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Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety Part 5 IS&RDSS Application – Improved exemplary system operation and reliability characteristics prediction

Keywords

complex system, reliability, improve

Abstract

There is presented the IS&RDSS application to the reliability of the improved exemplary complex technical system prediction. There are considered three ways of the exemplary system reliability improvement, a hot single reservation of its components, a cold single reservation of its components and replacing its components by the improved components with reduced intensities of departure from the reliability state subsets. The evaluations of this ways improved exemplary system unconditional multistate reliability function, the expected values and the standard deviations of its unconditional lifetimes in the reliability state subsets and the mean values of its lifetimes in the particular reliability states are performed. Moreover, in the case when the improved system is repairable, its renewal and availability characteristics are estimated.

11. The improved exemplary system reliability modelling

11.1. Reliability improvement of the exemplary system

Considering the results of the system components reliability modeling from Section 3 [3] concerned with the fixed system reliability structures and their shape parameters and with the assumed the exponential models of the reliability functions of the system components in various operation states and the results of the evaluations of the system components intensities of departures from the reliability state subsets from Section 4 [3], we may to perform the improvement of the system reliability.

In order to improve the reliability of the considered exemplary system there are used the following methods:
- a hot single reservation of system components,
- a cold single reservation of system components,
- replacing the system components by improved components with reduced intensities of departure from the reliability state subsets.

We assume that the reserve components of the subsystems S_υ, $\upsilon = 1,2$, are identical with the basic components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2$, in reliability sense, i.e. they have the same four-state reliability functions

$$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)}$$
$$= [1,[R_{ij}^{(\upsilon)}(t,1)]^{(b)},[R_{ij}^{(\upsilon)}(t,2)]^{(b)}$$

$$,[R_{ij}^{(\upsilon)}(t,3)]^{(b)}], \quad (110)$$
$$t \geq 0, \ b = 1,2,3,4, \ \upsilon = 1,2,$$

with the exponential co-ordinates

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,3)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(3)]^{(b)} t],$$
$$t \geq 0, \ b = 1,2,3,4, \ \upsilon = 1,2, \quad (111)$$

different in various operation states z_b, $b = 1,2,3,4,$ where $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$, $b = 1,2,3,4,$ $\upsilon = 1,2,$ are the subsystems components unknown intensities of departures respectively from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, determined in [2] and partly presented in [3].

Moreover, we assume that the improved system components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,$ have reduced intensities of departure from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, at the operation state z_b, $b = 1,2,3,4,$ by multiplying them by the factors $[\rho_{ij}^{(\upsilon)}(u)]^{(b)}$, $0 < [\rho_{ij\ i}^{(\upsilon)}(u)]^{(b)} \leq 1.$ Consequently, the improved components have the four-state reliability functions

$$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)} = [1,$$

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)}, [R_{ij}^{(\upsilon)}(t,2)]^{(b)}, [R_{ij}^{(\upsilon)}(t,3)]^{(b)}], \quad (112)$$
$$t \geq 0, \ b = 1,2,3,4, \ \upsilon = 1,2,$$

with the exponential co-ordinates

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)} [\rho_{ij}^{(\upsilon)}(1)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)} [\rho_{ij}^{(\upsilon)}(2)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,3)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(3)]^{(b)} [\rho_{ij}^{(\upsilon)}(3)]^{(b)} t],$$
$$t \geq 0, \ b = 1,2,3,4, \ \upsilon = 1,2. \quad (113)$$

In the case, when we improve the reliability of the considered exemplary system by replacing the system components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,$ by improved components with reduced intensities of departure from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, we assume that the fixed factors $[\rho_{ij}^{(\upsilon)}(u)]^{(b)}$, $\upsilon = 1,2,$ $u = 1,2,3,$ $b = 1,2,3,4$, reducing intensities of components departure from the reliability sate subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, are as follow:

- for components $E_{i1}^{(1)}$, $i = 1,2,$ of subsystem S_1

$$[\rho_{i1}^{(1)}(u)]^{(b)} = 1.0, \ i = 1,2, \ u = 1,2,3, \ b = 1,2,3,4,$$

- for components $E_{i2}^{(1)}$, $i = 1,2,$ of subsystem S_1

$$[\rho_{i2}^{(1)}(u)]^{(b)} = 0.9, \ i = 1,2, \ u = 1,2,3, \ b = 1,2,3,4,$$

- for components $E_{i3}^{(1)}$, $i = 1,2,$ of subsystem S_1

$$[\rho_{i3}^{(1)}(u)]^{(b)} = 0.8, \ i = 1,2, \ u = 1,2,3, \ b = 1,2,3,4,$$

- for components $E_{i1}^{(1)}$, $i = 1,2,3,4$, of subsystem S_2

$$[\rho_{i1}^{(2)}(u)]^{(b)} = 0.9, \ i = 1,2,3,4, \ u = 1,2,3,$$
$$b = 1,2,3,4,$$

- for components $E_{i2}^{(1)}$, $i = 1,2,3,4$, of subsystem S_2

$$[\rho_{i2}^{(2)}(u)]^{(b)} = 0.8, \ i = 1,2,3,4 \ \ u = 1,2,3,$$
$$b = 1,2,3,4.$$

Thus, as we fixed in Section 4 [3], at the operational state z_1, the system is identical with the subsystem S_1 that is a four-state series-parallel system with its reliability structure shape parameters $k = 2$, $l_1 = 3$, $l_2 = 3$, and its four-state reliability function is given respectively by the vector:

- for the exemplary system with a hot single reservation of its components, according to (12.57)-(12.58) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(1)}, [\boldsymbol{R}^{(1)}(t,2)]^{(1)},$$
$$[\boldsymbol{R}^{(1)}(t,3)]^{(1)}], \ t \geq 0, \quad (114)$$

with the coordinates:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(1)} = \boldsymbol{R}_{2;3,3}^{(1)}(t,1)$$

$$= 1 - \prod_{i=1}^{2} [1 - \prod_{j=1}^{3} [2\exp[-[\lambda_{ij}^{(1)}(1)^{(1)} t]$$

$$- \exp[-2[\lambda_{ij}^{(1)}(1)^{(1)} t]]],$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(1)} = \boldsymbol{R}_{2;3,3}^{(1)}(t,2)$$

$$= 1 - \prod_{i=1}^{2} [1 - \prod_{j=1}^{3} [2\exp[-[\lambda_{ij}^{(1)}(2)^{(1)} t]$$

$$- \exp[-2[\lambda_{ij}^{(1)}(2)^{(1)} t]]],$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(1)} = \boldsymbol{R}_{2;3,3}^{(1)}(t,3)$$

$$= 1 - \prod_{i=1}^{2} [1 - \prod_{j=1}^{3} [2\exp[-[\lambda_{ij}^{(1)}(3)^{(1)} t]$$

$$- \exp[-2[\lambda_{ij}^{(1)}(3)^{(1)} t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2] and partly presented in [3], we get:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(1)}$$

$$= 1 - [[1 - [2\exp[-0.0008t] - \exp[-2 \cdot 0.0008t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]]^2, \qquad (115)$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(1)}$$

$$= 1 - [[1 - [2\exp[-0.0009t] - \exp[-2 \cdot 0.0009t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]]^2, \qquad (116)$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(1)}$$

$$= 1 - [[1 - [2\exp[-0.0009t] - \exp[-2 \cdot 0.0009t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]]^2 \qquad (117)$$

- for the exemplary system with a cold single reservation of its components, according to (12.59)-(12.60) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}^{(2)}(t,1)]^{(1)}, [\boldsymbol{R}^{(2)}(t,2)]^{(1)},$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(1)}], \ t \ge 0, \qquad (118)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(1)} = \boldsymbol{R}_{2;3,3}^{(2)}(t,1) =$$

$$1 - \prod_{i=1}^{2} [1 - \prod_{j=1}^{3} [(1 + [\lambda_{ij}^{(1)}(1)]^{(1)} t) \exp[-[\lambda_{ij}^{(1)}(1)]^{(1)} t]]],$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(1)} = \boldsymbol{R}_{2;3,3}^{(2)}(t,2) =$$

$$1 - \prod_{i=1}^{2} [1 - \prod_{j=1}^{3} [(1 + [\lambda_{ij}^{(1)}(2)]^{(1)} t) \exp[-[\lambda_{ij}^{(1)}(2)]^{(1)} t]]],$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(1)} = \boldsymbol{R}_{2;3,3}^{(2)}(t,3) =$$

$$1 - \prod_{i=1}^{2} [1 - \prod_{j=1}^{3} [(1 + [\lambda_{ij}^{(1)}(3)]^{(1)} t) \exp[-[\lambda_{ij}^{(1)}(3)]^{(1)} t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(1)}$$

$$= 1 - [1 - [(1 + 0.0008t)\exp[-0.0008t]] \cdot$$

$$[(1 + 0.0011t)\exp[-0.0011t]] \cdot$$

$$[(1 + 0.0011t)\exp[-0.0011t]]]^2, \qquad (119)$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(1)}$$

$$= 1 - [1 - [(1 + 0.0009t)\exp[-0.0009t]] \cdot$$

$$[(1 + 0.0011t)\exp[-0.0011t]] \cdot$$

$$[(1 + 0.0011t)\exp[-0.0011t]]]^2, \qquad (120)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(1)}$$

$$= 1 - [1 - [(1 + 0.0009t)\exp[-0.0009t]] \cdot$$

$$[(1 + 0.0012t)\exp[-0.0012t]] \cdot$$

$$[(1+0.0011t)\exp[-0.0011t]]]^2, \qquad (121)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.61)-(12.62) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(3)}(t,\cdot)]^{(1)} = [1, [\boldsymbol{R}^{(3)}(t,1)]^{(1)}, [\boldsymbol{R}^{(3)}(t,2)]^{(1)}, [\boldsymbol{R}^{(3)}(t,3)]^{(1)}],\ t \geq 0, \qquad (122)$$

with the coordinates:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(1)} = \boldsymbol{R}^{(3)}_{2;3,3}(t,1) =$$

$$1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(1)]^{(1)}[\rho_{ij}^{(1)}(1)]^{(1)}t]],$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(1)} = \boldsymbol{R}^{(3)}_{2;3,3}(t,2) =$$

$$1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(2)]^{(1)}[\rho_{ij}^{(1)}(2)]^{(1)}t]],$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(1)} = \boldsymbol{R}^{(3)}_{2;3,3}(t,3) =$$

$$1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(3)]^{(1)}[\rho_{ij}^{(1)}(3)]^{(1)}t]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], and the fixed value of factors $[\rho_{ij}^{(1)}(u)]^{(1)}$, $i=1,2$, $j=1,2,3$, we get:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(1)}$$

$$= 1-[1-\exp[-[1.0\cdot 0.0008$$

$$+0.9\cdot 0.0011+0.8\cdot 0.0011]t]^2$$

$$=1-[1-\exp[-0.00267t]]^2, \qquad (123)$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(1)}$$

$$= 1-[1-\exp[-[1.0\cdot 0.0009$$

$$+0.9\cdot 0.0011+0.8\cdot 0.0011]t]^2$$

$$=1-[1-\exp[-0.00277t]]^2, \qquad (124)$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(1)}$$

$$= 1-[1-\exp[-[1.0\cdot 0.0009$$

$$+0.9\cdot 0.0012+0.8\cdot 0.0011]t]^2$$

$$=1-[1-\exp[-0.0032t]]^2. \qquad (125)$$

The expected values of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_1, calculated from the above results given by (114)-(117), (118)-(121), (122)-(1125) respectively, according to (12.63) from [1], are:

- for the exemplary system with a hot single reservation of its components

$$\mu_1^{(1)}(1) \cong 939.53,\ \ \mu_1^{(1)}(2) \cong 911.15,\ \ \mu_1^{(1)}(3) \cong 881.87, \qquad (126)$$

- for the exemplary system with a cold single reservation of its components

$$\mu_1^{(2)}(1) \cong 1285.90,\ \ \mu_1^{(2)}(2) \cong 1247.58,\ \ \mu_1^{(2)}(3) \cong 1207.25, \qquad (127)$$

- for the exemplary system with reduced rates of departure of its components

$$\mu_1^{(3)}(1) \cong 557.11,\ \ \mu_1^{(3)}(2) \cong 536.99,\ \ \mu_1^{(3)}(3) \cong 520.10, \qquad (128)$$

and further, using (7.8) from [1] and (126), (127), (128) respectively, it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_1, are:

- for the exemplary system with a hot single reservation of its components

$$\bar{\mu}_1^{(1)}(1) \cong 28.38,\ \bar{\mu}_1^{(1)}(2) \cong 29.28,\ \bar{\mu}_1^{(1)}(3) \cong 881.87, \qquad (129)$$

- for the exemplary system with a cold single reservation of its components

$$\bar{\mu}_1^{(2)}(1) \cong 38.32,\ \bar{\mu}_1^{(2)}(2) \cong 40.33,\ \bar{\mu}_1^{(2)}(3) \cong 1207.25, \qquad (130)$$

- for the exemplary system with reduced rates of departure of its components

$$\bar{\mu}_1^{(3)}(1) \cong 22.12,\ \bar{\mu}_1^{(3)}(2) \cong 16.89,$$
$$\bar{\mu}_1^{(3)}(3) \cong 520.10. \quad (131)$$

