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Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety
Part 3
IS&RDSS Application – Exemplary system operation and reliability characteristics prediction

Keywords

reliability, operation process, prediction

Abstract

There is presented the IS&RDSS application to the operation and reliability of an exemplary complex technical system prediction. There are performed, the unconditional mean sojourn times and the limit transient probabilities of the exemplary system operation process at the particular operation states evaluations. The evaluations of the exemplary system unconditional multistate reliability function, the expected values and the standard deviations of its unconditional lifetimes in the reliability state subsets and the mean values of its lifetimes in the particular reliability states are performed as well. Moreover, in the case when the system is repairable, its renewal and availability characteristics are estimated.

5. The exemplary system operation prediction

After considering the results (2) given in [3] and applying the formulae (4.5) from [1], we conclude that the unconditional mean sojourn times of the exemplary system at the particular operation states are given by:

$$M_1 = E[\theta_1] = p_{12}M_{12} + p_{13}M_{13} + p_{14}M_{14}$$

$$= 0.22 \cdot 192 + 0.32 \cdot 480 + 0.46 \cdot 200$$

$$= 287.84, \tag{12}$$

$$M_2 = E[\theta_2] = p_{21}M_{21} + p_{23}M_{23} + p_{24}M_{24}$$

$$= 0.20 \cdot 96 + 0.30 \cdot 81 + 0.50 \cdot 55$$

$$= 71.00, \tag{13}$$

$$M_3 = E[\theta_3] = p_{31}M_{31} + p_{32}M_{32} + p_{34}M_{34}$$

$$= 0.12 \cdot 870 + 0.16 \cdot 480 + 0.72 \cdot 300$$

$$= 397.20, \tag{14}$$

$$M_4 = E[\theta_4] = p_{41}M_{41} + p_{42}M_{42} + p_{43}M_{43}$$

$$= 0.48 \cdot 325 + 0.22 \cdot 510 + 0.30 \cdot 438$$

$$= 399.60. \tag{15}$$

Since, according to (4.7) from [1], the system of equations

$$\begin{cases}[\pi_1,\pi_2,\pi_3,\pi_4]=[\pi_1,\pi_2,\pi_3,\pi_4][p_{bl}]_{4x4}\\ \pi_1+\pi_2+\pi_3+\pi_4=1,\end{cases}$$

after considering (1), takes the form

$$\begin{cases}\pi_1=0.20\pi_2+0.12\pi_3+0.48\pi_4\\ \pi_2=0.22\pi_1+0.16\pi_3+0.22\pi_4\\ \pi_3=0.32\pi_1+0.30\pi_2+0.30\pi_4\\ \pi_4=0.46\pi_1+0.50\pi_2+0.72\pi_3\\ \pi_1+\pi_2+\pi_3+\pi_4=1,\end{cases}$$

then its approximate solutions are

$$\pi_1\cong 0.236,\ \pi_2\cong 0.169,$$
$$\pi_3\cong 0.234,\ \pi_4\cong 0.361. \qquad (16)$$

Hence, after considering the above result and (12)-(15), we have

$$\sum_{l=1}^{4}\pi_l M_l\cong 0.236\cdot 287.84+0.169\cdot 71.00$$

$$+0.234\cdot 397.20+0.361\cdot 399.60=317.13.$$

Thus, according to (4.6) from [1], the limit values of the exemplary system operation process transient probabilities $p_b(t)$ at the operation states z_b are given by

$$p_1=\frac{0.236\cdot 287.84}{317.13}\cong 0.214,$$

$$p_2=\frac{0.169\cdot 71.00}{317.13}\cong 0.038,$$

$$p_3=\frac{0.234\cdot 397.20}{317.13}\cong 0.293,$$

$$p_4=\frac{0.361\cdot 399.60}{317.13}\cong 0.455. \qquad (17)$$

Afterwards, the expected values of the total sojourn times $\hat{\theta}_b$, $b=1,2,3,4$, of the system operation process at the particular operation states z_b, $b=1,2,3,4$, during the fixed operation time

$\theta=1$ year = 365 days,

after applying (4.8) from [1], amount:

$E[\hat{\theta}_1]=0.214\cdot 1$ = 0.214 year = 78.1 days,

$E[\hat{\theta}_2]=0.039\cdot 1$ = 0.038 year = 13.9 days,

$E[\hat{\theta}_3]=0.293\cdot 1$ = 0.293 year = 106.9 days,

$E[\hat{\theta}_4]=0.455$ = 0.455 year = 166.1 days. (18)

6. The exemplary system reliability prediction

Considering the results of the system components reliability modeling from Section 3 [3] concerned with the fixed system reliability structures and their shape parameters and with the assumed the exponential models of the reliability functions of the system components in various operation states and the results of the evaluations of the system components intensities of departures from the reliability state subsets from Section 4 [3], we may to perform the prediction of the system reliability characteristics.

