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Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety
Part 4
IS&RDSS Application – Exemplary system operation and reliability optimization

Keywords

reliability, operation process, optimization

Abstract

There is presented the IS&RDSS application to the operation and reliability of an exemplary complex technical system optimization. There are determined, the optimal limit transient probabilities of the exemplary system operation process at the particular operation states maximizing the system lifetime in the reliability states not worse than the critical reliability state and its optimal sojourn times at the particular operation states. There are evaluated the exemplary system optimal unconditional multistate reliability function, the optimal expected values and the standard deviations of its unconditional lifetimes in the reliability state subsets and the optimal mean values of its lifetimes in the particular reliability states are. Moreover, in the case when the system is repairable, its optimal renewal and availability characteristics are found.

8. The exemplary system operation process optimization

8.1. Optimal transient probabilities of the system operation process at operation states

Considering the equations (63)-(66) given in [3], it is natural to assume that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equations (67), (69), (71) in [3] for the mean values of the system unconditional lifetimes in the reliability state subsets that can be used for the system operation process optimization performed in the accordance with the procedure proposed in Section 9.2.1 of [1].

The objective function defined by (9.1) from [1], in this case as the system critical state is $r = 2$, takes the form

$$\mu(2) = p_1 \cdot 483.87 + p_2 \cdot 694.44$$

$$+ p_3 \cdot 383.04 + p_4 \cdot 253.88. \tag{78}$$

Arbitrarily assumed, the lower $\breve{p}_b$ and upper $\hat{p}_b$ bounds of the unknown transient probabilities p_b, $b = 1,2,3,4,$ defined in [1] by (9.5), respectively are:

$\breve{p}_1 = 0.201$, $\breve{p}_2 = 0.03$, $\breve{p}_3 = 0.245$,
$\breve{p}_4 = 0.309$;

$\hat{p}_1 = 0.351$, $\hat{p}_2 = 0.105$, $\hat{p}_3 = 0.395$,
$\hat{p}_4 = 0.459$.

Therefore, according to (9.2)-(9.3) from [1], we assume the following bound constraints

$$0.201 \le p_1 \le 0.351,\ \ 0.03 \le p_2 \le 0.105,$$

$$0.245 \le p_3 \le 0.395,\ \ 0.309 \le p_4 \le 0.459,$$

$$\sum_{b=1}^{4} p_b = 1. \tag{80}$$

Now, in order to find the optimal values $\dot{p}_b$ of the transient probabilities p_b, $b = 1,2,3,4$ that maximize the objective function (78), we arrange the system conditional lifetimes mean values $\mu_b(2)$, $b = 1,2,3,4$ in non-increasing order

$$\mu_2(2) \ge \mu_1(2) \ge \mu_3(2) \ge \mu_4(2).$$

Next, according to (9.6) from [1], we substitute

$$x_1 = p_2 = 0.038 \quad x_2 = p_1 = 0.214$$

$$x_3 = p_3 = 0.293,\ x_4 = p_4 = 0.455 \tag{81}$$

and

$$\breve{x}_2 = \breve{p}_1 = 0.201,\ \breve{x}_1 = \breve{p}_2 = 0.03,$$
$$\breve{x}_3 = \breve{p}_3 = 0.245,\ \breve{x}_4 = \breve{p}_4 = 0.309,$$

$$\hat{x}_2 = \hat{p}_1 = 0.351,\ \hat{x}_1 = \hat{p}_2 = 0.105,$$
$$\hat{x}_3 = \hat{p}_3 = 0.395,\ \hat{x}_4 = \hat{p}_4 = 0.459. \tag{82}$$

and we maximize with respect to x_i, $i = 1,2,3,4$ the linear form (78) that according to (9.7)-(9.10) from [1] takes the form