At the operation state z_2, the system is identical with the subsystem S_2 that is a four-state series-parallel system with its structure shape parameters $k = 4$, $l_1 = 2$, $l_2 = 2$, $l_3 = 2$, $l_4 = 2$ and its four-state reliability function is given by the vector:

- for the exemplary system with a hot single reservation of its components, according to (12.57)-(12.58) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(2)}, [\boldsymbol{R}^{(1)}(t,2)]^{(2)},$$
$$[\boldsymbol{R}^{(1)}(t,3)]^{(2)}],\ t \ge 0, \quad (132)$$

with the coordinates:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(2)} = \boldsymbol{R}^{(1)}_{4;2,2,2,2}(t,1)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[2\exp[-[\lambda_{ij}^{(2)}(1)^{(2)}t]$$

$$- \exp[-2[\lambda_{ij}^{(2)}(1)^{(2)}t]]],$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(2)} = \boldsymbol{R}^{(1)}_{4;2,2,2,2}(t,2)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[2\exp[-[\lambda_{ij}^{(2)}(2)^{(2)}t]$$

$$- \exp[-2[\lambda_{ij}^{(2)}(2)^{(2)}t]]],$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(2)} = \boldsymbol{R}^{(1)}_{4;2,2,2,2}(t,3)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[2\exp[-[\lambda_{ij}^{(2)}(3)^{(2)}t]$$

$$- \exp[-2[\lambda_{ij}^{(2)}(3)^{(2)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(2)}$$

$$= 1 - [[1 - [2\exp[-0.0013t] - \exp[-2 \cdot 0.0013t]]$$

$$[2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]]^4, \quad (133)$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(2)}$$

$$= 1 - [[1 - [2\exp[-0.0014t] - \exp[-2 \cdot 0.0014t]]$$

$$[2\exp[-0.0016t] - \exp[-2 \cdot 0.0016t]]]^4, \quad (134)$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(2)}$$

$$= 1 - [[1 - [2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]$$

$$[2\exp[-0.0017t] - \exp[-2 \cdot 0.0017t]]]^4, \quad (135)$$

- for the exemplary system with a cold single reservation of its components, according to (12.59)-(12.60) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(2)}(t,1)]^{(2)}, [\boldsymbol{R}^{(2)}(t,2)]^{(2)},$$
$$[\boldsymbol{R}^{(2)}(t,3)]^{(2)}],\ t \ge 0, \quad (136)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(2)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,1) =$$

$$1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[(1 + [\lambda_{ij}^{(2)}(1)]^{(2)}t)\exp[-[\lambda_{ij}^{(2)}(1)]^{(2)}t]]],$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(2)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,2) =$$

$$1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[(1 + [\lambda_{ij}^{(2)}(2)]^{(2)}t)\exp[-[\lambda_{ij}^{(2)}(2)]^{(2)}t]]],$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(2)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,3) =$$

$$1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[(1 + [\lambda_{ij}^{(2)}(3)]^{(2)}t)\exp[-[\lambda_{ij}^{(2)}(3)]^{(2)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(2)}$$

$$= 1 - [1 - [(1 + 0.0013t)\exp[-0.0013t]] \cdot$$
$$[(1 + 0.0015t)\exp[-0.0015t]]]^4, \quad (137)$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(2)}$$

$$= 1 - [1 - [(1 + 0.0014t)\exp[-0.0014t]] \cdot$$

$$[(1 + 0.0016t)\exp[-0.0016t]]]^4, \qquad (138)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(2)}$$

$$1 - [1 - [(1 + 0.0015t)\exp[-0.0015t]] \cdot$$

$$[(1 + 0.0017t)\exp[-0.0017t]]]^4, \qquad (139)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.61)-(12.62) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(3)}(t,\cdot)]^{(2)} = [1, [\boldsymbol{R}^{(3)}(t,1)]^{(2)}, [\boldsymbol{R}^{(3)}(t,2)]^{(2)},$$
$$[\boldsymbol{R}^{(3)}(t,3)]^{(2)}], \ t \geq 0, \qquad (140)$$

with the coordinates:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(2)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,1) =$$

$$1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(1)]^{(2)}[\rho_{ij}^{(2)}(1)]^{(2)}t]],$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(2)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,2) =$$

$$1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(2)}(2)]^{(1)}[\rho_{ij}^{(2)}(2)]^{(2)}t]],$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(2)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,3) =$$

$$1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(2)}(3)]^{(1)}[\rho_{ij}^{(2)}(3)]^{(2)}t]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], and the fixed value of factors $[\rho_{ij}^{(2)}(u)]^{(2)}$, $i = 1,2,3,4$, $j = 1,2$ we get:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(2)}$$
$$= 1 - [1 - \exp[-[0.9 \cdot 0.0013$$

$$+ 0.8 \cdot 0.0015]t]^4$$

$$= 1 - [1 - \exp[-0.00237t]]^4, \qquad (141)$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(2)}$$

$$= 1 - [1 - \exp[-[0.9 \cdot 0.0014$$

$$+ 0.8 \cdot 0.0016]t]^4$$

$$= 1 - [1 - \exp[-0.00254t]]^4, \qquad (142)$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(2)}$$

$$= 1 - [1 - \exp[-[0.9 \cdot 0.0015$$

$$+ 0.8 \cdot 0.0017]t]^4$$

$$= 1 - [1 - \exp[-0.00271t]]^4. \qquad (143)$$

The expected values of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_2, calculated from the above results given by (132)-(135), (136)-(139), (140)-(143) respectively, according to (12.63) from [1], are:

- for the exemplary system with a hot single reservation of its components

$$\mu_2^{(1)}(1) \cong 1140.59, \quad \mu_2^{(1)}(2) \cong 1064.63,$$
$$\mu_2^{(1)}(3) \cong 998.14, \qquad (144)$$

- for the exemplary system with a cold single reservation of its components

$$\mu_2^{(2)}(1) \cong 1534.67, \quad \mu_2^{(2)}(2) \cong 1432.51,$$
$$\mu_2^{(2)}(3) \cong 1343.08, \qquad (145)$$

- for the exemplary system with reduced rates of departure of its components

$$\mu_2^{(3)}(1) \cong 837.76, \quad \mu_2^{(3)}(2) \cong 815.28,$$
$$\mu_1^{(3)}(3) \cong 764.14, \qquad (146)$$

and further, using (7.8) from [1] and (144). (145), (146) respectively, it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_2, are:

- for the exemplary system with a hot single reservation of its components

$$\bar{\mu}_2^{(1)}(1) \cong 75.96,\ \bar{\mu}_2^{(1)}(2) \cong 66.49,$$
$$\bar{\mu}_2^{(1)}(3) \cong 998.14, \quad (147)$$

- for the exemplary system with a cold single reservation of its components

$$\bar{\mu}_2^{(2)}(1) \cong 102.16,\ \bar{\mu}_2^{(2)}(2) \cong 89.43,$$
$$\bar{\mu}_2^{(2)}(3) \cong 1343.08, \quad (148)$$

- for the exemplary system with reduced rates of departure of its components

$$\bar{\mu}_2^{(3)}(1) \cong 22.48,\ \bar{\mu}_2^{(3)}(2) \cong 51.14,$$
$$\bar{\mu}_2^{(3)}(3) \cong 764.14. \quad (149)$$

At the operation state z_3 the system is a four-state series system composed of subsystems S_1 and S_2. At this operation state, the subsystem S_1 is a four-state series-parallel system with its structure shape parameters $k = 2$, $l_1 = 3$, $l_2 = 3$, and its four-state reliability function is given by the vector:
- for the exemplary system with a hot single reservation of its components, according to (12.57)-(12.58) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(1)(1)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(1)(1)}(t,1)]^{(3)}, [\boldsymbol{R}^{(1)(1)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(1)(1)}(t,3)]^{(3)}],\ t \geq 0, \quad (150)$$

with the coordinates:

$$[\boldsymbol{R}^{(1)(1)}(t,1)]^{(3)} = \boldsymbol{R}^{(1)}_{2;3,3}(t,1)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[2\exp[-[\lambda_{ij}^{(1)}(1)^{(3)}t]$$

$$- \exp[-2[\lambda_{ij}^{(1)}(1)^{(3)}t]]],$$

$$[\boldsymbol{R}^{(1)(1)}(t,2)]^{(3)} = \boldsymbol{R}^{(1)}_{2;3,3}(t,2)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[2\exp[-[\lambda_{ij}^{(1)}(2)^{(3)}t]$$

$$- \exp[-2[\lambda_{ij}^{(1)}(2)^{(3)}t]]],$$

$$[\boldsymbol{R}^{(1)(1)}(t,3)]^{(3)} = \boldsymbol{R}^{(1)}_{2;3,3}(t,3)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[2\exp[-[\lambda_{ij}^{(1)}(3)^{(3)}t]$$

$$- \exp[-2[\lambda_{ij}^{(1)}(3)^{(3)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)(1)}(t,1)]^{(3)}$$

$$= 1 - [[1 - [2\exp[-0.0009t] - \exp[-2\cdot 0.0009t]]$$

$$[2\exp[-0.0012t] - \exp[-2\cdot 0.0012t]]$$

$$[2\exp[-0.0011t] - \exp[-2\cdot 0.0011t]]]^2, \quad (151)$$

$$[\boldsymbol{R}^{(1)(1)}(t,2)]^{(3)}$$

$$= 1 - [[1 - [2\exp[-0.001t] - \exp[-2\cdot 0.001t]]$$

$$[2\exp[-0.0012t] - \exp[-2\cdot 0.0012t]]$$

$$[2\exp[-0.0012t] - \exp[-2\cdot 0.0012t]]]^2, \quad (152)$$

$$[\boldsymbol{R}^{(1)(1)}(t,3)]^{(3)}$$

$$= 1 - [[1 - [2\exp[-0.001t] - \exp[-2\cdot 0.001t]]$$

$$[2\exp[-0.0013t] - \exp[-2\cdot 0.0013t]]$$

$$[2\exp[-0.0012t] - \exp[-2\cdot 0.0012t]]]^2, \quad (153)$$

- for the exemplary system with a cold single reservation of its components, according to (12.59)-(12.60) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(2)(1)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(2)(1)}(t,1)]^{(3)}, [\boldsymbol{R}^{(2)(1)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(2)(1)}(t,3)]^{(3)}],\ t \geq 0, \quad (154)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)(1)}(t,1)]^{(3)} = \boldsymbol{R}^{(2)}_{2;3,3}(t,1) =$$
$$1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[(1 + [\lambda_{ij}^{(1)}(1)]^{(3)}t)\exp[-[\lambda_{ij}^{(1)}(1)]^{(3)}t]]],$$
$$[\boldsymbol{R}^{(2)(1)}(t,2)]^{(3)} = \boldsymbol{R}^{(2)}_{2;3,3}(t,2) =$$

$$1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[(1 + [\lambda_{ij}^{(1)}(2)]^{(3)}t)\exp[-[\lambda_{ij}^{(1)}(2)]^{(3)}t]]],$$

$$[\boldsymbol{R}^{(2)(1)}(t,3)]^{(3)} = \boldsymbol{R}^{(2)}_{2;3,3}(t,3) =$$

$$1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[(1+[\lambda_{ij}^{(1)}(3)]^{(3)}t)\exp[-[\lambda_{ij}^{(1)}(3)]^{(3)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)(1)}(t,1)]^{(3)}$$

$$=1-[1-[(1+0.0009t)\exp[-0.0009t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0011t)\exp[-0.0011t]]]^{2}, \quad (155)$$

$$[\boldsymbol{R}^{(2)(1)}(t,2)]^{(3)}$$

$$=1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^{2}, \quad (156)$$

$$[\boldsymbol{R}^{(2)(1)}(t,3)]^{(3)}$$

$$=1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0013t)\exp[-0.0013t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^{2}, \quad (157)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.61)-(12.62) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(3)(1)}(t,\cdot)]^{(3)}=[1,[\boldsymbol{R}^{(3)(1)}(t,1)]^{(3)},[\boldsymbol{R}^{(3)(1)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(3)(1)}(t,3)]^{(3)}],\ t\geq 0, \quad (158)$$

with the coordinates:

$$[\boldsymbol{R}^{(3)(1)}(t,1)]^{(3)}=\boldsymbol{R}^{(3)}_{2;3,3}(t,1)$$

$$=1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(1)]^{(3)}[\rho_{ij}^{(1)}(1)]^{(3)}t]],$$

$$[\boldsymbol{R}^{(3)(1)}(t,2)]^{(3)}=\boldsymbol{R}^{(3)}_{2;3,3}(t,2)$$

$$=1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(2)]^{(3)}[\rho_{ij}^{(1)}(2)]^{(3)}t]],$$

$$[\boldsymbol{R}^{(3)(1)}(t,3)]^{(3)}=\boldsymbol{R}^{(3)}_{2;3,3}(t,3)$$

$$=1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(3)]^{(3)}[\rho_{ij}^{(1)}(3)]^{(3)}t]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], and the fixed value of factors $[\rho_{ij}^{(1)}(u)]^{(3)}$, $i=1,2,\ \ j=1,2,3,$ we get:

$$[\boldsymbol{R}^{(3)(1)}(t,1)]^{(3)}$$

$$=1-[1-\exp[-[1.0\cdot 0.0009$$

$$+0.9\cdot 0.0012+0.8\cdot 0.0011]t]^{2}$$

$$=1-[1-\exp[-0.00286t]]^{2}, \quad (159)$$

$$[\boldsymbol{R}^{(3)(1)}(t,2)]^{(3)}$$

$$=1-[1-\exp[-[1.0\cdot 0.001$$

$$+0.9\cdot 0.0012+0.8\cdot 0.0012]t]^{2}$$

$$=1-[1-\exp[-0.00304t]]^{2}, \quad (160)$$

$$[\boldsymbol{R}^{(3)(1)}(t,3)]^{(3)}$$

$$=1-[1-\exp[-[1.0\cdot 0.001$$

$$+0.9\cdot 0.0013+0.8\cdot 0.0012]t]^{2}$$

$$=1-[1-\exp[-0.00313t]]^{2}. \quad (161)$$

The subsystem S_2, at the operation state z_3, is a four-state series-parallel system with its structure shape parameters $k=4$, $l_1=2,\ l_2=2,\ l_3=2,\ l_4=2$, and its four-state reliability function is given by the vector:

- for the exemplary system with a hot single reservation of its components, according to (12.57)-(12.58) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(1)(2)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(1)(2)}(t,1)]^{(3)}, [\boldsymbol{R}^{(1)(2)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(1)(2)}(t,3)]^{(3)}], \ t \geq 0, \quad (162)$$

with the coordinates:

$$[\boldsymbol{R}^{(1)(2)}(t,1)]^{(3)} = \boldsymbol{R}^{(1)}_{4;2,2,2,2}(t,1)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[2\exp[-[\lambda_{ij}^{(2)}(1)^{(3)}t]$$

$$- \exp[-2[\lambda_{ij}^{(2)}(1)^{(3)}t]]],$$

$$[\boldsymbol{R}^{(1)(2)}(t,2)]^{(3)} = \boldsymbol{R}^{(1)}_{4;2,2,2,2}(t,2)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[2\exp[-[\lambda_{ij}^{(2)}(2)^{(3)}t]$$

$$- \exp[-2[\lambda_{ij}^{(2)}(2)^{(3)}t]]],$$

$$[\boldsymbol{R}^{(1)(2)}(t,3)]^{(3)} = \boldsymbol{R}^{(1)}_{4;2,2,2,2}(t,3)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[2\exp[-[\lambda_{ij}^{(2)}(3)^{(3)}t]$$

$$- \exp[-2[\lambda_{ij}^{(2)}(3)^{(3)}t]]],$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)(2)}(t,1)]^{(3)}$$

$$= 1 - [[1 - [2\exp[-0.0009t] - \exp[-2 \cdot 0.0009t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]]^4, \quad (163)$$

$$[\boldsymbol{R}^{(1)(2)}(t,2)]^{(3)}$$

$$= 1 - [[1 - [2\exp[-0.001t] - \exp[-2 \cdot 0.001t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]]^4, \quad (164)$$

$$[\boldsymbol{R}^{(1)(2)}(t,3)]^{(3)}$$

$$= 1 - [[1 - [2\exp[-0.001t] - \exp[-2 \cdot 0.001t]]$$

$$[2\exp[-0.0013t] - \exp[-2 \cdot 0.0013t]]]^4, \quad (165)$$

- for the exemplary system with a cold single reservation of its components, according to (12.59)-(12.60) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(2)(2)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(2)(2)}(t,1)]^{(3)}, [\boldsymbol{R}^{(2)(2)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(2)(2)}(t,3)]^{(3)}], \ t \geq 0, \quad (166)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)(2)}(t,1)]^{(3)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,1) =$$

$$1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[(1 + [\lambda_{ij}^{(2)}(1)]^{(3)}t)\exp[-[\lambda_{ij}^{(2)}(1)]^{(3)}t]]],$$

$$[\boldsymbol{R}^{(2)(2)}(t,2)]^{(3)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,2) =$$

$$1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[(1 + [\lambda_{ij}^{(2)}(2)]^{(3)}t)\exp[-[\lambda_{ij}^{(2)}(2)]^{(3)}t]]],$$

$$[\boldsymbol{R}^{(2)(2)}(t,3)]^{(3)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,3) =$$

$$1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[(1 + [\lambda_{ij}^{(2)}(3)]^{(3)}t)\exp[-[\lambda_{ij}^{(2)}(3)]^{(3)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)(2)}(t,1)]^{(3)}$$

$$= 1 - [1 - [(1 + 0.0009t)\exp[-0.0009t]] \cdot$$

$$[(1 + 0.0012t)\exp[-0.0012t]]]^4, \quad (167)$$

$$[\boldsymbol{R}^{(2)(2)}(t,2)]^{(3)}$$

$$= 1 - [1 - [(1 + 0.001t)\exp[-0.001t]] \cdot$$

$$[(1 + 0.0012t)\exp[-0.0012t]]]^4, \quad (168)$$

$$[\boldsymbol{R}^{(2)(2)}(t,3)]^{(3)}$$
$$= 1 - [1 - [(1 + 0.001t)\exp[-0.001t]] \cdot$$

$$[(1 + 0.0013t)\exp[-0.0013t]]]^4, \quad (169)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.61)-(12.62) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(3)(2)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(3)(2)}(t,1)]^{(3)}, [\boldsymbol{R}^{(3)(2)}(t,2)]^{(3)}, [\boldsymbol{R}^{(3)(2)}(t,3)]^{(3)}],\ t \geq 0, \tag{170}$$

with the coordinates:

$$[\boldsymbol{R}^{(3)(2)}(t,1)]^{(3)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,1)$$

$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(1)]^{(3)}[\rho_{ij}^{(2)}(1)]^{(3)} t]],$$

$$[\boldsymbol{R}^{(3)(2)}(t,2)]^{(3)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,2)$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(2)}(2)]^{(3)}[\rho_{ij}^{(2)}(2)]^{(3)} t]],$$

$$[\boldsymbol{R}^{(3)(2)}(t,3)]^{(3)} = \boldsymbol{R}^{(2)}_{4;2,2,2,2}(t,3)$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(2)}(3)]^{(3)}[\rho_{ij}^{(2)}(3)]^{(3)} t]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], and the fix value of factors $[\rho_{ij}^{(2)}(u)]^{(3)}$, $i = 1,2,3,4,\quad j = 1,2$ we get:

$$[\boldsymbol{R}^{(3)(2)}(t,1)]^{(3)}$$

$$= 1 - [1 - \exp[-[0.9 \cdot 0.0009 + 0.8 \cdot 0.0012]t]^4$$

$$= 1 - [1 - \exp[-0.00177t]]^4, \tag{171}$$

$$[\boldsymbol{R}^{(3)(2)}(t,2)]^{(3)}$$

$$= 1 - [1 - \exp[-[0.9 \cdot 0.001 + 0.8 \cdot 0.0012]t]^4$$

$$= 1 - [1 - \exp[-0.00186t]]^4, \tag{172}$$

$$[\boldsymbol{R}^{(3)(2)}(t,3)]^{(3)}$$
$$= 1 - [1 - \exp[-[0.9 \cdot 0.001 + 0.8 \cdot 0.0013]t]^4$$

$$= 1 - [1 - \exp[-0.00194t]]^4. \tag{173}$$

Considering that the system at the operation state z_3 is a four-state series system composed of subsystems S_1 and S_2, its conditional four-state reliability function is given by the vector:

- for the exemplary system with a hot single reservation of its components, according to (12.17)-(12.18) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(3)}, [\boldsymbol{R}^{(1)}(t,2)]^{(3)}, [\boldsymbol{R}^{(1)}(t,3)]^{(3)}],\ t \geq 0, \tag{174}$$

with the coordinates:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(3)} = \overline{\boldsymbol{R}}^{(1)}_{2}(t,1)$$

$$= [\boldsymbol{R}^{(1)(1)}(t,1)]^{(3)}\ [\boldsymbol{R}^{(1)(2)}(t,1)]^{(3)},$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(3)} = \overline{\boldsymbol{R}}^{(1)}_{2}(t,2)$$

$$= [\boldsymbol{R}^{(1)(1)}(t,2)]^{(3)}\ [\boldsymbol{R}^{(1)(2)}(t,2)]^{(3)},$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(3)} = \overline{\boldsymbol{R}}^{(1)}_{2}(t,3)$$

$$= [\boldsymbol{R}^{(1)(1)}(t,3)]^{(3)}\ [\boldsymbol{R}^{(1)(2)}(t,3)]^{(3)}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(3)}$$

$$= [1 - [[1 - [2\exp[-0.0009t] - \exp[-2 \cdot 0.0009t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]]^2] \cdot$$

$$[1 - [[1 - [2\exp[-0.0009t] - \exp[-2 \cdot 0.0009t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]]^4], \tag{175}$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(3)}$$

$$= [1 - [[1 - [2\exp[-0.001t] - \exp[-2 \cdot 0.001t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]]^2] \cdot$$

$$[1 - [[1 - [2\exp[-0.001t] - \exp[-2 \cdot 0.001t]]$$

$$[2\exp[-0.0012t]-\exp[-2\cdot 0.0012t]]]^4, \quad (176)$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(3)}$$

$$= [1-[[1-[2\exp[-0.001t]-\exp[-2\cdot 0.001t]]$$

$$[2\exp[-0.0013t]-\exp[-2\cdot 0.0013t]]$$

$$[2\exp[-0.0012t]-\exp[-2\cdot 0.0012t]]]^2]\cdot$$

$$[1-[[1-[2\exp[-0.001t]-\exp[-2\cdot 0.001t]]$$

$$[2\exp[-0.0013t]-\exp[-2\cdot 0.0013t]]]^4], \quad (177)$$

- for the exemplary system with a cold single reservation of its components, according to (12.19)-(12.20) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(2)}(t,1)]^{(3)}, [\boldsymbol{R}^{(2)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(2)}(t,3)]^{(3)}],\ t \ge 0, \quad (178)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(3)} = \overline{\boldsymbol{R}}_2^{(2)}(t,1)$$

$$= [\boldsymbol{R}^{(2)(1)}(t,1)]^{(3)}\, [\boldsymbol{R}^{(2)(2)}(t,1)]^{(3)},$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(3)} = \overline{\boldsymbol{R}}_2^{(2)}(t,2)$$

$$= [\boldsymbol{R}^{(2)(1)}(t,2)]^{(3)}\, [\boldsymbol{R}^{(2)(2)}(t,2)]^{(3)},$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(3)} = \overline{\boldsymbol{R}}_2^{(2)}(t,3)$$

$$= [\boldsymbol{R}^{(2)(1)}(t,3)]^{(3)}\, [\boldsymbol{R}^{(2)(2)}(t,3)]^{(3)}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(3)}$$

$$= [1-[1-[(1+0.0009t)\exp[-0.0009t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0011t)\exp[-0.0011t]]]^2]\cdot$$

$$[1-[1-[(1+0.0009t)\exp[-0.0009t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^4], \quad (179)$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(3)}$$

$$= [1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^2]\cdot$$

$$[1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^4], \quad (180)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(3)}$$

$$= [1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0013t)\exp[-0.0013t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^2]\cdot$$

$$[1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0013t)\exp[-0.0013t]]]^4], \quad (181)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.21)-(12.22) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(3)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{R}^{(3)}(t,1)]^{(3)}, [\boldsymbol{R}^{(3)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(3)}(t,3)]^{(3)}],\ t \ge 0, \quad (182)$$

with the coordinates:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(3)} = \overline{\boldsymbol{R}}_2^{(3)}(t,1)$$

$$= [\boldsymbol{R}^{(3)(1)}(t,1)]^{(3)}\, [\boldsymbol{R}^{(3)(2)}(t,1)]^{(3)},$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(3)} = \overline{\boldsymbol{R}}_2^{(3)}(t,2)$$

$$= [\boldsymbol{R}^{(3)(1)}(t,2)]^{(3)}\, [\boldsymbol{R}^{(3)(2)}(t,2)]^{(3)},$$
$$[\boldsymbol{R}^{(3)}(t,3)]^{(3)} = \overline{\boldsymbol{R}}_2^{(3)}(t,3)$$

$$= [\boldsymbol{R}^{(3)(1)}(t,3)]^{(3)}\, [\boldsymbol{R}^{(3)(2)}(t,3)]^{(3)}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(3)}$$

$$=[1-[1-\exp[-0.00286t]]^2]\cdot$$

$$[1-[1-\exp[-0.00177t]]^4], \quad (183)$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(3)}$$

$$=[1-[1-\exp[-0.00304t]]^2]\cdot$$

$$[1-[1-\exp[-0.00186t]]^4], \quad (184)$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(3)}$$
$$=[1-[1-\exp[-0.00313t]]^2]\cdot$$

$$[1-[1-\exp[-0.00194t]]^4]. \quad (185)$$

The expected values of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_3, calculated from the above results given by (174)-(177), (178)-(181), (182)-(185) respectively, according to (12.23) from [1], are:

- for the exemplary system with a hot single reservation of its components

$$\mu_3^{(1)}(1) \cong 808.64, \quad \mu_3^{(1)}(2) \cong 764.63,$$
$$\mu_3^{(1)}(3) \cong 739.45, \quad (186)$$

