 0.038 \cdot 100 \cdot 8 + 0.293 \cdot 100 \cdot 14 + 0.455 \cdot 100 \cdot 14 = 1206 \text{ PLN.}$$

(279)

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, from (76) [3], we know that the mean value of the number of exceeding by the system the critical reliability state during the operation time $\theta = 1$ year is

$$H(1,2) = \frac{1}{357.68} \equiv 0.0028.$$

Thus, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time $\theta = 1$ year, according to (13.3) from [1], amounts

$$C_{ig}(1) \equiv 1206 + 1000 \cdot 0.0028 = 1206 + 2.8 = 1208.8 \text{ PLN.}$$

(280)

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is not ignored, from (77) [3], we know that the mean value of the number of system renovations after exceeding the critical reliability state during the operation time $\theta = 1$ year is given by

$$\bar{H}(1,2) \equiv \frac{1}{357.68 + 10} = \frac{1}{367.68} = 0.0027.$$ 

Thus, the total operation cost of the renewed exemplary system with ignored its renovation time during the operation time $\theta = 1$ year, according to (13.5) from [1], amounts

$$C_{nig}(1) \equiv 1206 + 1500 \cdot 0.0028 = 1206 + 4.05 = 1210.05 \text{ PLN.}$$

(281)

Proceeding similarly as before, the total operation cost of the non-failed exemplary system during the operation time $\theta = 1$ year, according to (13.7) from [1] and after considering (90) [4], is given by

$$\hat{C}(1) \equiv 0.341 \cdot 100 \cdot 6 + 0.105 \cdot 100 \cdot 8 + 0.245 \cdot 100 \cdot 14 + 0.309 \cdot 100 \cdot 14 = 1064.2 \text{ PLN.}$$

(282)

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is ignored, from (108) [4], we know that the mean value of the number of exceeding by the system the critical reliability state during the operation time $\theta = 1$ year is

$$\hat{H}(1,2) = \frac{1}{410.20} \equiv 0.0024.$$

Thus, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time $\theta = 1$ year, according to (13.8) from [1], amounts

$$\hat{C}_{ig}(1) \equiv 1064.2 + 1000 \cdot 0.0024 = 1064.2 + 2.4 = 1066.6 \text{ PLN.}$$

(283)

In the case when the exemplary system is repaired after exceeding the critical reliability state $r = 2$ and its renewal time is not ignored, from (109) [4], we know that the mean value of the number of system renovations after exceeding the critical reliability state during the operation time $\theta = 1$ year are is given by

$$\hat{\bar{H}}(1,2) \equiv \frac{1}{410.20 + 10} = \frac{1}{420.20} = 0.0024.$$ 

Thus, the total operation cost of the renewed exemplary system with ignored its renovation time during the operation time $\theta = 1$ year, according to (13.10) from [1], amounts

$$\hat{C}_{nig}(\theta) \equiv 1064.2 + 1500 \cdot 0.0024 = 1064.2 + 3.6 = 1067.8 \text{ PLN.}$$

(284)
The comparison of the results (279)-(281) with the results (282)-(284) justifies the sensibility of the system operation process optimization.

12.2. Operation cost analysis of the improved exemplary system

As in Section 12.1 we fixed that the exemplary system is composed of \( n = 14 \) components and that the numbers of the system components operating in various operation states \( z_b \), \( b = 1, 2, 3, 4 \), are different. Namely, there are operating 6 system components at the operation states \( z_1 \), 8 system components at the operation states \( z_2 \) and 14 system components at the operation states \( z_3 \) and \( z_4 \). According to the arbitrary assumption, the approximate mean operation cost of the single basic, reserve and improved components of the considered exemplary system that are used during the operation time \( \theta = 1000 \) days, independently of the operation state \( z_b \), \( b = 1, 2, 3, 4 \), amount:

- the operation costs of basic components of the system with non-improved components in the operation state \( z_b \), \( b = 1, 2, 3, 4 \), during the system operation time \( \theta \)

\[ c_i^{(0)}(1000, b) = 1000 \text{ PLN}, \quad i = 1, 2, ..., 14, \]

- the operation costs of basic and reserve components of the system with a hot single reservation of components in the operation state \( z_b \), \( b = 1, 2, ..., v \), during the system operation time \( \theta \)

\[ c_i^{(1)}(1000, b) = 1000 \text{ PLN}, \quad i = 1, 2, ..., 14, \]

- the operation costs of basic components of the system with a cold single reservation of components in the operation state \( z_b \), \( b = 1, 2, ..., v \), during the system operation time \( \theta \)

\[ c_i^{(2)}(1000, b) = 1000 \text{ PLN}, \quad i = 1, 2, ..., 14, \]

and the operation costs of reserve components of this system in the operation state \( z_b \), \( b = 1, 2, 3, 4 \), during the system operation time \( \theta \)

\[ c_i^{(3)}(1000, b) = 500 \text{ PLN}, \quad i = 1, 2, ..., 14, \]

- the operation costs of basic system components of the system with improved components by reduction their rates of departures from the reliability state subsets in the operation state \( z_b \), \( b = 1, 2, 3, 4 \), during the system operation time \( \theta \)

\[ c_i^{(1)}(1000, \rho(2), b) = 1500 \text{ PLN}, \quad i = 1, 2, ..., 14, \]

whereas, the cost of each system singular basic component that is not used is equal to 0.

In the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is ignored, we assume that the approximate cost of the system singular renovation are:

- the cost of the singular renovation of the repairable system with non-improved components is

\[ c_u^{(0)} = 50 \text{ PLN}, \]

- the cost of the singular renovation of the repairable system with non-ignored renovation time with non-improved components is

\[ c_u^{(0)} = 100 \text{ PLN}, \]

- the cost of the singular renovation of the repairable system with ignored renovation time with a hot single reservation of components is

\[ c_u^{(2)} = 75 \text{ PLN}, \]

- the cost of the singular renovation of the repairable system with ignored renovation time with a cold single reservation of components is

\[ c_u^{(2)} = 75 \text{ PLN}, \]

and the operation costs of improved components by reduction the rates of departures from the reliability state subsets

\[ c_i^{(3)} = 150 \text{ PLN}. \]

Similarly, in the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is not ignored, we assume that the approximate cost of the system singular renovation are:

- the cost of the singular renovation of the repairable system with non-ignored renovation time with non-improved components is

\[ c_u^{(0)} = 500 \text{ PLN}, \]

- the cost of the singular renovation of the repairable system with non-ignored renovation time with a hot single reservation of components is

\[ c_u^{(2)} = 75 \text{ PLN}, \]
\[ c_{\text{ni}g}^{(1)} = 1000 \text{ PLN,} \]

- the cost of the singular renovation of the repairable system with non-ignored renovation time with a cold single reservation of components is

\[ c_{\text{ni}g}^{(2)} = 750 \text{ PLN,} \]

- the cost of the singular renovation of the repairable system with non-ignored renovation time with improved components by reduction the rates of departures from the reliability state subsets

\[ c_{\text{ni}g}^{(3)} = 150 \text{ PLN.} \]

First, we will analyze the system operation cost before its operation process optimization. Thus, under the assumptions, the total operation cost of the non-failed exemplary system during the operation time \( \theta = 1000 \) days, according to (13.17), (13.18), (13.19) or (13.20) from [1], respectively amount:

- the total operation cost of the non-repairable system with non-improved components

\[
C_{\text{ni}}^{(0)}(1000) \equiv 0.214 \cdot 1000 \cdot 6 \\
+ 0.038 \cdot 1000 \cdot 8 + 0.293 \cdot 1000 \cdot 14 \\
+ 0.455 \cdot 1000 \cdot 14 = 12 060 \text{ PLN,} \quad (285)
\]