$$\mu(2) = x_1 \cdot 694.44 + x_2 \cdot 483.87 + x_3 \cdot 383.04$$

$$+ x_4 \cdot 253.88, \tag{83}$$

with the following bound constraints

$$0.03 \le x_1 \le 0.105,\ \ 0.201 \le x_2 \le 0.351,$$

$$0.245 \le x_3 \le 0.395,\ \ 0.309 \le x_4 \le 0.459, \tag{84}$$

$$\sum_{i=1}^{4} x_i = 1. \tag{85}$$

According to (9.11) from [1] we calculate

$$\breve{x} = \sum_{i=1}^{4} \breve{x}_i = 0.785,\ \ \hat{y} = 1 - \breve{x}$$

$$= 1 - 0.785 = 0.215 \tag{86}$$

and according to (9.12) from [1], we find (8.2)

$$\breve{x}^0 = 0,\ \hat{x}^0 = 0,\ \hat{x}^0 - \breve{x}^0 = 0,$$

$$\breve{x}^1 = 0.03,\ \hat{x}^1 = 0.105,\ \hat{x}^1 - \breve{x}^1 = 0.075,$$

$$\breve{x}^2 = 0.231,\ \hat{x}^2 = 0.456,\ \hat{x}^2 - \breve{x}^2 = 0.225,$$

$$\breve{x}^3 = 0.476.\ \hat{x}^3 = 0.851,\ \hat{x}^3 - \breve{x}^3 = 0.375,$$

$$\breve{x}^4 = 0.785\ \ \hat{x}^4 = 1.31\ \ \hat{x}^4 - \breve{x}^4 = 0.525. \tag{87}$$

From the above, the inequality (9.13) from [1] takes the form

$$\hat{x}^I - \breve{x}^I < 0.215. \tag{88}$$

Thus, from the above and from (87), it follows that the largest value $I \in \{0,1,2,3,4\}$ such that the inequality (88) is satisfied, is $I = 1$.
Therefore, we fix the optimal solution that maximize linear function (83) according to the rule (9.15) from [1]. Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.215,$$

$$\dot{x}_2 = \hat{y} - \hat{x}^1 + \breve{x}^1 + \breve{x}_2$$

$$= 0.215 - 0.105 + 0.03 + 0.201 = 0.341,$$

$$\dot{x}_3 = \breve{x}_3 = 0.245,$$

$$\dot{x}_4 = \breve{x}_4 = 0.309. \tag{89}$$

Finally, after making the inverse to (81) substitution, we get the optimal transient probabilities

$$\dot{p}_2 = \dot{x}_1 = 0.105,\ \ \dot{p}_1 = \dot{x}_2 = 0.341,$$

$$\dot{p}_3 = \dot{x}_3 = 0.245,\ \ \dot{p}_4 = \dot{x}_4 = 0.309, \quad (8.8) \tag{90}$$

that maximize the system mean lifetime in the reliability state subset {2,3} expressed by the linear form (78) giving, according to (9.18) from [1] and (9.14), its optimal value

$$\dot{\mu}(2) = \dot{p}_1 \cdot 483.87 + \dot{p}_2 \cdot 694.44 + \dot{p}_3 \cdot 383.04$$
$$+ \dot{p}_4 \cdot 253.88$$
$$= 0.341 \cdot 483.87 + 0.105 \cdot 694.44 +$$
$$+ 0.245 \cdot 383.04 + 0.309 \cdot 253.88$$
$$= 410.20 \quad (91)$$

8.2. Optimal sojourn times of the system operation process at operation states

Having the values of the optimal transient probabilities determined by (90), it is possible to find the optimal unconditional and conditional mean values of the sojourn times of the exemplary system operation process at the operation states and the optimal mean values of the total unconditional sojourn times of the exemplary system operation process at the operation states during the fixed operation time as well.