- for the exemplary system with a cold single reservation of its components

$$\mu_3^{(2)}(1) \cong 1111.18, \quad \mu_3^{(2)}(2) \cong 1050.95,$$
$$\mu_3^{(2)}(3) \cong 1016.19, \quad (187)$$

- for the exemplary system with reduced rates of departure of its components

$$\mu_3^{(3)}(1) \cong 457.41, \quad \mu_3^{(3)}(2) \cong 431.46,$$
$$\mu_3^{(3)}(3) \cong 417.81, \quad (188)$$

and further, using (7.8) from [1] and (186). (187), (188) respectively, it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_3, are:

- for the exemplary system with a hot single reservation of its components

$$\bar{\mu}_3^{(1)}(1) \cong 44.01, \; \bar{\mu}_3^{(1)}(2) \cong 25.18,$$
$$\bar{\mu}_3^{(1)}(3) \cong 739.45, \quad (189)$$

- for the exemplary system with a cold single reservation of its components

$$\bar{\mu}_3^{(2)}(1) \cong 60.23, \; \bar{\mu}_3^{(2)}(2) \cong 34.76,$$
$$\bar{\mu}_3^{(2)}(3) \cong 1016.19, \quad (190)$$

- for the exemplary system with reduced rates of departure of its components

$$\bar{\mu}_3^{(3)}(1) \cong 25.95, \; \bar{\mu}_3^{(3)}(2) \cong 13.65,$$
$$\bar{\mu}_3^{(3)}(3) \cong 417.81. \quad (191)$$

At the operation state z_4 the system is a four-state series system composed of subsystems S_1 and S_2, At this operation state, the subsystem S_1 is a four-state series-parallel system with its structure shape parameters k = 2, $l_1 = 3$, $l_2 = 3$, and its four-state reliability function is given by the vector:
- for the exemplary system with a hot single reservation of its components, according to (12.57)-(12.58) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(1)(1)}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}^{(1)(1)}(t,1)]^{(4)}, [\boldsymbol{R}^{(1)(1)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(1)(1)}(t,3)]^{(4)}], \; t \geq 0, \quad (192)$$

with the coordinates:

$$[\boldsymbol{R}^{(1)(1)}(t,1)]^{(4)} = \boldsymbol{R}^{(1)}_{2;3,3}(t,1)$$

$$=1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[2\exp[-[\lambda_{ij}^{(1)}(1)^{(4)}t]$$

$$-\exp[-2[\lambda_{ij}^{(1)}(1)^{(4)}t]]],$$

$$[\boldsymbol{R}^{(1)(1)}(t,2)]^{(4)} = \boldsymbol{R}^{(1)}_{2;3,3}(t,2)$$
$$=1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[2\exp[-[\lambda_{ij}^{(1)}(2)^{(4)}t]$$

$$-\exp[-2[\lambda_{ij}^{(1)}(2)^{(4)}t]]],$$

$$[\boldsymbol{R}^{(1)(1)}(t,3)]^{(4)} = \boldsymbol{R}^{(1)}_{2;3,3}(t,3)$$

$$=1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[2\exp[-[\lambda_{ij}^{(1)}(3)^{(4)}t]$$

$$-\exp[-2[\lambda_{ij}^{(1)}(3)^{(4)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)(1)}(t,1)]^{(4)}$$

$$=1-[[1-[2\exp[-0.0009t]-\exp[-2\cdot 0.0009t]]$$

$$[2\exp[-0.0012t]-\exp[-2\cdot 0.0012t]]$$

$$[2\exp[-0.0011t]-\exp[-2\cdot 0.0011t]]]^2, \qquad (193)$$

$$[\boldsymbol{R}^{(1)(1)}(t,2)]^{(4)}$$

$$=1-[[1-[2\exp[-0.001t]-\exp[-2\cdot 0.001t]]$$

$$[2\exp[-0.0012t]-\exp[-2\cdot 0.0012t]]$$

$$[2\exp[-0.0012t]-\exp[-2\cdot 0.0012t]]]^2, \qquad (194)$$

$$[\boldsymbol{R}^{(1)(1)}(t,3)]^{(4)}$$

$$=1-[[1-[2\exp[-0.001t]-\exp[-2\cdot 0.001t]]$$

$$[2\exp[-0.0013t]-\exp[-2\cdot 0.0013t]]$$

$$[2\exp[-0.0012t]-\exp[-2\cdot 0.0012t]]]^2, \qquad (195)$$

- for the exemplary system with a cold single reservation of its components, according to (12.59)-(12.60) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(2)(1)}(t,\cdot)]^{(4)}=[1,[\boldsymbol{R}^{(2)(1)}(t,1)]^{(4)},[\boldsymbol{R}^{(2)(1)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(2)(1)}(t,3)]^{(4)}],\ t\ge 0, \qquad (196)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)(1)}(t,1)]^{(4)}=\boldsymbol{R}^{(2)}_{2;3,3}(t,1)=$$

$$1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[(1+[\lambda_{ij}^{(1)}(1)]^{(4)}t)\exp[-[\lambda_{ij}^{(1)}(1)]^{(4)}t]]],$$

$$[\boldsymbol{R}^{(2)(1)}(t,2)]^{(4)}=\boldsymbol{R}^{(2)}_{2;3,3}(t,2)=$$

$$1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[(1+[\lambda_{ij}^{(1)}(2)]^{(4)}t)\exp[-[\lambda_{ij}^{(1)}(2)]^{(4)}t]]],$$

$$[\boldsymbol{R}^{(2)(1)}(t,3)]^{(4)}=\boldsymbol{R}^{(2)}_{2;3,3}(t,3)=$$

$$1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[(1+[\lambda_{ij}^{(1)}(3)]^{(4)}t)\exp[-[\lambda_{ij}^{(1)}(3)]^{(4)}t]]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)(1)}(t,1)]^{(4)}$$

$$=1-[1-[(1+0.0009t)\exp[-0.0009t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0011t)\exp[-0.0011t]]]^2, \qquad (197)$$

$$[\boldsymbol{R}^{(2)(1)}(t,2)]^{(4)}$$

$$=1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^2, \qquad (198)$$

$$[\boldsymbol{R}^{(2)(1)}(t,3)]^{(4)}$$

$$=1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0013t)\exp[-0.0013t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^2, \qquad (199)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.61)-(12.62) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(3)(1)}(t,\cdot)]^{(4)}=[1,[\boldsymbol{R}^{(3)(1)}(t,1)]^{(4)},[\boldsymbol{R}^{(3)(1)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(3)(1)}(t,3)]^{(4)}],\ t\ge 0, \qquad (200)$$

with the coordinates:

$$[\boldsymbol{R}^{(3)(1)}(t,1)]^{(4)}=\boldsymbol{R}^{(3)}_{2;3,3}(t,1)=$$

$$1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(1)]^{(4)}[\rho_{ij}^{(1)}(1)]^{(4)}t]],$$

$$[\boldsymbol{R}^{(3)(1)}(t,2)]^{(4)}=\boldsymbol{R}^{(3)}_{2;3,3}(t,2)=$$

$$1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(2)]^{(4)}[\rho_{ij}^{(1)}(2)]^{(4)}t]],$$

$$[\boldsymbol{R}^{(3)(1)}(t,3)]^{(4)}=\boldsymbol{R}_{2;3,3}^{(3)}(t,3)=$$

$$1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(3)]^{(4)}[\rho_{ij}^{(1)}(3)]^{(4)}t]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], and the fixed value of factors $[\rho_{ij}^{(1)}(u)]^{(4)}$, $i=1,2,$ $j=1,2,3,$ we get:

$$[\boldsymbol{R}^{(3)(1)}(t,1)]^{(4)}$$

$$=1-[1-\exp[-[1.0\cdot 0.0009$$

$$+0.9\cdot 0.0012+0.8\cdot 0.0011]t]^{2}$$

$$=1-[1-\exp[-0.00286t]]^{2}, \tag{201}$$

$$[\boldsymbol{R}^{(3)(1)}(t,2)]^{(4)}$$

$$1-[1-\exp[-[1.0\cdot 0.001$$

$$+0.9\cdot 0.0012+0.8\cdot 0.0012]t]^{2}$$

$$=1-[1-\exp[-0.00304t]]^{2}, \tag{202}$$

$$[\boldsymbol{R}^{(3)(1)}(t,3)]^{(4)}$$

$$1-[1-\exp[-[1.0\cdot 0.001$$

$$+0.9\cdot 0.0013+0.8\cdot 0.0012]t]^{2}$$
$$=1-[1-\exp[-0.00313t]]^{2}. \tag{203}$$

The subsystem S_2, at the operation state z_4, is a four-state series-"2 out of 4" system, with its structure shape parameters $k=4$, $m=2$, $l_1=2$, $l_2=2$, $l_3=2$, $l_4=2$, and its four-state reliability function is given by the vector:
- for the exemplary system with a hot single reservation of its components, according to (12.73)-(12.74) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(1)(2)}(t,\cdot)]^{(4)}=[1,[\boldsymbol{R}^{(1)(2)}(t,1)]^{(4)},[\boldsymbol{R}^{(1)(2)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(1)(2)}(t,3)]^{(4)}],\ t\geq 0, \tag{204}$$

with the coordinates:

$$[\boldsymbol{R}^{(1)(2)}(t,1)]^{(4)}=\boldsymbol{R}_{4;2,2,2,2}^{2\quad(1)}(t,1)$$

$$=1-$$
$$\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\leq 1}}^{1}\prod_{i=1}^{4}[[\prod_{j=1}^{2}[2\exp[-[\lambda_{ij}(1)]^{(4)}t]-\exp[-2[\lambda_{ij}(1)]^{(4)}t]]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}[2\exp[-[\lambda_{ij}(1)]^{(4)}t]-\exp[-2[\lambda_{ij}(1)]^{(4)}t]]]^{1-r_i},$$

$$[\boldsymbol{R}^{(1)(2)}(t,2)]^{(4)}=\boldsymbol{R}_{4;2,2,2,2}^{2\quad(1)}(t,2)$$

$$=1-$$
$$\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\leq 1}}^{1}\prod_{i=1}^{4}[[\prod_{j=1}^{2}[2\exp[-[\lambda_{ij}(2)]^{(4)}t]-\exp[-2[\lambda_{ij}(2)]^{(4)}t]]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}[2\exp[-[\lambda_{ij}(2)]^{(4)}t]-\exp[-2[\lambda_{ij}(2)]^{(4)}t]]]^{1-r_i}$$

$$[\boldsymbol{R}^{(1)(2)}(t,3)]^{(4)}=\boldsymbol{R}_{4;2,2,2,2}^{2\quad(1)}(t,3)$$

$$=1-$$
$$\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\leq 1}}^{1}\prod_{i=1}^{4}[[\prod_{j=1}^{2}[2\exp[-[\lambda_{ij}(3)]^{(4)}t]-\exp[-2[\lambda_{ij}(3)]^{(4)}t]]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}[2\exp[-[\lambda_{ij}(3)]^{(4)}t]-\exp[-2[\lambda_{ij}(3)]^{(4)}t]]]^{1-r_i}$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)(2)}(t,1)]^{(4)}=$$

$$=1-[[1-[2\exp[-0.0013t]-\exp[-2\cdot 0.0013t]]$$

$$[2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]]]^{4}$$

$$+4[1-[2\exp[-0.0013t]-\exp[-2\cdot 0.0013t]]$$

$$[2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]]]^3$$

$$\cdot[2\exp[-0.0013t]-\exp[-2\cdot 0.0013t]]$$

$$[2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]], \qquad (205)$$

$$[\boldsymbol{R}^{(1)(2)}(t,2)]^{(4)}$$

$$=1-[[1-[2\exp[-0.0014t]-\exp[-2\cdot 0.0014t]]$$

$$[2\exp[-0.0016t]-\exp[-2\cdot 0.0016t]]]^4$$

$$+4[1-[2\exp[-0.0014t]-\exp[-2\cdot 0.0014t]]$$

$$[2\exp[-0.0016t]-\exp[-2\cdot 0.0016t]]]^3$$

$$\cdot[2\exp[-0.0014t]-\exp[-2\cdot 0.0014t]]$$

$$[2\exp[-0.0016t]-\exp[-2\cdot 0.0016t]], \qquad (206)$$

$$[\boldsymbol{R}^{(1)(2)}(t,3)]^{(4)}$$

$$=1-[[1-[2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]]$$

$$[2\exp[-0.0018t]-\exp[-2\cdot 0.0018t]]]^4$$

$$+4[1-[2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]]$$

$$[2\exp[-0.0018t]-\exp[-2\cdot 0.0018t]]]^3$$

$$\cdot[2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]]$$

$$[2\exp[-0.0018t]-\exp[-2\cdot 0.0018t]], \qquad (207)$$

- for the exemplary system with a cold single reservation of its components, according to (12.77)-(12.78) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(2)(2)}(t,\cdot)]^{(4)}=[1,[\boldsymbol{R}^{(2)(2)}(t,1)]^{(4)},[\boldsymbol{R}^{(2)(2)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(2)(2)}(t,3)]^{(4)}],\ t\ge 0, \qquad (208)$$

with the coordinates:

$$[\boldsymbol{R}^{(2)(2)}(t,1)]^{(4)}=\boldsymbol{R}^{2\ \ (2)}_{4;2,2,2,2}(t,1)$$