Thus, as we fixed in Section 5, at the operational state z_1, the system is identical with the subsystem S_1 that is a four-state series-parallel system with its reliability structure shape parameters $k=2,\ l_1=3,\ l_2=3,$ and according to (1.36)-(1.37) from [1], its four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(1)}$$

$$=[1,[\boldsymbol{R}(t,1)]^{(1)},[\boldsymbol{R}(t,2)]^{(1)},[\boldsymbol{R}(t,3)]^{(1)}], \qquad (19)$$

$$t\geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(1)}=\boldsymbol{R}_{2;3,3}(t,1)=1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[R_{ij}^{(1)}(t,1)]^{(1)}]$$
$$=1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(1)]^{(1)}t]],$$

$$[\boldsymbol{R}(t,2)]^{(1)}=\boldsymbol{R}_{2;3,3}(t,2)=1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[R_{ij}^{(1)}(t,2)]^{(1)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(2)]^{(1)} t]],$$

$$[\boldsymbol{R}(t,3)]^{(1)} = \boldsymbol{R}_{2;3,3}(t,3) = 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[R_{ij}^{(1)}(t,3)]^{(1)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(3)]^{(1)} t]].$$

After substituting in the above expressions for the coordinates, the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2] and partly presented in Section 4 [3], we get:

$$[\boldsymbol{R}(t,1)]^{(1)}$$

$$= 1 - [1 - \exp[-[0.0008 + 0.0011 + 0.0011]t]^2$$

$$= 1 - [1 - \exp[-0.003t]]^2$$

$$= 2\exp[-0.003t] - \exp[-0.006t], \quad (20)$$

$$[\boldsymbol{R}(t,2)]^{(1)}$$

$$= 1 - [1 - \exp[-[0.0009 + 0.0011 + 0.0011]t]^2$$

$$= 1 - [1 - \exp[-0.0031t]]^2$$

$$= 2\exp[-0.0031t] - \exp[-0.0062t], \quad (21)$$

$$[\boldsymbol{R}(t,3)]^{(1)}$$

$$= 1 - [1 - \exp[-[0.0009 + 0.0012 + 0.0011]t]^2$$

$$= 1 - [1 - \exp[-0.0032t]]^2$$

$$= 2\exp[-0.0032t] - \exp[-0.0064t]. \quad (22)$$

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_1, calculated from the above results given by (19)-(22), according to (7.5)-(7.7) from [1], respectively are:

$$\mu_1(1) \cong 505, \quad \mu_1(2) \cong 483.87,$$

$$\mu_1(3) \cong 468.73, \quad (23)$$

$$\sigma_1(1) \cong 365.87, \ \sigma_1(2) \cong 360.66,$$

$$\sigma_1(3) \cong 349.41, \quad (24)$$

and further, using (7.8) from [1] and (23), it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_1, respectively are:

$$\bar{\mu}_1(1) \cong 21.13, \ \bar{\mu}_1(2) \cong 15.14, \ \bar{\mu}_1(3) \cong 468.7. \quad (25)$$

At the operation state z_2, the system is identical with the subsystem S_2 that is a four-state series-parallel system with its structure shape parameters $k = 4$, $l_1 = 2$, $l_2 = 2$, $l_3 = 2$, $l_4 = 2$ and according to (1.36)-(1.37) from [1], its four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(2)}$$

$$= [1, [\boldsymbol{R}(t,1)]^{(2)}, [\boldsymbol{R}(t,2)]^{(2)}, [\boldsymbol{R}(t,3)]^{(2)}], \quad (26)$$

$$t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(2)} = \boldsymbol{R}_{4;2,2,2,2}(t,1)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[R_{ij}^{(2)}(t,1)]^{(2)}]$$

$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(1)]^{(2)} t]],$$

$$[\boldsymbol{R}(t,2)]^{(2)} = \boldsymbol{R}_{4;2,2,2,2}(t,2)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[R_{ij}^{(2)}(t,2)]^{(2)}]$$

$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(2)]^{(2)} t]],$$

$$[\boldsymbol{R}(t,3)]^{(2)} = \boldsymbol{R}_{4;2,2,2,2}(t,3)$$

$$= 1 - \prod_{i=1}^{4}[1 - \prod_{j=1}^{2}[R_{ij}^{(2)}(t,3)]^{(2)}]$$

$$= 1 - \prod_{i=1}^{4}[1 - \exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(3)]^{(2)} t]].$$