Substituting the optimal transient probabilities at operation states

$$\dot{p}_1 = 0.341, \ \dot{p}_2 = 0.105,$$
$$\dot{p}_3 = 0.245, \ \dot{p}_4 = 0.309,$$

determined in (90) and the steady probabilities

$$\pi_1 \cong 0.236, \ \pi_2 \cong 0.169,$$
$$\pi_3 \cong 0.234, \ \pi_4 \cong 0.361,$$

determined by (16) in Section 5 [3] into (9.20) from [1], we get the following system of equations

$$-0.155524\dot{M}_1 + 0.057629\dot{M}_2 + 0.079794\dot{M}_3 + 0.123101\dot{M}_4 = 0$$

$$0.02478\dot{M}_1 - 0.151255\dot{M}_2 + 0.02457\dot{M}_3 + 0.037905\dot{M}_4 = 0$$

$$0.05782\dot{M}_1 + 0.041405\dot{M}_2 - 0.17667\dot{M}_3 + 0.088445\dot{M}_4 = 0$$

$$0.072924\dot{M}_1 + 0.052221\dot{M}_2 + 0.072306\dot{M}_3 - 0.249451\dot{M}_4 = 0 \quad (92)$$

with the unknown optimal mean values $\dot{M}_b$ of the system unconditional sojourn times in the operation states we are looking for.

Since the determinant of the main matrix of the system of equations (92) is equal to 0, then its rank is less than 4 and there are non-zero solutions of this system of equations that are ambiguous and dependent on one or more parameters. We may suppose that, for instance, we are arbitrarily interested in the fixed value of $\dot{M}_4$ and we put

$$\dot{M}_4 = 400.$$

Consequently, from (92), we get the system of equations

$$-0.155524\dot{M}_1 + 0.057629\dot{M}_2 + 0.079794\dot{M}_3 = -49.2404$$

$$0.02478\dot{M}_1 - 0.151255\dot{M}_2 + 0.02457\dot{M}_3 = -15.1620$$

$$0.05782\dot{M}_1 + 0.041405\dot{M}_2 - 0.17667\dot{M}_3 = -35.3780$$

$$0.072924\dot{M}_1 + 0.052221\dot{M}_2 + 0.072306\dot{M}_3 = 99.7804,$$

and we solve it with respect to $\dot{M}_1$, $\dot{M}_2$ and $\dot{M}_3$. This way obtained the solutions of the system of equations (92), are

$$\dot{M}_1 \cong 675, \ \dot{M}_2 \cong 290,$$
$$\dot{M}_3 \cong 490, \ \dot{M}_4 = 400. \quad (93)$$

It can be seen that these solutions differ much from the values M_1, M_2, M_3 and M_4 estimated in Section 5 [3] by (12)-(1).

Having these solutions, it is also possible to look for the optimal values $\dot{M}_{bl}$ of the mean values M_{bl} of the conditional sojourn times at operation states. Namely, substituting the probabilities

$$[p_{bl}] = \begin{bmatrix} 0 & 0.22 & 0.32 & 0.46 \\ 0.20 & 0 & 0.30 & 0.50 \\ 0.12 & 0.16 & 0 & 0.72 \\ 0.48 & 0.22 & 0.30 & 0 \end{bmatrix}$$

of the system operation process transitions between the operation states, determined by (1) in Section 3 [2], and the optimal mean values $\dot{M}_b$ given by

(93) into (9.21) from [1], we get the following system of equations

$$0.22\dot{M}_{12} + 0.32\dot{M}_{13} + 0.46\dot{M}_{14} = 675$$

$$0.20\dot{M}_{21} + 0.30\dot{M}_{23} + 0.50\dot{M}_{24} = 290$$

$$0.12\dot{M}_{31} + 0.16\dot{M}_{32} + 0.72\dot{M}_{34} = 490$$

$$0.48\dot{M}_{41} + 0.22\dot{M}_{42} + 0.30\dot{M}_{14} = 400$$

with the unknown optimal values $\dot{M}_{bl}$ we want to find.
As the solutions of the above system of equations are ambiguous, then we arbitrarily fix some of them, for instance because of practically important reasons, and we find the remaining ones.
In this case, we proceed as follows:
- we fix in the first equation $\dot{M}_{12} = 200$, $\dot{M}_{13} = 500$ and we find $\dot{M}_{14} \cong 1024$;
- we fix in the second equation $\dot{M}_{21} = 100$, $\dot{M}_{23} = 100$ and we find $\dot{M}_{24} \cong 480$;
- we fix in the third equation $\dot{M}_{31} = 900$, $\dot{M}_{32} = 500$ and we find $\dot{M}_{34} \cong 419$;
- we fix in the fourth equation $\dot{M}_{41} = 300$, $\dot{M}_{42} = 500$ and we find $\dot{M}_{43} \cong 487$. (94)