$$=1-\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1}\prod_{i=1}^{4}[\prod_{j=1}^{2}[(1+[\lambda_{ij}(1)]^{(4)}t)\exp[-[\lambda_{ij}(1)]^{(4)}t]]]^{r_i}$$
$$\cdot[1-\prod_{j=1}^{2}[(1+[\lambda_{ij}(1)]^{(4)}t)\exp[-[\lambda_{ij}(1)]^{(4)}t]]]^{1-r_i},$$

$$[\boldsymbol{R}^{(2)(2)}(t,2)]^{(4)}=\boldsymbol{R}^{2\ \ (2)}_{4;2,2,2,2}(t,2)$$

$$=1-\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1}\prod_{i=1}^{4}[\prod_{j=1}^{2}[(1+[\lambda_{ij}(2)]^{(4)}t)\exp[-[\lambda_{ij}(2)]^{(4)}t]]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}[(1+[\lambda_{ij}(2)]^{(4)}t)\exp[-[\lambda_{ij}(2)]^{(4)}t]]]^{1-r_i},$$

$$[\boldsymbol{R}^{(2)(2)}(t,3)]^{(4)}=\boldsymbol{R}^{2\ \ (2)}_{4;2,2,2,2}(t,3)$$

$$=1-\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_4\le 1}}^{1}\prod_{i=1}^{4}[\prod_{j=1}^{2}[(1+[\lambda_{ij}(3)]^{(4)}t)\exp[-[\lambda_{ij}(3)]^{(4)}t]]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}[(1+[\lambda_{ij}(3)]^{(4)}t)\exp[-[\lambda_{ij}(3)]^{(4)}t]]]^{1-r_i}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)(2)}(t,1)]^{(4)}=$$

$$=1-[[1-[(1+0.0013t)\exp[-0.0013t]]$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]]]^4$$

$$+4[1-[(1+0.0013t)\exp[-0.0013t]]$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]]]^3$$

$$\cdot[(1+0.0013t)\exp[-0.0013t]]$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]]]], \qquad (209)$$

$$[\boldsymbol{R}^{(2)(2)}(t,2)]^{(4)}$$

$$=1-[[1-[(1+0.0014t)\exp[-0.0014t]]$$

$$\cdot[(1+0.0016t)\exp[-0.0016t]]]^4$$

$$+4[1-[(1+0.0014t)\exp[-0.0014t]]$$

$$\cdot[(1+0.0016t)\exp[-0.0016t]]]^3$$

$$\cdot[(1+0.0014t)\exp[-0.0014t]]$$

$$\cdot[(1+0.0016t)\exp[-0.0016t]]]], \qquad (210)$$

$$[\boldsymbol{R}^{(2)(2)}(t,3)]^{(4)}$$

$$=1-[[1-[(1+0.0015t)\exp[-0.0015t]]$$

$$\cdot[(1+0.0018t)\exp[-0.0018t]]]^4$$

$$+4[1-[(1+0.0015t)\exp[-0.0015t]]$$

$$\cdot[(1+0.0018t)\exp[-0.0018t]]]^3$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]]$$

$$\cdot[(1+0.0018t)\exp[-0.0018t]]]]. \qquad (211)$$

- for the exemplary system with reduced rates of departure of its components, according to (12.81)-(12.82) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(3)(2)}(t,\cdot)]^{(4)}=[1,[\boldsymbol{R}^{(3)(2)}(t,1)]^{(4)},[\boldsymbol{R}^{(3)(2)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(3)(2)}(t,3)]^{(4)}],\ t\geq 0, \qquad (12.102)$$

with the coordinates1

$$[\boldsymbol{R}^{(3)(2)}(t,1)]^{(4)}=\boldsymbol{R}^{2\ \ (3)}_{4;2,2,2,2}(t,1)$$

$$=1-\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_{4k}\leq 1}}^{1}\prod_{i=1}^{4}[[\prod_{j=1}^{2}\exp[-[\lambda_{ij}(1)]^{(4)}[\rho_{ij}(1)]^{(4)}t]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}\exp[-[\lambda_{ij}(1)]^{(4)}[\rho^{(2)}_{ij}(u)]^{(4)}t]]^{1-r_i},$$

$$[\boldsymbol{R}^{(3)(2)}(t,2)]^{(4)}=\boldsymbol{R}^{2\ \ (3)}_{4;2,2,2,2}(t,2)$$

$$=1-\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_{4k}\leq 1}}^{1}\prod_{i=1}^{4}[[\prod_{j=1}^{2}\exp[-[\lambda_{ij}(2)]^{(4)}[\rho_{ij}(2)]^{(4)}t]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}\exp[-[\lambda_{ij}(2)]^{(4)}[\rho^{(2)}_{ij}(2)]^{(4)}t]]^{1-r_i},$$

$$[\boldsymbol{R}^{(3)(2)}(t,3)]^{(4)}=\boldsymbol{R}^{2\ \ (3)}_{4;2,2,2,2}(t,3)$$

$$=1-\sum_{\substack{r_1,r_2,r_3,r_4=0\\ r_1+r_2+r_3+r_{4k}\leq 1}}^{1}\prod_{i=1}^{4}[[\prod_{j=1}^{2}\exp[-[\lambda_{ij}(3)]^{(4)}[\rho_{ij}(3)]^{(4)}t]]^{r_i}$$

$$\cdot[1-\prod_{j=1}^{2}\exp[-[\lambda_{ij}(3)]^{(4)}[\rho^{(2)}_{ij}(3)]^{(4)}t]]^{1-r_i}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], and the fixed value of factors $[\rho^{(2)}_{ij}(u)]^{(4)}$, $i=1,2,3,4,$ $j=1,2$ we get:

$$[\boldsymbol{R}^{(3)(2)}(t,1)]^{(4)}$$

$$=1-[[1-\exp[-[0.9\cdot 0.0013+0.8\cdot 0.0015]t]]^4$$

$$+4\exp[-[0.9\cdot 0.0013+0.8\cdot 0.0015]t]$$

$$[1-\exp[-[0.9\cdot 0.0013+0.8\cdot 0.0015]t]]^3]$$

$$=1-[[1-\exp[-0.237t]]^4$$

$$+4\exp[-237t][1-\exp[-237t]]^3], \qquad (213)$$

$$[\boldsymbol{R}^{(3)(2)}(t,2)]^{(4)}$$

$$=1-[[1-\exp[-[0.9\cdot 0.0014+0.8\cdot 0.0016]t]]^4$$

$$+4\exp[-[0.9\cdot 0.0014+0.8\cdot 0.0016]t]$$

$$[1-\exp[-[0.9\cdot 0.0014+0.8\cdot 0.0016]t]]^3]$$

$$=1-[[1-\exp[-0.254t]]^4$$

$$+4\exp[-254t][1-\exp[-254t]]^3], \qquad (214)$$

$$[\boldsymbol{R}^{(3)(2)}(t,3)]^{(4)}$$

$$= 1 - [[1 - \exp[-[0.9 \cdot 0.0015 + 0.8 \cdot 0.0018]t]]^4$$

$$+ 4\exp[-[0.9 \cdot 0.0015 + 0.8 \cdot 0.0018]t]$$

$$[1 - \exp[-[0.9 \cdot 0.0015 + 0.8 \cdot 0.0018]t]]^3]$$

$$= 1 - [[1 - \exp[-0.279t]]^4$$

$$+ 4\exp[-279t][1 - \exp[-279t]]^3]. \qquad (215)$$

Considering that the system at the operation state z_4 is a four-state series system composed of subsystems S_1 and S_2, its conditional four-state reliability function is given by the vector:
- for the exemplary system with a hot single reservation of its components, according to (12.17)-(12.18) from [1] for $v = 1$ and $p_1 = 1$,

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}^{(1)}(t,1)]^{(4)}, [\boldsymbol{R}^{(1)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(1)}(t,3)]^{(4)}], \; t \geq 0, \qquad (216)$$

with the coordinates:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(4)} = \overline{\boldsymbol{R}}_2^{(1)}(t,1)$$

$$= [\boldsymbol{R}^{(1)(1)}(t,1)]^{(4)} \; [\boldsymbol{R}^{(1)(2)}(t,1)]^{(4)},$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(4)} = \overline{\boldsymbol{R}}_2^{(1)}(t,2)$$

$$= [\boldsymbol{R}^{(1)(1)}(t,2)]^{(4)} \; [\boldsymbol{R}^{(1)(2)}(t,2)]^{(4)},$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(4)} = \overline{\boldsymbol{R}}_2^{(1)}(t,3)$$

$$= [\boldsymbol{R}^{(1)(1)}(t,3)]^{(4)} \; [\boldsymbol{R}^{(1)(2)}(t,3)]^{(4)}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(4)}$$

$$= [1 - [[1 - [2\exp[-0.0009t] - \exp[-2 \cdot 0.0009t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]$$

$$[2\exp[-0.0011t] - \exp[-2 \cdot 0.0011t]]]^2] \cdot$$

$$[1 - [[1 - [2\exp[-0.0013t] - \exp[-2 \cdot 0.0013t]]$$

$$[2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]]^4$$

$$+ 4[1 - [2\exp[-0.0013t] - \exp[-2 \cdot 0.0013t]]$$

$$[2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]]^3$$

$$\cdot [2\exp[-0.0013t] - \exp[-2 \cdot 0.0013t]]$$
$$, \qquad (217)$$
$$[2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]]$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(4)}$$

$$= [1 - [[1 - [2\exp[-0.001t] - \exp[-2 \cdot 0.001t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]$$

$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]]^2] \cdot$$

$$[1 - [[1 - [2\exp[-0.0014t] - \exp[-2 \cdot 0.0014t]]$$

$$[2\exp[-0.0016t] - \exp[-2 \cdot 0.0016t]]]^4$$

$$+ 4[1 - [2\exp[-0.0014t] - \exp[-2 \cdot 0.0014t]]$$

$$[2\exp[-0.0016t] - \exp[-2 \cdot 0.0016t]]]^3$$

$$\cdot [2\exp[-0.0014t] - \exp[-2 \cdot 0.0014t]]$$
$$, \qquad (218)$$
$$[2\exp[-0.0016t] - \exp[-2 \cdot 0.0016t]]$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(4)} =$$

$$= [1 - [[1 - [2\exp[-0.001t] - \exp[-2 \cdot 0.001t]]$$

$$[2\exp[-0.0013t] - \exp[-2 \cdot 0.0013t]]$$
$$[2\exp[-0.0012t] - \exp[-2 \cdot 0.0012t]]]^2] \cdot$$

$$[1 - [[1 - [2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]$$

$$[2\exp[-0.0018t] - \exp[-2 \cdot 0.0018t]]]^4$$

$$+ 4[1 - [2\exp[-0.0015t] - \exp[-2 \cdot 0.0015t]]$$

$$[2\exp[-0.0018t] - \exp[-2 \cdot 0.0018t]]]^3$$

$$\cdot [2\exp[-0.0015t]-\exp[-2\cdot 0.0015t]] \quad , \quad [2\exp[-0.0018t]-\exp[-2\cdot 0.0018t]]] \tag{219}$$

- for the exemplary system with a cold single reservation of its components, according to (12.19)-(12.20) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}^{(2)}(t,1)]^{(4)}, [\boldsymbol{R}^{(2)}(t,2)]^{(4)}, [\boldsymbol{R}^{(2)}(t,3)]^{(4)}], \ t \geq 0, \tag{220}$$

with the coordinates:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(4)} = \overline{\boldsymbol{R}}_2^{(2)}(t,1)$$

$$=[\boldsymbol{R}^{(2)(1)}(t,1)]^{(4)}\ [\boldsymbol{R}^{(2)(2)}(t,1)]^{(4)},$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(4)} = \overline{\boldsymbol{R}}_2^{(2)}(t,2)$$

$$=[\boldsymbol{R}^{(2)(1)}(t,2)]^{(4)}\ [\boldsymbol{R}^{(2)(2)}(t,2)]^{(4)},$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(4)} = \overline{\boldsymbol{R}}_2^{(2)}(t,3)$$

$$=[\boldsymbol{R}^{(2)(1)}(t,3)]^{(4)}\ [\boldsymbol{R}^{(2)(2)}(t,3)]^{(4)}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(4)}$$

$$=[1-[1-[(1+0.0009t)\exp[-0.0009t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0011t)\exp[-0.0011t]]]^2]$$

$$\cdot[1-[[1-[(1+0.0013t)\exp[-0.0013t]]$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]]]^4$$

$$+4[1-[(1+0.0013t)\exp[-0.0013t]]$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]]]^3$$

$$\cdot[(1+0.0013t)\exp[-0.0013t]] \quad , \quad \cdot[(1+0.0015t)\exp[-0.0015t]]]]] \tag{221}$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(4)}$$

$$=[1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^2]$$

$$\cdot[1-[[1-[(1+0.0014t)\exp[-0.0014t]]$$

$$\cdot[(1+0.0016t)\exp[-0.0016t]]]^4$$

$$+4[1-[(1+0.0014t)\exp[-0.0014t]]$$

$$\cdot[(1+0.0016t)\exp[-0.0016t]]]^3$$

$$\cdot[(1+0.0014t)\exp[-0.0014t]] \quad , \quad \cdot[(1+0.0016t)\exp[-0.0016t]]]]] \tag{222}$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(4)}$$

$$=[1-[1-[(1+0.001t)\exp[-0.001t]]\cdot$$

$$[(1+0.0013t)\exp[-0.0013t]]\cdot$$

$$[(1+0.0012t)\exp[-0.0012t]]]^2]$$

$$\cdot[1-[[1-[(1+0.0015t)\exp[-0.0015t]]$$

$$\cdot[(1+0.0018t)\exp[-0.0018t]]]^4$$

$$+4[1-[(1+0.0015t)\exp[-0.0015t]]$$

$$\cdot[(1+0.0018t)\exp[-0.0018t]]]^3$$

$$\cdot[(1+0.0015t)\exp[-0.0015t]] \quad , \quad \cdot[(1+0.0018t)\exp[-0.0018t]]]]] \tag{223}$$