After substituting in the above expressions for the coordinates the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}(t,1)]^{(2)}$$

$$= 1-[1-\exp[-[0.0013+0.0015]t]^4$$

$$=1-[1-\exp[-0.0028t]]^4$$

$$= 4\exp[-0.0028t]-6\exp[-0.0056t]$$

$$+4\exp[-0.0084t]-\exp[-0.0112t], \quad (27)$$

$$[\boldsymbol{R}(t,2)]^{(2)}$$
$$= 1-[1-\exp[-[0.0014+0.0016]t]^4$$

$$=1-[1-\exp[-0.003t]]^4$$

$$= 4\exp[-0.003t]-6\exp[-0.006t]$$

$$+4\exp[-0.009t]-\exp[-0.012t], \quad (28)$$

$$[\boldsymbol{R}(t,3)]^{(2)}$$

$$= 1-[1-\exp[-[0.0015+0.0017]t]^4$$

$$=1-[1-\exp[-0.0032t]]^4$$

$$= 4\exp[-0.0032t]-6\exp[-0.0064t]$$

$$+4\exp[-0.0096t]-\exp[-0.0128t]. \quad (29)$$

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_2, calculated from the above results given by (26)-(29), according to (7.5)-(7.7) from [1], respectively are:

$$\mu_2(1) \cong 744.05, \ \ \mu_2(2) \cong 694.44,$$

$$\mu_2(3) \cong 651.04, \quad (30)$$

$$\sigma_2(1) \cong 426.12, \ \sigma_2(2) \cong 397.76,$$

$$\sigma_2(3) \cong 372.86, \quad (31)$$

and further, using (7.8) from [1] and (30), it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_2, respectively are:

$$\bar{\mu}_2(1) \cong 49.61, \ \bar{\mu}_2(2) \cong 43.4, \ \bar{\mu}_2(3) \cong 651.04.$$

At the operation state z_3 the system is a four-state series system composed of subsystems S_1 and S_2. At this operation state, the subsystem S_1 is a four-state series-parallel system with its structure shape parameters (6.9) $k = 2$, $l_1 = 3$, $l_2 = 3$, and according to (1.36)-(1.37) from [1], its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(3)}$$

$$= [1, [\boldsymbol{R}^{(1)}(t,1)]^{(3)}, [\boldsymbol{R}^{(1)}(t,2)]^{(3)}, [\boldsymbol{R}^{(1)}(t,3)]^{(3)}], \quad (33)$$

$$t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}^{(1)}(t,1)]^{(3)} = \boldsymbol{R}_{2;3,3}(t,1)$$

$$= 1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[R_{ij}^{(1)}(t,1)]^{(3)}]$$

$$= 1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(1)]^{(3)}t]],$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(3)} = \boldsymbol{R}_{2;3,3}(t,2)$$

$$= 1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[R_{ij}^{(1)}(t,2)]^{(3)}]$$

$$= 1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(2)]^{(3)}t]],$$

$$[\boldsymbol{R}(t,3)]^{(1)} = \boldsymbol{R}_{2;3,3}(t,3)$$

$$= 1-\prod_{i=1}^{2}[1-\prod_{j=1}^{3}[R_{ij}^{(1)}(t,3)]^{(1)}]$$

$$= 1-\prod_{i=1}^{2}[1-\exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(3)]^{(1)}t]].$$

After substituting in the above expressions for the coordinates the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(3)}$$

$$= 1-[1-\exp[-[0.0009+0.0012+0.0011]t]^2$$

$$= 1-[1-\exp[-0.0032t]]^2$$

$$= 2\exp[-0.0032t]-\exp[-0.0064t], \tag{34}$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(3)}$$

$$= 1-[1-\exp[-[0.001+0.0012+0.0012]t]^2$$

$$= 1-[1-\exp[-0.0034t]]^2$$

$$= 2\exp[-0.0034t]-\exp[-0.0068t], \tag{35}$$

$$[\boldsymbol{R}^{(1)}(t,3)]^{(3)}$$

$$= 1-[1-\exp[-[0.001+0.0013+0.0012]t]^2$$

$$= 1-[1-\exp[-0.0035t]]^2$$

$$= 2\exp[-0.0035t]-\exp[-0.007t]. \tag{36}$$

The subsystem S_2, at the operation state z_3, is a four-state series-parallel system with its structure shape parameters $k = 4$, $l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$, and according to (1.36)-(1.37) from [1], its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(3)}$$

$$= [1,\ [\boldsymbol{R}^{(2)}(t,1)]^{(3)},\ [\boldsymbol{R}^{(2)}(t,2)]^{(3)},\ [\boldsymbol{R}^{(2)}(t,3)]^{(3)}], \tag{37}$$

$$t \geq 0,$$

with the coordinates

$$[\boldsymbol{R}^{(2)}(t,1)]^{(3)} = \boldsymbol{R}_{4;2,2,2,2}(t,1)$$

$$= 1-\prod_{i=1}^{4}[1-\prod_{j=1}^{2}[R_{ij}^{(2)}(t,1)]^{(3)}]$$

$$= 1-\prod_{i=1}^{4}[1-\exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(1)]^{(3)}t]],$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(3)} = \boldsymbol{R}_{4;2,2,2,2}(t,2)$$

$$= 1-\prod_{i=1}^{4}[1-\prod_{j=1}^{2}[R_{ij}^{(2)}(t,2)]^{(3)}]$$

$$= 1-\prod_{i=1}^{4}[1-\exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(2)]^{(3)}t]],$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(3)} = \boldsymbol{R}_{4;2,2,2,2}(t,3)$$

$$= 1-\prod_{i=1}^{4}[1-\prod_{j=1}^{2}[R_{ij}^{(2)}(t,3)]^{(3)}] \quad (6.16)$$

$$= 1-\prod_{i=1}^{4}[1-\exp[-\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(3)]^{(3)}t]].$$