Other very useful and much easier to be applied in practice tool that can help in more reliable and safe operation process of the complex technical systems planning are the system operation process optimal mean values of the total sojourn times at the particular operation states during the the system operation time θ.
Assuming as in Section 5 [3] the system operation time $\theta = 1$ year = 365 days, after aplying (9.22) from [1], we get their values

$$\dot{E}[\hat{\theta}_1] = \dot{p}_1\theta = 0.341 \cdot 365 = 125.5,$$

$$\dot{E}[\hat{\theta}_2] = \dot{p}_2\theta = 0.105 \cdot 365 = 38.3,$$

$$\dot{E}[\hat{\theta}_3] = \dot{p}_3\theta = 0.245 \cdot 365 = 89.4,$$

$$\dot{E}[\hat{\theta}_4] = \dot{p}_4\theta = 0.309 \cdot 365 = 112.8, \quad (95)$$

that differ much from the values of $E[\hat{\theta}_1]$, $E[\hat{\theta}_2]$, $E[\hat{\theta}_3]$, $E[\hat{\theta}_4]$ determined by (18) in [3].
In practice, these differences can be very helpful for the system operation process planning and reorganizing.

9. The exemplary system reliability optimization

To make the optimization of the reliability of the exemplary system we need the optimal values $\dot{p}_1$, $\dot{p}_2$, $\dot{p}_3$, $\dot{p}_4$, of the transient probabilities p_b, $b = 1,2,3,4$, in particular operation states determined by (90). Substituting these optimal solutions into the formula (10.2) from [1], we obtain the optimal mean values of the system unconditional lifetimes in the reliability state subsets $\{1,2,3\}$ and $\{3\}$, that respectively are

$$\dot{\mu}(1) = \dot{p}_1 \cdot 505 + \dot{p}_2 \cdot 744.05 + \dot{p}_3 \cdot 405.56$$
$$+ \dot{p}_4 \cdot 237.05$$
$$= 0.341 \cdot 505 + 0.105 \cdot 744.05$$
$$+ 0.245 \cdot 405.56 + 0.309 \cdot 237.05$$
$$\cong 422.94, \quad (96)$$

$$\dot{\mu}(3) = \dot{p}_1 \cdot 468.73 + \dot{p}_2 \cdot 651.04 + \dot{p}_3 \cdot 370.67$$
$$+ \dot{p}_4 \cdot 237.05$$
$$= 0.341 \cdot 468.73 + 0.105 \cdot 651.04$$
$$+ 0.245 \cdot 370.67 + 0.309 \cdot 237.05$$
$$\cong 392.25. \quad (97)$$

According to (10.6) from [1], the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are

$$\dot{\bar{\mu}}(1) = \dot{\mu}(1) - \dot{\mu}(2) = 12.74$$

$$\dot{\bar{\mu}}(2) = \dot{\mu}(2) - \dot{\mu}(3) = 17.95$$

$$\dot{\mu}(3) = \mu(3) = 392.25. \quad (98)$$

Moreover, according to (10.3)-(10.4) from [1], the corresponding optimal unconditional multistate reliability function of the system is given by the vector

$$\dot{\boldsymbol{R}}(t,\cdot) = [1, \dot{\boldsymbol{R}}(t,1), \dot{\boldsymbol{R}}(t,2), \dot{\boldsymbol{R}}(t,3)], t \geq 0, \quad (99)$$