- the total operation cost of the non-repairable system with a hot single reservation of components

\[
C_{\text{nih}}^{(1)}(1000) \equiv 2[0.214 \cdot 1000 \cdot 6 \\
+ 0.038 \cdot 1000 \cdot 8 + 0.293 \cdot 1000 \cdot 14 \\
+ 0.455 \cdot 1000 \cdot 14 ]= 24 120 \text{ PLN,} \quad (286)
\]

- the total operation cost of the non-repairable system with a cold single reservation of components

\[
C_{\text{nic}}^{(2)}(1000) \equiv 0.214 \cdot (1000 + 500) \cdot 6 \\
+ 0.038 \cdot (1000 + 500) \cdot 8 \\
+ 0.293 \cdot (1000 + 500) \cdot 14
\]

+ 0.455 \cdot (1000 + 500) \cdot 14 = 18 090 \text{ PLN,} \quad (287)

- the total operation cost of the non-repairable system with improved components by reduction their rates of departures from the reliability state subsets is

\[
C_{\text{ni}i}^{(3)}(1000) \equiv 0.214 \cdot 1500 \cdot 6 \\
+ 0.038 \cdot 1500 \cdot 8 + 0.293 \cdot 1500 \cdot 14 \\
+ 0.455 \cdot 1500 \cdot 14 = 18 090 \text{ PLN.} \quad (287)
\]

In the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is ignored, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time \( \theta = 1000 \) days, according to (13.27), (13.29), (13.31) or (13.33) from [1], respectively amount:

- the total operation cost of the repairable system with ignored renovation time with non-improved components

\[
C_{\text{ri}}^{(0)}(1000) \equiv 12 060 + 50 \cdot 2.80 \\
= 12 060 + 140 = 12 200 \text{ PLN} \quad (288)
\]
as according to (76) [3]

\[
H^{(0)}(1000,2) = \frac{1000}{357.68} \equiv 2.80,
\]

- the total operation cost of the repairable system with ignored renovation time with a hot single reservation of components is

\[
C_{\text{rh}}^{(1)}(1000) \equiv 24 120 + 100 \cdot 1.40 \\
= 24 120 + 140 = 24 260 \text{ PLN} \quad (289)
\]
as according to (273) [5]

\[
H^{(1)}(1000,2) = \frac{1000}{712.01} \equiv 1.40,
\]

- the total operation cost of the repairable system with ignored renovation time with a cold single reservation of components is

\[
C_{\text{rc}}^{(2)}(1000) \equiv 18 090 + 75 \cdot 1.02
\]
as according to (274) [5]

\[ H^{(2)}(1000,2) = \frac{1000}{977.87} \approx 1.02, \]

- the total operation cost of the repairable system with ignored renovation time with components improved by reduction their rates of departures is

\[ C^{(3)}_{\text{ig}}(1000) \equiv 18\,090 + 150 \cdot 2.47 \]
\[ = 18\,090 + 370.5 = 18\,460.5 \text{ PLN} \] (291)
as according to (275) [5]

\[ H^{(3)}(1000,2) = \frac{1000}{404.89} \approx 2.47. \]

In the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is non-ignored and has the distribution function with the mean value \( \mu_n(2) = 10 \), the total operation cost of the renewed exemplary system with non-ignored its renovation time during the operation time \( \theta = 1000 \) days, according to (13.42), (13.44), (13.46) or (13.48) from [1], respectively amount:

- the total operation cost of the repairable system with non-ignored renovation time with non-improved components

\[ C^{(0)}_{\text{ig}}(1000) \equiv 12\,060 + 500 \cdot 2.72 \]
\[ = 12\,060 + 1360 = 13\,420 \text{ PLN} \] (292)
as according to (77) [3]

\[ H^{(0)}(1000,2) = \frac{1000}{357.68 + 10} \approx 2.72, \]