- for the exemplary system with reduced rates of departure of its components, according to (12.21)-(12.22) from [1] for $v=1$ and $p_1=1$,

$$[\boldsymbol{R}^{(3)}(t,\cdot)]^{(4)} = [1, [\boldsymbol{R}^{(3)}(t,1)]^{(4)}, [\boldsymbol{R}^{(3)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(3)}(t,3)]^{(4)}], \ t \ge 0, \quad (224)$$

with the coordinates:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(4)} = \overline{\boldsymbol{R}}_2^{(3)}(t,1)$$
$$= [\boldsymbol{R}^{(3)(1)}(t,1)]^{(4)} \ [\boldsymbol{R}^{(3)(2)}(t,1)]^{(4)},$$
$$[\boldsymbol{R}^{(3)}(t,2)]^{(4)} = \overline{\boldsymbol{R}}_2^{(3)}(t,2)$$
$$= [\boldsymbol{R}^{(3)(1)}(t,2)]^{(4)} \ [\boldsymbol{R}^{(3)(2)}(t,2)]^{(4)},$$
$$[\boldsymbol{R}^{(3)}(t,3)]^{(4)} = \overline{\boldsymbol{R}}_2^{(3)}(t,3)$$
$$= [\boldsymbol{R}^{(3)(1)}(t,3)]^{(4)} \ [\boldsymbol{R}^{(3)(2)}(t,3)]^{(4)}.$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(3)}(t,1)]^{(4)}$$
$$= [1 - [1 - \exp[-0.00286t]]^2] \cdot$$
$$[1 - [[1 - \exp[-0.237t]]^4$$
$$+ 4\exp[-237t][1 - \exp[-237t]]^3]], \quad (225)$$

$$[\boldsymbol{R}^{(3)}(t,2)]^{(4)}$$
$$= [1 - [1 - \exp[-0.00304t]]^2] \cdot$$
$$[1 - [[1 - \exp[-0.254t]]^4$$
$$+ 4\exp[-254t][1 - \exp[-254t]]^3]], \quad (226)$$

$$[\boldsymbol{R}^{(3)}(t,3)]^{(4)}$$
$$= [1 - [1 - \exp[-0.00313t]]^2] \cdot$$
$$[1 - [[1 - \exp[-0.279t]]^4$$
$$+ 4\exp[-279t][1 - \exp[-279t]]^3]]. \quad (227)$$

The expected values of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_4, calculated from the above results given by (216)-(219), (220)-(223), (224)-(227) respectively, according to (12.23) from [1], are:

- for the exemplary system with a hot single reservation of its components

$$\mu_4^{(1)}(1) \cong 592.37, \ \mu_4^{(1)}(2) \cong 555.01,$$
$$\mu_4^{(1)}(3) \cong 516.24, \quad (228)$$

- for the exemplary system with a cold single reservation of its components

$$\mu_4^{(2)}(1) \cong 817.52, \ \mu_4^{(2)}(2) \cong 765.99,$$
$$\mu_4^{(2)}(3) \cong 712.18, \quad (229)$$

- for the exemplary system with reduced rates of departure of its components

$$\mu_4^{(3)}(1) \cong 311.18, \ \mu_4^{(3)}(2) \cong 291.39,$$
$$\mu_4^{(3)}(3) \cong 272.63, \quad (230)$$

and further, using (7.8) from [1] and (228). (229), (230) respectively, it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_4, are:

- for the exemplary system with a hot single reservation of its components

$$\overline{\mu}_4^{(1)}(1) \cong 37.36, \ \overline{\mu}_4^{(1)}(2) \cong 38.77,$$
$$\overline{\mu}_4^{(1)}(3) \cong 516.24, \quad (231)$$

- for the exemplary system with a cold single reservation of its components

$$\overline{\mu}_4^{(2)}(1) \cong 51.53, \ \overline{\mu}_4^{(2)}(2) \cong 53.81,$$
$$\overline{\mu}_4^{(2)}(3) \cong 712.18, \quad (232)$$

- for the exemplary system with reduced rates of departure of its components

$$\overline{\mu}_4^{(3)}(1) \cong 19.79, \ \overline{\mu}_4^{(3)}(2) \cong 18.76,$$
$$\overline{\mu}_4^{(3)}(3) \cong 272.63. \quad (233)$$

In the case when the system operation time is large enough, its improved unconditional four-state reliability function of the exemplary system depending on the kind of its components redundancy are as follows:

- for a system with a hot single reservation of its components

$$\boldsymbol{R}^{(1)}(t,\cdot) = [1, \boldsymbol{R}^{(1)}(t,1),\ \boldsymbol{R}^{(1)}(t,2),\ \boldsymbol{R}^{(1)}(t,3)], \quad (234)$$
$t \ge 0,$

where according to (12.12) from [1], considering 17 [4], the vector co-ordinates are given respectively by

$$\boldsymbol{R}^{(1)}(t,1) = p_1[\boldsymbol{R}^{(1)}(t,1)]^{(1)} + p_2[\boldsymbol{R}^{(1)}(t,1)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(1)}(t,1)]^{(3)} + p_4[\boldsymbol{R}^{(1)}(t,1)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(1)}(t,1)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(1)}(t,1)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(1)}(t,1)]^{(3)} + 0.0.455 \cdot [\boldsymbol{R}^{(1)}(t,1)]^{(4)} \quad (235)$$
for $t \ge 0,$

$$\boldsymbol{R}^{(1)}(t,2) = p_1[\boldsymbol{R}^{(1)}(t,2)]^{(1)} + p_2[\boldsymbol{R}^{(1)}(t,2)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(1)}(t,2)]^{(3)} + p_4[\boldsymbol{R}^{(1)}(t,2)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(3)}$$
$$+ 0.0.455 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(4)} \ \text{for } t \ge 0, \quad (236)$$

$$\boldsymbol{R}^{(1)}(t,3) = p_1[\boldsymbol{R}^{(1)}(t,3)]^{(1)} + p_2[\boldsymbol{R}^{(1)}(t,3)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(1)}(t,3)]^{(3)} + p_4[\boldsymbol{R}^{(1)}(t,3)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(1)}(t,3)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(1)}(t,3)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(1)}(t,3)]^{(3)}$$
$$+ 0.0.455 \cdot [\boldsymbol{R}^{(1)}(t,3)]^{(4)} \ \text{for } t \ge 0, \quad (237)$$

and the coordinates $[\boldsymbol{R}^{(1)}(t,1)]^{(1)}$, $[\boldsymbol{R}^{(1)}(t,1)]^{(2)}$, $[\boldsymbol{R}^{(1)}(t,1)]^{(3)}$, $[\boldsymbol{R}^{(1)}(t,1)]^{(4)}$ are given by (115), (133), (175), (217), $[\boldsymbol{R}^{(1)}(t,2)]^{(1)}$, $[\boldsymbol{R}^{(1)}(t,2)]^{(2)}$, $[\boldsymbol{R}^{(1)}(t,2)]^{(3)}$, $[\boldsymbol{R}^{(1)}(t,2)]^{(4)}$ are given by (116), (134), (176), (218) and $[\boldsymbol{R}^{(1)}(t,3)]^{(1)}$, $[\boldsymbol{R}^{(1)}(t,3)]^{(2)}$, $[\boldsymbol{R}^{(1)}(t,3)]^{(3)}$, $[\boldsymbol{R}^{(1)}(t,3)]^{(4)}$ are given by (117), (135), (177), (219),

- for a system with a cold single reservation of its components

$$\boldsymbol{R}^{(2)}(t,\cdot) = [1, \boldsymbol{R}^{(2)}(t,1),\ \boldsymbol{R}^{(2)}(t,2),\ \boldsymbol{R}^{(2)}(t,3)], \quad (238)$$
$t \ge 0,$

where according to (12.14) from [1], considering 17 [4], the vector co-ordinates are given respectively by

$$\boldsymbol{R}^{(2)}(t,1) = p_1[\boldsymbol{R}^{(2)}(t,1)]^{(1)} + p_2[\boldsymbol{R}^{(2)}(t,1)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(2)}(t,1)]^{(3)} + p_4[\boldsymbol{R}^{(2)}(t,1)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(2)}(t,1)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(2)}(t,1)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(2)}(t,1)]^{(3)}$$
$$+ 0.455 \cdot [\boldsymbol{R}^{(2)}(t,1)]^{(4)} \ \text{for } t \ge 0, \quad (239)$$

$$\boldsymbol{R}^{(2)}(t,2) = p_1[\boldsymbol{R}^{(2)}(t,2)]^{(1)} + p_2[\boldsymbol{R}^{(2)}(t,2)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(2)}(t,2)]^{(3)} + p_4[\boldsymbol{R}^{(2)}(t,2)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(3)}$$
$$+ 0.455 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(4)} \ \text{for } t \ge 0, \quad (240)$$

$$\boldsymbol{R}^{(2)}(t,3) = p_1[\boldsymbol{R}^{(2)}(t,3)]^{(1)} + p_2[\boldsymbol{R}^{(2)}(t,3)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(2)}(t,3)]^{(3)} + p_4[\boldsymbol{R}^{(2)}(t,3)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(2)}(t,3)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(2)}(t,3)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(2)}(t,3)]^{(3)}$$
$$+ 0.455 \cdot [\boldsymbol{R}^{(2)}(t,3)]^{(4)} \ \text{for } t \ge 0, \quad (241)$$

and the coordinates $[\boldsymbol{R}^{(2)}(t,1)]^{(1)}$, $[\boldsymbol{R}^{(2)}(t,1)]^{(2)}$, $[\boldsymbol{R}^{(2)}(t,1)]^{(3)}$, $[\boldsymbol{R}^{(2)}(t,1)]^{(4)}$ are given by (119), (137), (179), (221), $[\boldsymbol{R}^{(2)}(t,2)]^{(1)}$, $[\boldsymbol{R}^{(2)}(t,2)]^{(2)}$, $[\boldsymbol{R}^{(2)}(t,2)]^{(3)}$, $[\boldsymbol{R}^{(2)}(t,2)]^{(4)}$ are given by (120), (138), (180), (222) and $[\boldsymbol{R}^{(2)}(t,3)]^{(1)}$, $[\boldsymbol{R}^{(2)}(t,3)]^{(2)}$, $[\boldsymbol{R}^{(2)}(t,3)]^{(3)}$,

$[\boldsymbol{R}^{(2)}(t,3)]^{(4)}$ are given by (121), (139), (181), (223),

- for a system with reduced rates of departure of its components

$$\boldsymbol{R}^{(3)}(t,\cdot) = [1, \boldsymbol{R}^{(3)}(t,1), \boldsymbol{R}^{(3)}(t,2), \boldsymbol{R}^{(3)}(t,3)], \quad (242)$$
$$t \geq 0,$$

where according to (12.16) from [1], considering 17 [4], the vector co-ordinates are given respectively by

$$\boldsymbol{R}^{(3)}(t,1) = p_1[\boldsymbol{R}^{(3)}(t,1)]^{(1)} + p_2[\boldsymbol{R}^{(3)}(t,1)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(3)}(t,1)]^{(3)} + p_4[\boldsymbol{R}^{(3)}(t,1)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(3)}(t,1)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(3)}(t,1)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(3)}(t,1)]^{(3)}$$
$$+ 0.0.455 \cdot [\boldsymbol{R}^{(3)}(t,1)]^{(4)} \text{ for } t \geq 0, \quad (243)$$

$$\boldsymbol{R}^{(3)}(t,2) = p_1[\boldsymbol{R}^{(3)}(t,2)]^{(1)} + p_2[\boldsymbol{R}^{(3)}(t,2)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(3)}(t,2)]^{(3)} + p_4[\boldsymbol{R}^{(3)}(t,2)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(3)}$$
$$+ 0.0.455 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(4)} \text{ for } t \geq 0, \quad (244)$$

$$\boldsymbol{R}^{(3)}(t,3) = p_1[\boldsymbol{R}^{(3)}(t,3)]^{(1)} + p_2[\boldsymbol{R}^{(3)}(t,3)]^{(2)}$$
$$+ p_3[\boldsymbol{R}^{(3)}(t,3)]^{(3)} + p_4[\boldsymbol{R}^{(3)}(t,3)]^{(4)}$$
$$= 0.214 \cdot [\boldsymbol{R}^{(3)}(t,3)]^{(1)} + 0.038 \cdot [\boldsymbol{R}^{(3)}(t,3)]^{(2)}$$
$$+ 0.293 \cdot [\boldsymbol{R}^{(3)}(t,3)]^{(3)}$$
$$+ 0.0.455 \cdot [\boldsymbol{R}^{(3)}(t,3)]^{(4)} \text{ for } t \geq 0, \quad (245)$$

and the coordinates $[\boldsymbol{R}^{(3)}(t,1)]^{(1)}$, $[\boldsymbol{R}^{(3)}(t,1)]^{(2)}$, $[\boldsymbol{R}^{(3)}(t,1)]^{(3)}$, $[\boldsymbol{R}^{(3)}(t,1)]^{(4)}$ are given by (123), (141), (183), (225), $[\boldsymbol{R}^{(3)}(t,2)]^{(1)}$, $[\boldsymbol{R}^{(3)}(t,2)]^{(2)}$, $[\boldsymbol{R}^{(3)}(t,2)]^{(3)}$, $[\boldsymbol{R}^{(3)}(t,2)]^{(4)}$ are given by (124), (142), (184), (226) and $[\boldsymbol{R}^{(3)}(t,3)]^{(1)}$, $[\boldsymbol{R}^{(3)}(t,3)]^{(2)}$, $[\boldsymbol{R}^{(3)}(t,3)]^{(3)}$, $[\boldsymbol{R}^{(3)}(t,3)]^{(4)}$ are given by (125), (143), (185), (227).