with the coordinates given by

$$\dot{\boldsymbol{R}}(t,1) = 0.341 \cdot [\boldsymbol{R}(t,1)]^{(1)} + 0.105 \cdot [\boldsymbol{R}(t,1)]^{(2)} + 0.245 \cdot [\boldsymbol{R}(t,1)]^{(3)} + 0.309 \cdot [\boldsymbol{R}(t,1)]^{(4)} \text{ for } t \geq 0, \quad (100)$$

$$\dot{\boldsymbol{R}}(t,2) = 0.341 \cdot [\boldsymbol{R}(t,2)]^{(1)} + 0.105 \cdot [\boldsymbol{R}(t,2)]^{(2)} + 0.245 \cdot [\boldsymbol{R}(t,2)]^{(3)} + 0.309 \cdot [\boldsymbol{R}(t,2)]^{(4)} \text{ for } t \geq 0, \quad (101)$$

$$\dot{\boldsymbol{R}}(t,3) = 0.341 \cdot [\boldsymbol{R}(t,3)]^{(1)} + 0.105 \cdot [\boldsymbol{R}(t,3)]^{(2)} + 0.245 \cdot [\boldsymbol{R}(t,3)]^{(3)} + 0.309 \cdot [\boldsymbol{R}(t,3)]^{(4)} \text{ for } t \geq 0, \quad (102)$$

where $[\boldsymbol{R}(t,1)]^{(b)}$, $[\boldsymbol{R}(t,2)]^{(b)}$, $[\boldsymbol{R}(t,3)]^{(b)}$, $b = 1,2,3,4,$ are fixed in Section 6 [3], respectively by (20)-(22), (27)-(29), (42)-(44), (57)-(59).

The coordinates of the exemplary system optimal unconditional four-state reliability function are illustrated in *Figure 9*.

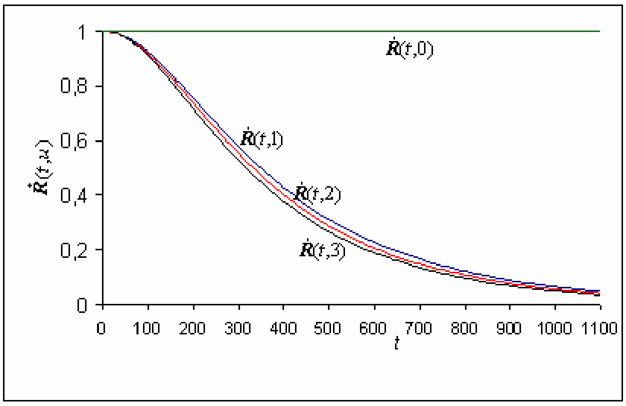

Figure 9. The graph of the exemplary system optimal reliability function $[\dot{\boldsymbol{R}}(t,\cdot)]$ coordinates

Further, by (10.5) from [1], the corresponding optimal variances and standard deviations of the system unconditional lifetime in the system reliability state subsets are

$$\dot{\sigma}^2(1) = 2\int_0^\infty t \ \dot{\boldsymbol{R}}(t,1)dt - [\dot{\mu}(1)]^2 \cong 119374.76,$$
$$\dot{\sigma}(1) \cong 345.51, \quad (9.4)(103)$$

$$\dot{\sigma}^2(2) = 2\int_0^\infty t \ \dot{\boldsymbol{R}}(t,2)dt - [\dot{\mu}(2)]^2 \cong 101665.04,$$
$$\dot{\sigma}(2) \cong 318.85, \quad (104)$$

$$\dot{\sigma}^2(3) = 2\int_0^\infty t \ \dot{\boldsymbol{R}}(t,3)dt - [\dot{\mu}(3)]^2 \cong 93424.64,$$
$$\dot{\sigma}(3) \cong 305.65, \quad (105)$$

where $\dot{\boldsymbol{R}}(t,1)$, $\dot{\boldsymbol{R}}(t,2)$, $\dot{\boldsymbol{R}}(t,3)$ are given by (10.5)-(10.7) and $\dot{\mu}(1)$, $\dot{\mu}(2)$, $\dot{\mu}(3)$ are given by (8.15) and (9.1)-(9.2).