- the total operation cost of the repairable system with non-ignored renovation time with a hot single reservation of components

\[ C^{(1)}_{\text{ig}}(\theta) \equiv 24\,120 + 1000 \cdot 1.39 \]
\[ = 24\,120 + 1390 = 25\,510 \text{ PLN}, \] (293)
as according to (276) [5]

\[ \overline{H}^{(0)}(1000,2) = \frac{1000}{712.01 + 10} \equiv 1.39, \]

- the total operation cost of the repairable system with non-ignored renovation time with a cold single reservation of components

\[ C^{(2)}_{\text{ig}}(\theta) \equiv 18\,090 + 750 \cdot 1.01 \]
\[ = 18\,090 + 757.5 = 18\,847.50 \text{ PLN} \] (12.17)
as according to (11.167) [5]

\[ \overline{H}^{(2)}(1000,2) = \frac{1000}{977.87 + 10} \equiv 1.01, \]

- the total operation cost of the repairable system with non-ignored renovation time with improved components by reduction the rates of departures in the reliability state subsets

\[ C^{(3)}_{\text{ig}}(1000, \rho(2)) \equiv 18\,090 + 1500 \cdot 2.41 \]
\[ = 18\,090 + 3615 = 21\,705 \text{ PLN} \] (296)
as according to (278) [5]

\[ \overline{H}^{(3)}(1000,2) = \frac{1000}{404.89 + 10} \equiv 2.41. \]

Now, we will analyze the improved exemplary system operation cost after its operation process optimization. Proceeding similarly as before, the total operation cost of the non-failed exemplary system during the operation time \( \theta = 1000 \) days, according to (13.52), (13.53), (13.54) or (13.55) from [1], respectively amount:

- the total operation cost of the non-repairable system with non-improved components

\[ C^{(0)}_{\text{nig}}(1000) \equiv 12\,060 + 500 \cdot 2.72 \]
\[ = 12\,060 + 1360 = 13\,420 \text{ PLN} \] (297)
as according to (77) [3]

\[ \overline{H}^{(0)}(1000,2) = \frac{1000}{357.68 + 10} \approx 2.72, \]

- the total operation cost of the non-repairable system with a hot single reservation of components is

\[ \hat{C}^{(0)}(1000) \equiv \hat{C}^{(0)}(1000) = 0.341 \cdot 1000 \cdot 6 \]
\[ + 0.105 \cdot 1000 \cdot 8 + 0.245 \cdot 1000 \cdot 14 \]
\[ + 0.309 \cdot 1000 \cdot 14 = 10\,642 \text{ PLN}, \] (297)

- the total operation cost of the non-repairable system with a hot single reservation of components is

\[ \hat{C}^{(1)}(1000) \equiv 2 \cdot 0.341 \cdot 1000 \cdot 6 \]
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+ 0.105 \cdot 1000 \cdot 8 + 0.245 \cdot 1000 \cdot 14
+ 0.309 \cdot 1000 \cdot 14 \right] = 21 284 \text{ PLN,}
(298)

- the total operation cost of the non-repairable system with a cold single reservation of components is

\[ \hat{C}_{(2)}^{(1000)} \equiv 0.341 \cdot (1000 + 500) \cdot 6 \]
+ 0.105 \cdot (1000 + 500) \cdot 8
+ 0.245 \cdot (1000 + 500) \cdot 14
+ 0.309 \cdot (1000 + 500) \cdot 14 = 15 963 \text{ PLN,}
(299)

- the total operation cost of the non-repairable system with improved components by reduction their rates of departures from the reliability state subsets is

\[ \hat{C}_{(3)}^{(1000)} \equiv 0.341 \cdot 1500 \cdot 6 + 0.105 \cdot 1500 \cdot 8 \]
+ 0.245 \cdot 1500 \cdot 14 + 0.309 \cdot 1500 \cdot 14
= 15 963 \text{ PLN.}
(300)