The expected values and standard deviations of the exemplary system unconditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, calculated from the above results given by (234)-(237), (238)-(241), (242)-(245) respectively , according to (7.5)-(7.7) from [1] and considering (126), (144), (186), (228) or (127), (145), (187), (229) or (128), (146), (188), (230) respectively are:

- for a system with a hot single reservation of its components

$$\mu^{(1)}(1) = p_1\mu_1^{(1)}(1) + p_2\mu_2^{(1)}(1)$$
$$+ p_3\mu_3^{(1)}(1) + p_4\mu_4^{(1)}(1)$$
$$= 0.214 \cdot 939.53 + 0.038 \cdot 1140.59$$
$$+ 0.293 \cdot 808.64 + 0.455 \cdot 592.37 \cong 750.86, \quad (246)$$

$$\sigma^{(1)}(1) \cong 357.75, \quad (247)$$

$$\mu^{(1)}(2) = p_1\mu_1^{(1)}(2) + p_2\mu_2^{(1)}(2)$$
$$+ p_3\mu_3^{(1)}(2) + p_4\mu_4^{(1)}(2)$$
$$= 0.214 \cdot 911.15 + 0.038 \cdot 1064.63$$
$$+ 0.293 \cdot 764.63 + 0.455 \cdot 555.01 \cong 712.01, \quad (248)$$

$$\sigma^{(1)}(2) \cong 342.99, \quad (249)$$

$$\mu^{(1)}(3) = p_1\mu_1^{(1)}(3) + p_2\mu_2^{(1)}(3)$$
$$+ p_3\mu_3^{(1)}(3) + p_4\mu_4^{(1)}(3)$$
$$= 0.214 \cdot 881.87 + 0.038 \cdot 998.14$$
$$+ 0.293 \cdot 739.45 + 0.455 \cdot 516.24 \cong 678. \quad (250)$$

$$\sigma^{(1)}(3) \cong 332.00, \quad (251)$$

- for a system with a cold single reservation of its components

$$\mu^{(2)}(1) = p_1\mu_1^{(2)}(1) + p_2\mu_2^{(2)}(1)$$

$$+ p_3\mu_3^{(2)}(1) + p_4\mu_4^{(2)}(1)$$

$$= 0.214 \cdot 1285.90 + 0.038 \cdot 1534.67$$

$$+ 0.293 \cdot 1111.18 + 0.455 \cdot 817.52$$

$$\cong 1031.05, \qquad (252)$$

$$\sigma^{(2)}(1) \cong 478.56, \qquad (253)$$

$$\mu^{(2)}(2) = p_1\mu_1^{(2)}(2) + p_2\mu_2^{(2)}(2)$$

$$+ p_3\mu_3^{(2)}(2) + p_4\mu_4^{(2)}(2)$$

$$= 0.214 \cdot 1247.58 + 0.038 \cdot 1432.51$$

$$+ 0.293 \cdot 1050.95 + 0.455 \cdot 765.99$$

$$\cong 977.87, \qquad (254)$$

$$\sigma^{(2)}(2) \cong 459.16, \qquad (255)$$

$$\mu^{(2)}(3) = p_1\mu_1^{(2)}(3) + p_2\mu_2^{(2)}(3)$$

$$+ p_3\mu_3^{(2)}(3) + p_4\mu_4^{(2)}(3)$$

$$= 0.214 \cdot 1207.25 + 0.038 \cdot 1343.08$$

$$+ 0.293 \cdot 1016.19 + 0.455 \cdot 712.18$$

$$\cong 931.17, \qquad (256)$$

$$\sigma^{(2)}(3) \cong 444.58, \qquad (257)$$

- for a system with reduced rates of departure of its components

$$\mu^{(3)}(1) = p_1\mu_1^{(3)}(1) + p_2\mu_2^{(3)}(1)$$

$$+ p_3\mu_3^{(3)}(1) + p_4\mu_4^{(3)}(1)$$

$$= 0.214 \cdot 557.1 + 0.038 \cdot 837.76$$

$$+ 0.293 \cdot 457.41 + 0.455 \cdot 311.18$$

$$\cong 428.03, \qquad (258)$$

$$\sigma^{(3)}(1) \cong 304.79, \qquad (259)$$

$$\mu^{(3)}(2) = p_1\mu_1^{(3)}(2) + p_2\mu_2^{(3)}(2)$$

$$+ p_3\mu_3^{(3)}(2) + p_4\mu_4^{(3)}(2)$$

$$= 0.214 \cdot 536.99 + 0.038 \cdot 815.28$$

$$+ 0.293 \cdot 431.46 + 0.455 \cdot 291.39$$

$$\cong 404.89, \qquad (260)$$

$$\sigma^{(3)}(2) \cong 289.99, \qquad (261)$$

$$\mu^{(3)}(3) = p_1\mu_1^{(3)}(3) + p_2\mu_2^{(3)}(3)$$

$$+ p_3\mu_3^{(3)}(3) + p_4\mu_4^{(3)}(3)$$

$$= 0.214 \cdot 520.10 + 0.038 \cdot 764.14$$

$$+ 0.293 \cdot 417.81 + 0.455 \cdot 272.63$$

$$\cong 386.80, \qquad (262)$$

$$\sigma^{(3)}(3) \cong 278.98, \qquad (263)$$

and further, considering (7.8) from [1] and (246), (248) and (250) or (252), (254) and (256) or (258), (260) and (262), it follows the mean values of that the unconditional lifetimes in the particular reliability states 1, 2, 3, respectively are:
- for a system with a hot single reservation of its components

$$\bar{\mu}^{(1)}(1) = \mu^{(1)}(1) - \mu^{(1)}(2) = 38.85,$$

$$\bar{\mu}^{(1)}(2) = \mu^{(1)}(2) - \mu^{(1)}(3) = 33.08,$$

$$\bar{\mu}^{(1)}(3) = \mu^{(1)}(3) = 678.19, \qquad (264)$$

- for a system with a cold single reservation of its components

$$\bar{\mu}^{(2)}(1) = \mu^{(2)}(1) - \mu^{(2)}(2) = 53.18,$$

$$\bar{\mu}^{(2)}(2) = \mu^{(2)}(2) - \mu^{(2)}(3) = 64.7,$$

$$\bar{\mu}^{(2)}(3) = \mu^{(2)}(3) = 931.17, \qquad (265)$$

- for a system with reduced rates of departure of its components

$$\bar{\mu}^{(3)}(1) = \mu^{(3)}(1) - \mu^{(3)}(2) = 23.14,$$

$$\bar{\mu}^{(3)}(2) = \mu^{(3)}(2) - \mu^{(3)}(3) = 18.9,$$

$$\bar{\mu}^{(3)}(3) = \mu^{(3)}(3) = 386.80. \qquad (266)$$

Since the critical reliability state is r =2, then the system risk function, according to (7.9) from [1], is given by

- for a system with a hot single reservation of its components

$$\boldsymbol{r}^{(1)}(t) = 1 - \boldsymbol{R}^{(1)}(t,2) = 1 - [0.214 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(1)}$$

$$+ 0.038 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(2)} + 0.293 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(3)}$$

$$+ 0.0.455 \cdot [\boldsymbol{R}^{(1)}(t,2)]^{(4)}] \quad \text{for } t \geq 0, \qquad (267)$$

- for a system with a cold single reservation of its components

$$\boldsymbol{r}^{(2)}(t)$$

$$= 1 - \boldsymbol{R}^{(2)}(t,2) = 1 - [0.214 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(1)}$$

$$+ 0.038 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(2)} + 0.293 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(3)}$$

$$+ 0.0.455 \cdot [\boldsymbol{R}^{(2)}(t,2)]^{(4)}] \quad \text{for } t \geq 0, \qquad (268)$$

- for a system with reduced rates of departure of its components

$$\boldsymbol{r}^{(3)}(t)$$

$$= 1 - \boldsymbol{R}^{(3)}(t,2) = 1 - [0.214 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(1)}$$

$$+ 0.038 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(2)} + 0.293 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(3)}$$

$$+ 0.0.455 \cdot [\boldsymbol{R}^{(3)}(t,2)]^{(4)}] \quad \text{for } t \geq 0. \qquad (269)$$

Hence, by (7.10) from [1], the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, is

- for a system with a hot single reservation of its components

$$\tau^{(1)} = \boldsymbol{r}^{(1)-1}(\delta) \cong 272.68, \qquad (270)$$

- for a system with a cold single reservation of its components

$$\tau^{(2)} = \boldsymbol{r}^{(2)-1}(\delta) \cong 381.88, \qquad (271)$$

- for a system with reduced rates of departure of its components

$$\tau^{(3)} = \boldsymbol{r}^{(3)-1}(\delta) \cong 77.77. \qquad (272)$$

11.2. Renewal and availability characteristics of the improved exemplary system

To determine the renewal and availability characteristics of the improved exemplary system, we use the results of the system reliability characteristics evaluation performed in Sections 11.1 and the results of the Section 12.3.1. of [1].

If components of the repairable improved exemplary system with ignored time of renovation have exponential reliability functions at the operation states z_b, $b = 1,2,...,v$, with the coordinates given by (110)-(111) or respectively by (112)-(113) and the system reliability critical state is $r = 2$, applying *Proposition 12.1* from Section 12.3.1 of [1], we determine its following characteristics:

a) the time $S_N^{(k)}(2)$ $k = 1,2,3$, until the *Nth* exceeding by the system the reliability critical state 2, for sufficiently large N,

- for a system with a hot single reservation of its components has approximately normal distribution $N(712.01N, 342.99\sqrt{N})$, i.e.,

$$F^{(N)(1)}(t,2)$$

$$= P(S_N^{(1)}(2) < t) \cong F_{N(0,1)}(\frac{t - 712N}{342.99\sqrt{N}}),$$

$$t \in (-\infty, \infty),$$

- for a system with a cold single reservation of its components has approximately normal distribution $N(977.87N, 459.16\sqrt{N})$, i.e.,

$$F^{(N)(2)}(t,2)$$

$$= P(S_N^{(2)}(2) < t) \cong F_{N(0,1)}(\frac{t - 977.87N}{459.16\sqrt{N}}),$$

$$t \in (-\infty, \infty),$$

- for a system with reduced rates of departure of its components has approximately normal distribution $N(712.01N, 342.99\sqrt{N})$, i.e.,

$$F^{(N)(3)}(t,2)$$

$$= P(S_N^{(3)}(2) < t) \cong F_{N(0,1)}(\frac{t - 404.89N}{289.99\sqrt{N}}),$$

$t \in (-\infty, \infty)$,

b) the expected value and the variance of the time $\bar{S}_N^{(k)}(2)$, $k = 1,2,3,$ until the *Nth* exceeding by the system the reliability critical state 2, for sufficiently large *N*, are respectively given by
- for a system with a hot single reservation of its components

$$E[S_N^{(1)}(2)] = 712.01N,$$
$$D[S_N^{(1)}(2)] = 117642.14N,$$

- for a system with a cold single reservation of its components

$$E[S_N^{(2)}(2)] = 977.87N,$$
$$D[S_N^{(1)}(2)] = 210827.91N,$$

- for a system with reduced rates of departure of its components

$$E[S_N^{(3)}(2)] = 404.89N,$$
$$D[S_N^{(3)}(2)] = 84094.2N,$$

c) the number $\bar{N}^{(k)}(t,2)$, $k = 1,2,3,$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \geq 0,$ for sufficiently large t, has approximately distribution of the form
- for a system with a hot single reservation of its components

$$P(N^{(1)}(t,2) = N)$$
$$\cong F_{N(0,1)}\left(\frac{712.01(N+1) - t}{12.85\sqrt{t}}\right) - F_{N(0,1)}\left(\frac{712.01N - t}{12.85\sqrt{t}}\right),$$
$$N = 0,1,\ldots,$$

- for a system with a hot single reservation of its components

$$P(N^{(2)}(t,2) = N)$$
$$\cong F_{N(0,1)}\left(\frac{977.87(N+1) - t}{14.68\sqrt{t}}\right) - F_{N(0,1)}\left(\frac{977.87N - t}{14.68\sqrt{t}}\right),$$
$$N = 0,1,\ldots,$$