If the critical reliability state is $r = 2$, then the optimal system risk function, according to (10.7) from [1], is given by

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,2) \text{ for } t \geq 0, \quad (106)$$

where $\dot{\boldsymbol{R}}(t,2)$ is given by (101).

Hence, considering (10.8) from [1], the moment when the optimal system risk function exceeds a permitted level, for instance $\delta = 0.05$, is

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 80. \quad (107)$$

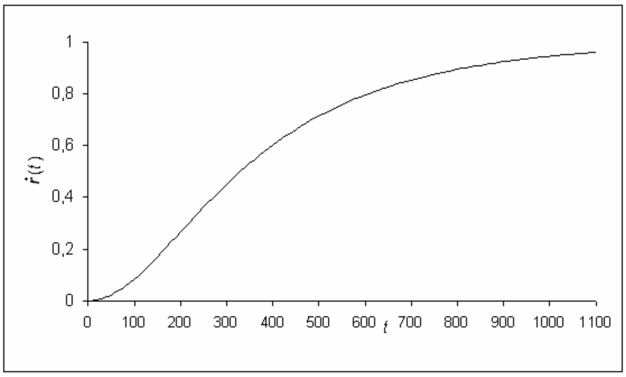

Figure 10. The graph of the exemplary system optimal risk function $r(t)$

10. The exemplary system renewal and availability optimization

To determine the optimal renewal and availability characteristics of the exemplary system after its operation process optimization, we use the results of the system reliability characteristics optimization performed in Sections 8 and 9 and the results of the Section 11.2 of [1].

In the case when the exemplary system renovation time is ignored, considering the optimal values $\dot{\mu}(2)$ determined by (91) and $\dot{\sigma}(2)$ determined by (104) and applying *Proposition 11.1* from Section 11.2.1 of [1], we determine its following optimal characteristics:

a) the optimal time $\dot{S}_N(2)$ until the *Nth* exceeding by the system of reliability critical state 2, for sufficiently large *N*, has approximately normal distribution $N(410.20N, 318.85\sqrt{N})$, i.e.,

$$\dot{F}^{(N)}(t,2) = P(\dot{S}_N(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 410.20N}{318.85\sqrt{N}}),\ t \in (-\infty, \infty),$$

b) the expected value and the variance of the optimal time $\dot{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state 2, for sufficiently large N, respectively are

$$E[\dot{S}_N(2)] = 410.20N,\ D[\dot{S}_N(2)] = 101665.32N\ ,$$

c) the optimal number $\dot{N}(t,2)$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \ge 0$, for sufficiently large *t*, has distribution approximately of the form

$$P(\dot{N}(t,2) = N) \cong F_{N(0,1)}(\frac{410.20(N+1) - t}{15.74\sqrt{t}})$$
$$- F_{N(0,1)}(\frac{410.20N - t}{15.74\sqrt{t}})\ \ N = 0,1,2,...,$$

d) the expected value and the variance of the optimal number $N(t,2)$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \ge 0$, for sufficiently large *t*, respectively are

$$H(t,2) = 0.00243t,\ D(t,2) = 0.0015t. \qquad (108)$$

To make the estimation of the renewal and availability of the exemplary system in the case when the time of renovation is non-ignored, considering the optimal values $\dot{\mu}(2)$ determined by (91) and $\dot{\sigma}(2)$ determined by (104), assuming the mean value of the system renovation time $\mu_0(2) = 10$ year and the standard deviation of the system renovation time $\sigma_0(2) = 5$ year and applying *Proposition 11.2* from Section 11.2.2 of [1], we determine its following optimal characteristics:

a) the optimal time $\dot{\bar{S}}_N(2)$ until the *Nth* exceeding by the system the reliability critical state 2, for sufficiently large N, has approximately normal distribution $N(410.20N + 10(N-1), \sqrt{101665.32N + 25(N-1)}$, i.e.,