In the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is ignored, the total operation cost of the repairable exemplary system with ignored its renovation time during the operation time \( \theta = 1000 \text{ days, according to (13.58), (13.60), (13.63) or (13.65) from [1], respectively amount:} \]

- the total operation cost of the repairable system with ignored renovation time with non-improved components

\[ \hat{C}_{(0)}^{(1000)} \equiv 10 642 + 50 \cdot 2.44 \]
= 10 642 + 122 = 10 764 \text{ PLN}
(301)
as according to (108) [4]

\[ \hat{H}^{(0)}(1000,2) = \frac{1000}{410.20} \equiv 2.44, \]

- the total operation cost of the repairable system with ignored renovation time with a hot single reservation of components is

\[ \hat{C}_{(0)}^{(1000)} \equiv 21 284 + 100 \cdot 1.28 \]
= 21 284 + 128 = 21 412 \text{ PLN}\]
(302)
as according to (108) [4]

\[ \hat{H}^{(0)}(1000,2) = \frac{1000}{781.33} \equiv 1.28, \]

- the total operation cost of the repairable system with ignored renovation time with a cold single reservation of components is

\[ \hat{C}_{(0)}^{(1000)} \equiv 15 963 + 75 \cdot 0.93 \]
= 15 963 + 69.75 = 16 032.75 \text{ PLN} \]
(303)
as according to (108) [4]

\[ \hat{H}^{(0)}(1000,2) = \frac{1000}{1070.01} \equiv 0.93, \]

- the total operation cost of the repairable system with ignored renovation time with components improved by reduction their rates of departures is

\[ \hat{C}_{(0)}^{(1000)} \equiv 15 963 + 150 \cdot 2.15 \]
= 15 963 + 322.5 = 16 285.5 \text{ PLN} \]
(304)
as according to (108) [4]

\[ \hat{H}^{(0)}(1000,2) = \frac{1000}{464.47} \equiv 2.15. \]

In the case when the exemplary system is repaired after exceeding the critical reliability state \( r = 2 \) and its renewal time is non-ignored and has the distribution function with the mean value \( \mu_2 = 10 \), the total operation cost of the renewed exemplary system with non-ignored its renovation time during the operation time \( \theta = 1000 \text{ days, according to (13.68), (13.70), (13.72) or (13.74) from [1], respectively amount:} \]

- the total operation cost of the repairable system with non-ignored renovation time with non-improved components

\[ \hat{C}_{(0)}^{(1000)} \equiv 10 642 + 500 \cdot 2.38 \]
= 10 642 + 1190 = 11 832 \text{ PLN}\]
(305)
as according to (109) [4]
\[
\tilde{H}^{(0)}(1000,2) = \frac{1000}{410.20 + 10} \equiv 2.38,
\]
- the total operation cost of the repairable system with non-ignored renovation time with a hot single reservation of components
\[
\tilde{C}_{\text{rel}}^{(1)}(1000) \equiv 21284 + 1000 \cdot 1.26
\]
\[
= 21284 + 1260 = 22544 \text{ PLN}, \quad (306)
\]
as according to (109) [4]
\[
\tilde{H}^{(1)}(1000,2) = \frac{1000}{781.33 + 10} \equiv 1.26,
\]
- the total operation cost of the repairable system with non-ignored renovation time with a cold single reservation of components
\[
\tilde{C}_{\text{rel}}^{(2)}(1000) \equiv 15963 + 750 \cdot 0.93
\]
\[
= 15963 + 697.5 = 16660.50 \text{ PLN} \quad (307)
\]
as according to (109) [4]
\[
\tilde{H}^{(2)}(1000,2) = \frac{1000}{1070.01 + 10} \equiv 0.93,
\]
- the total operation cost of the repairable system with non-ignored renovation time with improved components by reduction the rates of departures in the reliability state subsets
\[
\tilde{C}_{\text{rel}}^{(3)}(1000, \rho(2)) \equiv 15963 + 1500 \cdot 2.11
\]
\[
= 15963 + 3165 = 19123 \text{ PLN} \quad (308)
\]
as according to (109) [4]
\[
\tilde{H}^{(3)}(1000,2) = \frac{1000}{464.47 + 10} \equiv 2.11.
\]
The comparison of the results (285)-(296) with the results (297)-(308) justifies the sensibility of the system operation process optimization.