After substituting in the above expressions for the coordinates the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(3)}$$

$$= 1-[1-\exp[-[0.0009+0.0012]t]^4$$

$$= 1-[1-\exp[-0.0021t]]^4$$

$$= 4\exp[-0.0021t]-6\exp[-0.0042t]$$

$$+4\exp[-0.0063t]-\exp[-0.0084t], \tag{38}$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(3)}$$

$$= 1-[1-\exp[-[0.001+0.0012]t]^4$$

$$= 1-[1-\exp[-0.0022t]]^4$$

$$= 4\exp[-0.0022t]-6\exp[-0.0044t]$$

$$+4\exp[-0.0066t]-\exp[-0.0088t], \tag{39}$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(3)}$$

$$= 1-[1-\exp[-[0.001+0.0013]t]^4$$

$$= 1-[1-\exp[-0.0023t]]^4$$

$$= 4\exp[-0.0023t]-6\exp[-0.0046t]$$

$$+4\exp[-0.0069t]-\exp[-0.0092t]. \tag{40}$$

Considering that the system at the operation state z_3 is a four-state series system composed of subsystems S_1 and S_2, after applying (1.22)-(1.23) from [1], its conditional four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(3)}$$

$$= [1, [\boldsymbol{R}(t,1)]^{(3)}, [\boldsymbol{R}(t,2)]^{(3)}, [\boldsymbol{R}(t,3)]^{(3)}], \tag{41}$$

$t \geq 0,$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(3)} = \overline{\boldsymbol{R}}_2\,(t,1) = [\boldsymbol{R}^{(1)}(t,1)]^{(3)}\,[\boldsymbol{R}^{(2)}(t,1)]^{(3)},$$

$$[\boldsymbol{R}(t,2)]^{(3)} = \overline{\boldsymbol{R}}_2\,(t,2) = [\boldsymbol{R}^{(1)}(t,2)]^{(3)}\,[\boldsymbol{R}^{(2)}(t,2)]^{(3)},$$

$$[\boldsymbol{R}(t,3)]^{(3)} = \overline{\boldsymbol{R}}_2\,(t,3) = [\boldsymbol{R}^{(1)}(t,3)]^{(3)}\,[\boldsymbol{R}^{(2)}(t,3)]^{(3)}.$$

After substituting in the above expressions for the coordinates the results (34)-(36) and (38)-(40), we get:

$$[\boldsymbol{R}(t,1)]^{(3)}$$

$$= 8\exp[-0.0053t] - 12\exp[-0.0074t]$$

$$+ 8\exp[-0.0095t] - 2\exp[-0.0116t]$$

$$- 4\exp[-0.0085t] + 6\exp[-0.0106t]$$

$$- 4\exp[-0.0127t] + \exp[-0.0148t], \tag{42}$$

$$[\boldsymbol{R}(t,2)]^{(3)}$$

$$= 8\exp[-0.0056t] - 12\exp[-0.0078t]$$

$$+ 8\exp[-0.01t] - 2\exp[-0.0122t]$$

$$- 4\exp[-0.009t] + 6\exp[-0.0112t]$$

$$- 4\exp[-0.0134t] + \exp[-0.0156t], \tag{43}$$

$$[\boldsymbol{R}(t,3)]^{(3)}$$

$$= 8\exp[-0.0058t] - 12\exp[-0.0081t]$$

$$+ 8\exp[-0.0104t] - 2\exp[-0.0127t],$$

$$- 4\exp[-0.0093t] + 6\exp[-0.00116t]$$

$$- 4\exp[-0.0139t] + \exp[-0.0162t]. \tag{44}$$

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_3, calculated from the above results given by (41)-(44), according to (7.5)-(7.7) from [1], respectively are:

$$\mu_3(1) \cong 405.56, \quad \mu_3(2) \cong 383.04,$$

$$\mu_3(3) \cong 370.67, \tag{45}$$

$$\sigma_3(1) \cong 264.58, \quad \sigma_3(2) \cong 250.39,$$

$$\sigma_3(3) \cong 241.78, \tag{46}$$

and further, using (7.8) from [1] and (45), it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_3, respectively are:

$$\overline{\mu}_3(1) \cong 22.52, \quad \overline{\mu}_3(2) \cong 12.37, \quad \overline{\mu}_3(3) \cong 370.67.$$