$$\dot{\bar{F}}^{(N)}(t,2) = P(\dot{\bar{S}}_N(2) < t)$$

$$= F_{N(0,1)}(\frac{t - 420.20N + 10}{\sqrt{101690.32N - 25}}),\ t \in (-\infty, \infty)\ ;$$

b) the expected value and the variance of the optimal time $\dot{\bar{S}}_N(2)$ until the *Nth* exceeding by the system the reliability critical state 2, for sufficiently large N, respectively are

$$E[\dot{\bar{S}}_N(2)] \cong 410.20N + 10(N-1),$$

$$D[\dot{\bar{S}}(2)] \cong 101665.32N + 25(N-1),$$

c) the optimal number $\dot{\bar{N}}(t,2)$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \ge 0$, for sufficiently large t, has approximately distribution of the form

$$P(\dot{\bar{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{420.20(N+1) - t - 10}{15.56\sqrt{t+10}})$$

$$- F_{N(0,1)}(\frac{420.20N - t - 10}{15.56\sqrt{t+10}})\ \ N = 1,2,...,$$

d) the expected value and the variance of the optimal number $\dot{\bar{N}}(t,2)$ of exceeding by the system the reliability critical state 2 up to the moment $t, t \ge 0$, for sufficiently large t, respectively

$$\dot{\bar{H}}(t,2) \cong \frac{t+10}{420.20},\ \dot{\bar{D}}(t,2) \cong 0.0014(t+10),$$

e) the optimal time $\dot{\bar{\bar{S}}}_N(2)$ until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution $N(420.20N, 318.89\sqrt{N})$, i.e.,

$$\dot{\bar{\bar{F}}}^{(N)}(t,2) = P(\dot{\bar{\bar{S}}}_N(2) < t) \cong F_{N(0,1)}(\frac{t - 420.20N}{318.89\sqrt{N}}),$$
$$t \in (-\infty, \infty),\ N = 1,2,...,$$

f) the expected value and the variance of the optimal time $\dot{\bar{\bar{S}}}_N(2)$ until the *Nth* system's renovation, for sufficiently large N, respectively are

$$E[\dot{\bar{\bar{S}}}_N(2)] \cong 420.20N,$$
$$D[\dot{\bar{\bar{S}}}^{(N)}(2)] \cong 101690.32N,$$

g) the optimal number $\dot{\bar{\bar{N}}}(t,2)$ of system's renovations up to the moment $t, t \ge 0$, for sufficiently large t, has approximately distribution of the form

$$P(\dot{\bar{\bar{N}}}(t,2) = N) \cong F_{N(0,1)}(\frac{420.20(N+1) - t}{15.56\sqrt{t}}$$
$$- F_{N(0,1)}(\frac{420.20N - t}{15.56\sqrt{t}},\ N = 1,2,...,$$

h) the expected value and the variance of the optimal number $\dot{\bar{\bar{N}}}(t,2)$ of system's renovations up to the moment $t, t \ge 0$, for sufficiently large t, respectively are

$$\dot{\bar{\bar{H}}}(t,2) \cong 0.00238t,\ \dot{\bar{\bar{D}}}(t,2) \cong 0.0014t, \qquad (109)$$

i) the optimal steady availability coefficient of the system at the moment $t, t \ge 0$, for sufficiently large t, is

$$\dot{A}(t,2) \cong 0.98,\quad t \ge 0,$$

j) the optimal steady availability coefficient of the system in the time interval $< t, t+\tau),\ \tau > 0,\ t \ge 0$, for sufficiently large t, is

$$\dot{A}(t,\tau,2) \cong 0.0024\int_{\tau}^{\infty} \dot{\boldsymbol{R}}(t,2)dt,\quad t \ge 0,\quad \tau > 0,$$

where $\dot{\boldsymbol{R}}(t,2)$ is given by (101).

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