13. The exemplary system corrective and preventive maintenance policy optimization

13.1. Maintenance policy maximizing system availability

To optimize the exemplary system corrective and preventive maintenance policy maximizing its availability, we use its following reliability and renewal parameters:
- the number of the system and components reliability states $4 (z = 3)$,
- the system and components critical reliability state $r = 2$,
- the 2-nd coordinate of the system unconditional reliability function $R(t, \cdot)$
\[
R(t, 2) = 0.214 \cdot [R(t,2)]^{(1)} + 0.038 \cdot [R(t,2)]^{(2)}
\]
\[
+ 0.293 \cdot [R(t,2)]^{(3)} + 0.455 \cdot [R(t,2)]^{(4)} \quad (309)
\]
for $t \geq 0$, where $[R(t,2)]^{(b)}$, $b=1,2,3,4$, are respectively given by (21), (28), (43), (58) [3].
- the derivative of the 2-nd coordinate of the system unconditional reliability function $R'(t, \cdot)$
\[
R'(t, 2) = 0.214 \cdot [R'(t,2)]^{(1)} + 0.038 \cdot [R'(t,2)]^{(2)}
\]
\[
+ 0.293 \cdot [R'(t,2)]^{(3)} + 0.455 \cdot [R'(t,2)]^{(4)} \quad (310)
\]
for $t \geq 0$,
- the mean value of the system corrective maintenance (renovation) time $\mu_\text{c}(2) = 10$,
- the mean value of the system preventive maintenance (renovation) time $\mu_\text{p}(2) = 5$.
Moreover, to apply the algorithm proposed in [1], we fix:
- the measure of the method of secants accuracy $\varepsilon = 0.001$,
- the number of the values of the system preventive maintenance period $\eta_i$ for which we find the values of the availability coefficient of the exemplary system in the cases when there is no its optimal value $\kappa = 20$,
- the values of the system preventive maintenance period $\eta_i$ for which we find the values of the availability coefficient of the system in the cases when there is no its optimal value
\[
\eta_i = (i-1)0.2 \mu(2) = (i-1)0.2 \cdot 358, \quad (310)
\]
$i = 1,2,\ldots,20$. 

441
where $\mu(2)$, is given by (69) [3].

Since

$$\mu_0(2) = 10 > \mu(2) = 5$$

we are looking for the optimal value $\eta$ of the preventive maintenance period $\eta$ that maximizes the availability coefficient of the system $A(\eta, r)$ given by (14.6) from [1] by determining, if it exists, its approximate value from the equation (14.12) from [1] by applying the method of secants in the interval $< a, b >$ as follows:

- we define, obtained after the transformation of the equation (14.12) from [1], the function

$$f(\eta) = \lambda(\eta, 2) \int_0^\eta R(t, 2)dt + R(\eta, 2)$$

$$= \lambda(\eta, 2) \int_0^\eta R(t, 2)dt + R(\eta, 2) - 2$$

where $R(t, 2)$ is given by (309) and

$$\lambda(\eta, r) = -\frac{R'(\eta, 2)}{R(\eta, 2)},$$

- we define the interval $< a, b >$ assuming $a = 0$ and finding $b$ such that

$$f(b) > 0,$$

- we use the recurrent formula

$$\eta_0 = a,$$

$$\eta_{k+1} = \eta_k - \frac{f(\eta_k)}{f(b) - f(\eta_k)}(b - \eta_k)$$

(311)

for $k = 0, 1, ..., K$,

where $K$ is such that

$$f(\eta_{K+1}) < \varepsilon$$

and $\varepsilon = 0.001$ is the measure of the method of secants accuracy.