At the operation state z_4 the system is a four-state series system composed of subsystems S_1 and S_2, At this operation state, the subsystem S_1 is a four-state series-parallel system with its structure shape parameters $k = 2$, $l_1 = 3$, $l_2 = 3$, and according to (1.36)-(1.37) from [1], its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(1)}(t,\cdot)]^{(4)}$$

$$= [1, [\boldsymbol{R}^{(1)}(t,1)]^{(4)}, [\boldsymbol{R}^{(1)}(t,2)]^{(4)}, [\boldsymbol{R}^{(1)}(t,3)]^{(4)}], \tag{48}$$

$t \geq 0,$

with the coordinates

$$[\boldsymbol{R}^{(1)}(t,1)]^{(4)} = \boldsymbol{R}_{2;3,3}(t,1)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[R_{ij}^{(1)}(t,1)]^{(4)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(1)]^{(4)}t]],$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(4)} = \boldsymbol{R}_{2;3,3}(t,2)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[R_{ij}^{(1)}(t,2)]^{(4)}] =$$

$$1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(2)]^{(4)} t]],$$

$$[\boldsymbol{R}(t,3)]^{(4)} = \boldsymbol{R}_{2;3,3}(t,3)$$

$$= 1 - \prod_{i=1}^{2}[1 - \prod_{j=1}^{3}[R_{ij}^{(1)}(t,3)]^{(4)}]$$

$$= 1 - \prod_{i=1}^{2}[1 - \exp[-\sum_{j=1}^{3}[\lambda_{ij}^{(1)}(3)]^{(4)} t]].$$

After substituting in the above expressions for the coordinates the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(1)}(t,1)]^{(4)}$$

$$= 1 - [1 - \exp[-[0.0009 + 0.0012 + 0.0011]t]^2$$

$$= 1 - [1 - \exp[-0.0032t]]^2$$

$$= 2\exp[-0.0032t] - \exp[-0.0064t], \quad (49)$$

$$[\boldsymbol{R}^{(1)}(t,2)]^{(4)}$$

$$= 1 - [1 - \exp[-[0.001 + 0.0012 + 0.0012]t]^2$$

$$= 1 - [1 - \exp[-0.0034t]]^2$$

$$= 2\exp[-0.0034t] - \exp[-0.0068t], \quad (50)$$

$$[\boldsymbol{R}(t,3)]^{(4)}$$

$$= 1 - [1 - \exp[-[0.001 + 0.0013 + 0.0012]t]^2$$

$$= 1 - [1 - \exp[-0.0035t]]^2$$

$$= 2\exp[-0.0035t] - \exp[-0.007t]. \quad (51)$$

The subsystem S_2, at the operation state z_4, is a four-state series-“2 out of 4” system, with its structure shape parameters $k = 4$, $m = 2$, $l_1 = 2$, $l_2 = 2$, $l_3 = 2$, $l_4 = 2$, and according to (1.40)-(1.41) from [1], its four-state reliability function is given by the vector

$$[\boldsymbol{R}^{(2)}(t,\cdot)]^{(4)}$$

$$= [1, [\boldsymbol{R}^{(2)}(t,1)]^{(4)}, [\boldsymbol{R}^{(2)}(t,2)]^{(4)}, [\boldsymbol{R}^{(2)}(t,3)]^{(4)}], \quad (52)$$

$$t \geq 0,$$

where

$$[\boldsymbol{R}^{(2)}(t,1)]^{(4)}$$

$$= \boldsymbol{R}^{2}_{4;2,2,2,2}(t,1) = 1 -$$

$$\sum_{\substack{r_1,r_2,r_3,r_4=0 \\ r_1+r_2+r_3+r_4 \leq 1}}^{1} \prod_{i=1}^{4}[[\prod_{j=1}^{2}[R_{ij}^{(2)}(t,1)]^{(4)}]^{r_i}[1 - \prod_{j=1}^{2}[R_{ij}^{(2)}(t,1)]^{(4)}]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0 \\ r_1+r_2+r_3+r_4 \leq 1}}^{1} \prod_{i=1}^{4} \exp[-r_i \sum_{j=1}^{2}[\lambda_{ij}^{(2)}(1)]^{(4)} t]$$

$$[1 - \exp[\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(1)]^{(4)} t]]^{1-r_i},$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(4)} \quad (6.31)$$

$$= \boldsymbol{R}^{2}_{4;2,2,2,2}(t,2) = 1 -$$

$$\sum_{\substack{r_1,r_2,r_3,r_4=0 \\ r_1+r_2+r_3+r_4 \leq 1}}^{1} \prod_{i=1}^{4}[[\prod_{j=1}^{2}[R_{ij}^{(2)}(t,2)]^{(4)}]^{r_i}[1 - \prod_{j=1}^{2}[R_{ij}^{(2)}(t,2)]^{(4)}]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1,r_2,r_3,r_4=0 \\ r_1+r_2+r_3+r_4 \leq 1}}^{1} \prod_{i=1}^{4} \exp[-r_i \sum_{j=1}^{2}[\lambda_{ij}^{(2)}(2)]^{(4)} t]$$

$$[1 - \exp[\sum_{j=1}^{2}[\lambda_{ij}^{(2)}(2)]^{(4)} t]]^{1-r_i},$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(4)}$$