- for a system with reduced rates of departure of its components

$$P(N^{(3)}(t,2) = N)$$
$$\cong F_{N(0,1)}\left(\frac{404.89(N+1) - t}{14.41\sqrt{t}}\right) - F_{N(0,1)}\left(\frac{404.89N - t}{14.41\sqrt{t}}\right),$$
$$N = 0,1,\ldots,$$

d) the expected value and the variance of the number $\bar{N}^{(k)}(t,r)$, $k = 1,2,3,$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \geq 0,$ for sufficiently large t, are approximately given by
- for a system with a hot single reservation of its components

$$H^{(1)}(t,2) = 0.0014t, \;\; D^{(1)}(t,2) = 0.00033t, \quad (273)$$

- for a system with a cold single reservation of its components

$$H^{(2)}(t,2) = 0.0010t, \; D^{(2)}(t,2) = 0.00026t, \quad (274)$$

- for a system with reduced rates of departure of its components

$$H^{(3)}(t,2) = 0.0025t, \; D^{(3)}(t,2) = 0.00127t, \quad (275)$$

To make the estimation of the renewal and availability of the improved exemplary system in the case when the time of renovation is non-ignored, considering the values $\mu^{(k)}(2)$, $k = 1,2,3,$ determined by (248) or (254) or (260) respectively and $\sigma^{(k)}(2)$, $k = 1,2,3,$ determined by (249) or (255) or (261), assuming the mean value of the system renovation time $\mu_0(2) = 10$ year and the standard deviation of the system renovation time $\sigma_0(2) = 5$ year and if components of the repairable improved exemplary system have exponential reliability functions at the operation states z_b, $b = 1,2,\ldots,v,$ with the coordinates given by (110)-(111) or respectively by (112)-(113) and the system reliability critical state is $r = 2,$ applying *Proposition 11.2* from Section 12.3.2 of [1], we determine its following characteristics:
a) the time $\bar{S}_N^{(k)}(2)$, $k = 1,2,3,$ until the *Nth* exceeding by the system the reliability critical state 2, for sufficiently large *N*, are respectively given by
- for a system with a hot single reservation of its components

$$\bar{F}^{(N)(1)}(t,2) = P(\bar{S}_N^{(1)}(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 722.01N + 10}{\sqrt{117667.14N - 25}}), \; t \in (-\infty, \infty),$$

- for a system with a cold single reservation of its components

$$\bar{F}^{(N)(2)}(t,2) = P(\bar{S}_N^{(2)}(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 987.87N + 10}{\sqrt{210852.91N - 25}}), \; t \in (-\infty, \infty),$$

- for a system with reduced rates of departure of its components

$$\bar{F}^{(N)(3)}(t,2) = P(\bar{S}_N^{(3)}(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 414.89N + 10}{\sqrt{84119.2N - 25}}), \; t \in (-\infty, \infty),$$

b) the expected value and the variance of the time $\bar{S}_N^{(k)}(2)$, $k = 1,2,3,$ until the *Nth* exceeding by the system the reliability critical state 2 for sufficiently large *N*, are respectively given by
- for a system with a hot single reservation of its components

$$E[\bar{S}_N^{(1)}(2)] \cong 712.01N + 10(N-1),$$
$$D[\bar{S}_N^{(1)}(2)] \cong 117642.14N + 25(N-1),$$

- for a system with a cold single reservation of its components

$$E[\bar{S}_N^{(2)}(2)] \cong 977.87N + 10(N-1),$$
$$D[\bar{S}_N^{(2)}(2)] \cong 210827.91N + 25(N-1),$$

- for a system with reduced rates of departure of its components

$$E[\bar{S}_N^{(3)}(2)] \cong 404.89N + 10(N-1),$$
$$D[\bar{S}_N^{(3)}(2)] \cong 84094.2N + 25(N-1),$$

c) the number $\bar{N}^{(k)}(t,2)$, $k = 1,2,3,$ of exceeding by the system the reliability critical state 2 of this system up to the moment $t, t \geq 0,$ for sufficiently large $t,$ has approximately distribution of the form
- for a system with a hot single reservation of its components

$$P(\bar{N}^{(1)}(t,2) = N)$$

$$\cong F_{N(0,1)}(\frac{722.01(N+1) - t - 10}{12.77\sqrt{t+10}})$$

$$- F_{N(0,1)}(\frac{722.01N - t - 10}{12.77\sqrt{t+10}}), \; N = 0,1,\ldots,$$

- for a system with a cold single reservation of its components

$$P(\bar{N}^{(2)}(t,2) = N)$$

$$\cong F_{N(0,1)}(\frac{987.87(N+1) - t - 10}{14.60\sqrt{t+10}})$$

$$- F_{N(0,1)}(\frac{987.87N - t - 10}{14.60\sqrt{t+10}}), \; N = 0,1,\ldots,$$

- for a system with reduced rates of departure of its components

$$P(\bar{N}^{(3)}(t,2) = N) \cong$$

$$\cong F_{N(0,1)}(\frac{414.89(N+1) - t - 10}{14.23\sqrt{t+10}})$$

$$- F_{N(0,1)}(\frac{987.87N - t - 10}{14.23\sqrt{t+10}}), \; N = 0,1,\ldots,$$

d) the expected value and the variance of the number $\bar{N}^{(k)}(t,2)$, $k = 1,2,3,$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \geq 0,$ for sufficiently large $t,$ are respectively given by
- for a system with a hot single reservation of its components

$$\bar{H}^{(1)}(t,2) \cong 0.00138(t+10),$$
$$\bar{D}^{(1)}(t,2) \cong 0.00031(t+10),$$

- for a system with a cold single reservation of its components

$$\bar{H}^{(2)}(t,2) \cong 0.00101(t+10),$$
$$\bar{D}^{(2)}(t,2) \cong 0.00022(t+10),$$

- for a system with reduced rates of departure of its components

$$\overline{H}^{(3)}(t,2) \cong 0.00241(t+10),$$
$$\overline{D}^{(3)}(t,2) \cong 0.001178(t+10),$$

e) the time $\overline{\overline{S}}_N^{(k)}(2)$, $k = 1,2,3,$ until the *Nth* system's renovation, for sufficiently large *N*,
- for a system with a hot single reservation of its components has approximately normal distribution $N(722.01N, 343.03\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)(1)}(t,2) = P(\overline{\overline{S}}_N^{(1)}(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 722.01N}{343.03\sqrt{N}}), \ t \in (-\infty, \infty),$$

- for a system with a cold single reservation of its components has approximately normal distribution $N(987.87N, 459.18\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)(2)}(t,2) = P(\overline{\overline{S}}_N^{(2)}(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 987.87N}{459.18\sqrt{N}}), \ t \in (-\infty, \infty),$$

- for a system with reduced rates of departure of its components has approximately normal distribution $N(414.89N, 290.03\sqrt{N})$, i.e.,

$$\overline{\overline{F}}^{(N)(3)}(t,2) = P(\overline{\overline{S}}_N^{(3)}(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 414.89N}{290.03\sqrt{N}}), \ t \in (-\infty, \infty),$$

f) the expected value and the variance of the time $\overline{\overline{S}}_N^{(k)}(2)$, $k = 1,2,3,$ until the *Nth* system's renovation, for sufficiently large *N*, are respectively given by
- for a system with a hot single reservation of its components

$$E[\overline{\overline{S}}_N^{(1)}(2)] \cong 722.01N,$$
$$D[\overline{\overline{S}}_N^{(1)}(r)] \cong 117667.14N,$$

- for a system with a cold single reservation of its components

$$E[\overline{\overline{S}}_N^{(2)}(2)] \cong 987.87N,$$
$$D[\overline{\overline{S}}_N^{(2)}(r)] \cong 210852.91N,$$

- for a system with reduced rates of departure of its components

$$E[\overline{\overline{S}}_N^{(3)}(2)] \cong 414.89N,$$
$$D[\overline{\overline{S}}_N^{(2)}(r)] \cong 84119.2N,$$

g) the number $\overline{\overline{N}}^{(k)}(t,2)$, $k = 1,2,3,$ of system's renovations up to the moment $t, t \geq 0,$ for sufficiently large $t,$ has approximately distribution of the form
- for a system with a hot single reservation of its components

$$P(\overline{\overline{N}}^{(1)}(t,2) = N)$$

$$\cong F_{N(0,1)}(\frac{722.01(N+1) - t}{12.77\sqrt{t}})$$

$$- F_{N(0,1)}(\frac{722.01N - t}{12.77\sqrt{t}}), \ N = 1,2,...,$$

- for a system with a cold single reservation of its components

$$P(\overline{\overline{N}}^{(2)}(t,2) = N) \cong F_{N(0,1)}(\frac{987.87(N+1) - t}{14.60\sqrt{t}})$$
$$- F_{N(0,1)}(\frac{987.87N - t}{14.60\sqrt{t}}), \ N = 1,2,...,$$

- for a system with reduced rates of departure of its components

$$P(\overline{\overline{N}}^{(3)}(t,2) = N)$$

$$\cong F_{N(0,1)}(\frac{414.89(N+1) - t}{14.23\sqrt{t}})$$

$$- F_{N(0,1)}(\frac{414.89N - t}{14.23\sqrt{t}}), \ N = 1,2,...,$$

h) the expected value and the variance of the number $\overline{\overline{N}}^{(k)}(t,2)$, $k = 1,2,3,$ of system's renovations up to the moment $t, t \geq 0,$ for sufficiently large $t,$ are respectively given by

- for a system with a hot single reservation of its components

$$\overline{\overline{H}}^{(1)}(t,2) \cong 0.00138t\,,$$
$$\overline{\overline{D}}^{(1)}(t,2) \cong 0.00031t, \qquad (276)$$

- for a system with a cold single reservation of its components

$$\overline{\overline{H}}^{(2)}(t,2) \cong 0.00101t\,,$$
$$\overline{\overline{D}}^{(2)}(t,2) \cong 0.00022t, \qquad (277)$$

- for a system with reduced rates of departure of its components

$$\overline{\overline{H}}^{(3)}(t,2) \cong 0.00241t\,,$$
$$\overline{\overline{D}}^{(3)}(t,2) \cong 0.001178t, \qquad (278)$$

i) the steady availability coefficient of the system at the moment $t, t \ge 0,$ for sufficiently large $t,$ is given by
- for a system with a hot single reservation of its components

$$A^{(1)}(t,2) \cong 0.986\,, \quad t \ge 0, \quad \tau > 0,$$

- for a system with a cold single reservation of its components

$$A^{(2)}(t,2) \cong 0.988\,, \quad t \ge 0, \quad \tau > 0,$$

- for a system with reduced rates of departure of its components

$$A^{(3)}(t,2) \cong 0.976\,, \quad t \ge 0, \quad \tau > 0,$$

j) the steady availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ for sufficiently large $t,$ is given by
- for a system with a hot single reservation of its components

$$A^{(1)}(t,\tau,2) \cong 0.00138\int_{\tau}^{\infty} \boldsymbol{R}^{(1)}(t,2)dt, \quad t \ge 0, \quad \tau > 0,$$

- for a system with a cold single reservation of its components

$$A^{(2)}(t,\tau,2) \cong 0.00101\int_{\tau}^{\infty} \boldsymbol{R}^{(2)}(t,2)dt, \quad t \ge 0, \quad \tau > 0,$$

- for a system with reduced rates of departure of its components

$$A^{(3)}(t,\tau,2) \cong 0.00241\int_{\tau}^{\infty} \boldsymbol{R}^{(2)}(t,2)dt, \quad t \ge 0, \quad \tau > 0.$$

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References

[1] Kołowrocki, K. & Soszyńska, J. (2010). Integrated Safety and Reliability Decision Support System – IS&RDSS. Tasks 10.0-10.15 in WP10: Safety and Reliability Decision Support Systems for Various Maritime and Coastal Transport Sectors. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[2] Kołowrocki, K. & Soszyńska, J. (2010). The exemplary system operation, reliability, risk, availability and cost identification, prediction and optimization - Testing IS&RSS. Task 9.6 in WP9: Applications and Testing of Packages of Tools in Complex Maritime Transportation Systems and Processes Safety and Reliability Optimization. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0.Gdynia Maritime University, 2010.

[3] Kołowrocki, K., Soszyńska, J. & Ng Kien Ming (2011). Integrated package of tools supporting decision making on operation, identification, prediction and optimization of complex technical systems reliability and safety. Part 2. IS&RDSS Application – Exemplary system operation and reliability unknown parameters identification. *Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association*, Issue 5, Vol. 2, 373-386.

[4] Kołowrocki, K., Soszyńska, J. & Ng Kien Ming (2011). Integrated package of tools supporting decision making on operation, identification, prediction and optimization of complex technical systems reliability and safety. Part 3. IS&RDSS Application – Exemplary system

operation and reliability characteristics prediction. *Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association*, Issue 5, Vol. 2, 387-398.

[5] Kwiatuszewska-Sarnecka, B. (2003). *Reliability analysis of effectiveness of reservation in series systems* (in Polish). Systems Research Institute, Polish Academy of Sciences, Warszawa, 2003.

[6] Kwiatuszewska-Sarnecka, B. (2006). Reliability improvement of large multi-state series-parallel systems. *International Journal of Automation and Computing* Vol. 2,157-164.