- we fix the optimal value $\eta$ of the preventive maintenance period $\eta$ assuming

$$\eta = \eta_{K+1}.$$

As a result of the computer calculations, we recognize that there is no optimal value $\eta$ of the exemplary system preventive maintenance period $\eta$ that maximize the value of its availability coefficient. The values of the system preventive maintenance period $\eta$ defined by (13.2) and the values of the availability coefficient of the exemplary system are given in Table 4.

### Table 4. The values of the availability coefficient of the exemplary system

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$A(\eta, 2)$</th>
<th>$\eta$</th>
<th>$A(\eta, 2)$</th>
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<tbody>
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<td>0.0</td>
<td>0.0</td>
<td>716.0</td>
<td>0.97202</td>
</tr>
<tr>
<td>71.6</td>
<td>0.93036</td>
<td>787.6</td>
<td>0.97220</td>
</tr>
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<td>143.2</td>
<td>0.95742</td>
<td>859.2</td>
<td>0.97233</td>
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<td>214.8</td>
<td>0.96501</td>
<td>930.8</td>
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<td>286.4</td>
<td>0.96809</td>
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<td>501.2</td>
<td>0.97110</td>
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<td>0.97265</td>
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<td>572.8</td>
<td>0.97152</td>
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<td>644.4</td>
<td>0.97180</td>
<td>1360.4</td>
<td>0.97271</td>
</tr>
</tbody>
</table>

### 13.2. Maintenance policy minimizing system renovation cost

To optimize the exemplary system corrective and preventive maintenance policy minimizing its cost of renovation, we use its following reliability and operation cost parameters:

- the system and components critical reliability state $r = 2$,
- the 2-nd coordinate of the system unconditional reliability function $R(t, \cdot)$

$$R(t, 2) = 0.214 \cdot [R(t, 2)]^{(1)} + 0.038 \cdot [R(t, 2)]^{(2)}$$

$$+ 0.293 \cdot [R(t, 2)]^{(3)} + 0.455 \cdot [R(t, 2)]^{(4)}$$

(312)

for $t \geq 0$,

where $[R(t, 2)]^{(b)}$, $b = 1, 2, 3, 4$, are respectively given by (21), (28), (43), (58) [3].

- the derivative of the 2-nd coordinate of the system unconditional reliability function $R(t, \cdot)$
\[ R'(t, 2) = 0.214 \cdot [R'(t, 2)]^{(1)} + 0.038 \cdot [R'(t, 2)]^{(2)} + 0.293 \cdot [R'(t, 2)]^{(1)} + 0.455 \cdot [R'(t, 2)]^{(2)} \quad \text{for } t \geq 0, \]

- the mean value of the cost of the exemplary system corrective maintenance (renovation) \( c_0(1) = 1000 \text{ PLN}, \)
- the mean value of the cost of the exemplary system preventive maintenance (renovation) \( c_1(1) = 800 \text{ PLN}, \)
- the measure of the method of secants accuracy \( \varepsilon = 0.001, \)
- the number of the values of the system age \( \varsigma \) for which we find the values of the system renovation cost in the cases when there is no its optimal value \( \kappa = 20, \)
- the values of the system age \( \varsigma \) for which we find the values of the system renovation cost in the cases when there is no optimal value

\[
\varsigma_i = i \cdot 0.2 \mu(2) = i \cdot 0.2 \cdot 358, \quad i = 1, 2, ..., 20. \quad (313)
\]

where \( \mu(2) \) is given by (69) [3].

After fixing the above system reliability and operation cost input parameters, we use the procedure described in Section 14.2.2 of [1].