$$= \boldsymbol{R}^{2}_{4;2,2,2,2}(t,3) = 1 -$$

$$\sum_{\substack{r_1,r_2,r_3,r_4=0 \\ r_1+r_2+r_3+r_4 \leq 1}}^{1} \prod_{i=1}^{4}[[\prod_{j=1}^{2}[R_{ij}^{(2)}(t,3)]^{(4)}]^{r_i}[1 - \prod_{j=1}^{2}[R_{ij}^{(2)}(t,3)]^{(4)}]^{1-r_i}$$

$$= 1 - \sum_{\substack{r_1, r_2, r_3, r_4 = 0 \\ r_1 + r_2 + r_3 + r_4 \le 1}}^{1} \prod_{i=1}^{4} \exp[-r_i \sum_{j=1}^{2} [\lambda_{ij}^{(2)}(3)]^{(4)} t]$$

$$[1 - \exp[\sum_{j=1}^{2} [\lambda_{ij}^{(2)}(3)]^{(4)} t]]^{1-r_i}$$

After substituting in the above expressions for the coordinates the suitable evaluations of the system components intensities of departures from the reliability state subsets found in [2], we get:

$$[\boldsymbol{R}^{(2)}(t,1)]^{(4)}$$

$$= 1 \text{ - } [1 - \exp[-[0.0013 + 0.0015]t]]^4 \, 4$$

$$- 4\exp[-1[0.0013 + 0.0015]t]$$

$$[1 - \exp[-[0.0013 + 0.0015]t]^3$$

$$= 1 \text{ - } [1 - \exp[-0.0028t]]^4$$

$$- \; 4\exp[-0.0028t] \; [1 - \exp[-0.0028t]]^3$$

$$= 6\exp[-0.0056t] - 8\exp[-0.0084t]$$

$$+ 3\exp[-0.0112t], \qquad (53)$$

$$[\boldsymbol{R}^{(2)}(t,2)]^{(4)}$$

$$= 1 \text{ - } [1 - \exp[-[0.0014 + 0.0016]t]]^4$$

$$- 4\exp[-1[0.0014 + 0.0016]t]$$

$$[1 - \exp[-[0.0014 + 0.0016]t]^3$$

$$= 1 \text{ - } [1 - \exp[-0.003t]]^4$$

$$- \; 4\exp[-0.003t] \; [1 - \exp[-0.003t]]^3$$

$$= 6\exp[-0.006t] - 8\exp[-0.009t]$$

$$+ 3\exp[-0.012t], \qquad (54)$$

$$[\boldsymbol{R}^{(2)}(t,3)]^{(4)}$$

$$= 1 \text{ - } [1 - \exp[-[0.0015 + 0.0018]t]]^4$$

$$- 4\exp[-1[0.0015 + 0.0018]t]$$

$$[1 - \exp[-[0.0015 + 0.0018]t]^3$$

$$= 1 \text{ - } [1 - \exp[-0.0033t]]^4$$

$$- 4\exp[-0.0033t] \; [1 - \exp[-0.0033t]]^3$$

$$= 6\exp[-0.0066t] - 8\exp[-0.0099t]$$

$$+ 3\exp[-0.0132t]. \qquad (55)$$

Considering that the system at the operation state z_4 is a four-state series system composed of subsystems S_1 and S_2, after applying (1.22)–(1.23) from [1], its conditional four-state reliability function is given by the vector

$$[\boldsymbol{R}(t,\cdot)]^{(4)}$$

$$= [1, [\boldsymbol{R}(t,1)]^{(4)}, [\boldsymbol{R}(t,2)]^{(4)}, [\boldsymbol{R}(t,3)]^{(4)}], \qquad (56)$$

$$t \ge 0,$$

with the coordinates

$$[\boldsymbol{R}(t,1)]^{(4)} = \overline{\boldsymbol{R}}_2(t,1) = [\boldsymbol{R}^{(1)}(t,1)]^{(4)} \, [\boldsymbol{R}^{(2)}(t,1)]^{(4)},$$

$$[\boldsymbol{R}(t,2)]^{(4)} = \overline{\boldsymbol{R}}_2(t,2) = [\boldsymbol{R}^{(1)}(t,2)]^{(4)} \, [\boldsymbol{R}^{(2)}(t,2)]^{(4)},$$

$$[\boldsymbol{R}(t,3)]^{(4)} = \overline{\boldsymbol{R}}_2(t,3) = [\boldsymbol{R}^{(1)}(t,3)]^{(4)} \, [\boldsymbol{R}^{(2)}(t,3)]^{(4)}.$$