Since

\[ c_0(2) = 1000 > c_1(2) = 800, \]

we are looking for the optimal value \( \hat{\varsigma} \) of the system age \( \varsigma \) at which the system preventive renovation is performed that minimizes the system renovation cost per unit time \( C(\varsigma) \) given by (14.20) from [1] by determining, if it exists, its approximate value from the equation (14.23) from [1] by applying the method of secants in the interval \( <a, b> \) as follows:

- we define, obtained after the transformation of the equation (14.23) from [1], the function

\[
\psi(\varsigma) = R'(\varsigma, 2) - \varsigma R'(\varsigma, 2) R(\varsigma, 2) = \frac{c_0(2) R(\varsigma, 2) - c_1(2) R'(\varsigma, 2) L(\varsigma, 2)}{c_0(2) - c_1(2)} \]

for \( \varsigma \geq 0, \)

where \( R(t, 2) \) is given by (312) and

\[
L(\varsigma, 2) = \int_{0}^{t} \frac{f(t, 2)}{1 - R(\varsigma, 2)} dt = \frac{1}{1 - R(\varsigma, 2)} \int_{0}^{t} R(\varsigma, 2) dt \]


- we define the interval \( <a, b> \) assuming \( a = 0 \) and finding \( b \) such that \( \psi(b) > 0, \)

- we use the recurrent formula

\[
\varsigma_0 = a, \quad \varsigma_{k+1} = \varsigma_k - \frac{\psi(\varsigma_k)}{\psi(b) - \psi(\varsigma_k)} (b - \varsigma_k)
\]

for \( k = 0, 1, ..., K, \)

where \( K \) is such that

\[ \psi(\varsigma_{K+1}) < \varepsilon \]

and \( \varepsilon \) is the measure of the method of secants accuracy.

- we fix the optimal value \( \hat{\varsigma} \) of the system age \( \varsigma \) assuming

\[ \hat{\varsigma} = \varsigma_{K+1}, \]

As a result of the computer calculations, we recognize that there is no optimal value \( \hat{\varsigma} \) of the exemplary system age \( \varsigma \) at which the system preventive renovation is performed that minimize the system renovation cost. The exemplary values of the system age \( \varsigma \) at which the system preventive renovation is performed defined by (313) and the values of the renovation cost of the exemplary system are given in Table 5.

**Table 5.** The values of the renovation cost of the exemplary system

<table>
<thead>
<tr>
<th>( \varsigma )</th>
<th>( C(\varsigma, 1) )</th>
<th>( \hat{\varsigma} )</th>
<th>( C(\hat{\varsigma}, 1) )</th>
</tr>
</thead>
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<tr>
<td>71.6</td>
<td>11.53</td>
<td>787.6</td>
<td>2.92</td>
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<tr>
<td>143.2</td>
<td>6.27</td>
<td>859.2</td>
<td>2.90</td>
</tr>
<tr>
<td>214.8</td>
<td>4.68</td>
<td>930.8</td>
<td>2.87</td>
</tr>
<tr>
<td>286.4</td>
<td>3.96</td>
<td>1002.4</td>
<td>2.86</td>
</tr>
<tr>
<td>358.0</td>
<td>3.58</td>
<td>1074.0</td>
<td>2.84</td>
</tr>
<tr>
<td>429.6</td>
<td>3.34</td>
<td>1145.6</td>
<td>2.83</td>
</tr>
<tr>
<td>501.2</td>
<td>3.19</td>
<td>1217.2</td>
<td>2.83</td>
</tr>
</tbody>
</table>
Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation reliability and safety. Part 6 IS&RDSS Application – Exemplary system operation cost analysis and maintenance optimization

<table>
<thead>
<tr>
<th></th>
<th>Operation Cost</th>
<th>Maintenance Cost</th>
<th>Reliability</th>
<th>Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>572.8</td>
<td>3.09</td>
<td>1288.8</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>644.4</td>
<td>3.02</td>
<td>1360.4</td>
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<td></td>
<td>716.0</td>
<td>2.96</td>
<td>1432.0</td>
<td>2.81</td>
</tr>
</tbody>
</table>

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References