After substituting in the above expressions for the coordinates the results (49)-(51) and (53)-(55), we get:

$$[\boldsymbol{R}(t,1)]^{(4)}$$

$$= 12\exp[-0.0088t] - 16\exp[-0.0116t]$$

$$+ 6\exp[-0.0144t] - 6\exp[-0.012t]$$

$$+ 8\exp[-0.0148t] - 3\exp[-0.0176t], \qquad (57)$$

$$[\boldsymbol{R}(t,2)]^{(4)}$$

$$= 12\exp[-0.0094t] - 16\exp[-0.0124t]$$

$$+ 6\exp[-0.0154t] - 6\exp[-0.0128t]$$

$$+8\exp[-0.0158t]-3\exp[-0.0188t], \qquad (58)$$

$$[\boldsymbol{R}(t,3)]^{(4)}$$

$$= 12\exp[-0.0101t]-16\exp[-0.0134t]$$

$$+6\exp[-0.0167t]-6\exp[-0.0136t]$$

$$+8\exp[-0.0169t]-3\exp[-0.0202t]. \qquad (59)$$

The expected values and standard deviations of the system conditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ at the operation state z_4, calculated from the above results given by (56)-(59), according to (7.5)-(7.7) from [1], respectively are:

$$\mu_4(1) \cong 271.08, \ \ \mu_4(2) \cong 253.88,$$

$$\mu_4(3) \cong 237.05, \qquad (60)$$

$$\sigma_4(1) \cong 163.8, \ \sigma_4(2) \cong 153.35,$$

$$\sigma_4(3) \cong 142.58, \qquad (61)$$

and further, using (7.8) from [1] and (60), it follows that the mean values of the conditional lifetimes in the particular reliability states 1, 2, 3 at the operation state z_4, respectively are:

$$\bar{\mu}_4(1) \cong 17.20, \ \bar{\mu}_4(2) \cong 16.83, \ \bar{\mu}_4(3) \cong 237.05.$$

In the case when the system operation time is large enough, its unconditional four-state reliability function is given by the vector

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1), \boldsymbol{R}(t,2), \boldsymbol{R}(t,3)], \ t \ge 0, \qquad (62)$$

where according to (7.3)-(7.4) from [1], considering (4.6) from [1], the vector co-ordinates are given respectively by

$$\boldsymbol{R}(t,1) = p_1[\boldsymbol{R}(t,1)]^{(1)} + p_2[\boldsymbol{R}(t,1)]^{(2)}$$

$$+ p_3[\boldsymbol{R}(t,1)]^{(3)} + p_4[\boldsymbol{R}(t,1)]^{(4)}$$

$$= 0.214\cdot[\boldsymbol{R}(t,1)]^{(1)}$$

$$+0.038\cdot[\boldsymbol{R}(t,1)]^{(2)} + 0.293\cdot[\boldsymbol{R}(t,1)]^{(3)}$$

$$+0.0.455\cdot[\boldsymbol{R}(t,1)]^{(4)} \ \text{ for } t \ge 0, \qquad (64)$$

$$\boldsymbol{R}(t,2) = p_1[\boldsymbol{R}(t,2)]^{(1)} + p_2[\boldsymbol{R}(t,2)]^{(2)}$$

$$+ p_3[\boldsymbol{R}(t,2)]^{(3)} + p_4[\boldsymbol{R}(t,2)]^{(4)}$$

$$= 0.214\cdot[\boldsymbol{R}(t,2)]^{(1)} + 0.038\cdot[\boldsymbol{R}(t,2)]^{(2)}$$

$$+0.293\cdot[\boldsymbol{R}(t,2)]^{(3)}$$

$$+0.455\cdot[\overline{\boldsymbol{R}}(t,2)]^{(4)} \ \text{ for } t \ge 0, \qquad (65)$$

$$\boldsymbol{R}(t,3) = p_1[\boldsymbol{R}(t,3)]^{(1)} + p_2[\boldsymbol{R}(t,3)]^{(2)}$$

$$+ p_3[\boldsymbol{R}(t,3)]^{(3)} + p_4[\boldsymbol{R}(t,3)]^{(4)}$$

$$= 0.214\cdot[\boldsymbol{R}(t,3)]^{(1)} + 0.038\cdot[\boldsymbol{R}(t,3)]^{(2)}$$

$$+0.293\cdot[\boldsymbol{R}(t,3)]^{(3)}$$

$$+0.455\cdot[\boldsymbol{R}(t,3)]^{(4)} \ \text{ for } t \ge 0, \qquad (66)$$

and the coordinates $[\boldsymbol{R}(t,1)]^{(1)}$, $[\boldsymbol{R}(t,1)]^{(2)}$, $[\boldsymbol{R}(t,1)]^{(3)}$, $[\boldsymbol{R}(t,1)]^{(4)}$ are given by (20), (27), (42), (57), $[\boldsymbol{R}(t,2)]^{(1)}$, $[\boldsymbol{R}(t,2)]^{(2)}$, $[\boldsymbol{R}(t,2)]^{(3)}$, $[\boldsymbol{R}(t,2)]^{(4)}$ are given by (21), (28), (43), (58) and $[\boldsymbol{R}(t,3)]^{(1)}$, $[\boldsymbol{R}(t,3)]^{(2)}$, $[\boldsymbol{R}(t,3)]^{(3)}$, $[\boldsymbol{R}(t,3)]^{(4)}$ are given by (22), (29), (44), (59).

The coordinates of the exemplary system unconditional four-state reliability function are illustrated in *Figure 7*.

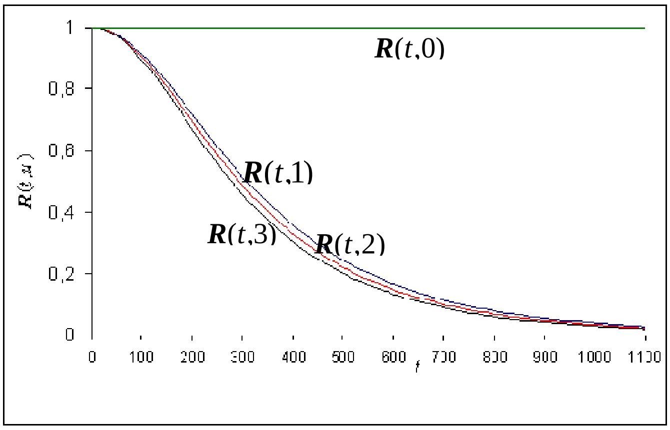

Figure 7. The graph of the exemplary system reliability function $[\boldsymbol{R}(t,\cdot)]$ coordinates

The expected values and standard deviations of the system unconditional lifetimes in the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, calculated from the above results given by (63)-(66), according to

(7.5)-(7.7) from [1] and considering (23), (30), (45), (60), respectively are:

$$\mu(1) = p_1\mu_1(1) + p_2\mu_2(1) + p_3\mu_3(1) + p_4\mu_4(1)$$

$$= 0.214 \cdot 505 + 0.038 \cdot 744.05$$

$$+ 0.293 \cdot 405.56 + 0.455 \cdot 271.08$$

$$\cong 378.51, \quad (67)$$

$$\sigma(1) \cong 286.77, \quad (68)$$

$$\mu(2) = p_1\mu_1(2) + p_2\mu_2(2) + p_3\mu_3(2)$$

$$+ p_4\mu_4(2)$$

$$= 0.214 \cdot 483.87 + 0.038 \cdot 694.44 +$$

$$+ 0.293 \cdot 383.04 + 0.455 \cdot 253.88$$

$$\cong 357.68, \quad (69)$$

$$\sigma(2) \cong 275.18, \quad (70)$$

$$\mu(3) = p_1\mu_1(3) + p_2\mu_2(3) + p_3\mu_3(3)$$

$$+ p_4\mu_4(3)$$

$$= 0.214 \cdot 468.73 + 0.038 \cdot 651.04 +$$

$$+ 0.293 \cdot 370.67 + 0.455 \cdot 237.05 \cong 341.51, \quad (71)$$

$$\sigma(3) \cong 264.78, \quad (72)$$

and further, considering (7.8) from [1] and (67), (69) and (71), it follows the mean values of that the unconditional lifetimes in the particular reliability states 1, 2, 3, respectively are:

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 20.83,$$

$$\bar{\mu}(2) = \mu(2) - \mu(3) = 16.17,$$

$$\bar{\mu}(3) = \mu(3) = 341.51. \quad (73)$$

Since the critical reliability state is r =2, then the system risk function, according to (7.9) from [1], is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}(t,2)$$

$$= 1 - [0.214 \cdot [\boldsymbol{R}(t,2)]^{(1)} + 0.038 \cdot [\boldsymbol{R}(t,2)]^{(2)}$$

$$+ 0.293 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(3)} + 0.455 \cdot [\overline{\boldsymbol{R}}(t,2)]^{(4)}\ ] \quad (74)$$

for $t \geq 0$.

Hence, by (7.10) from [1], the moment when the system risk function exceeds a permitted level, for instance δ = 0.05, is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 70.08. \quad (75)$$

The graph of the risk function $r(t)$ of the exemplary four-state system operating in variable conditions is given in *Figure 8.*

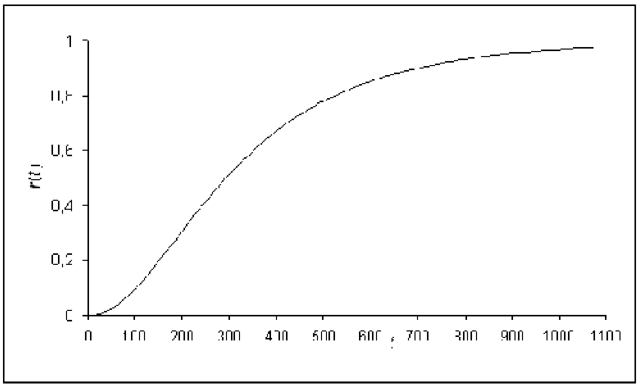

Figure 8. The graph of the exemplary system risk function $r(t)$

7. The exemplary system renewal and availability prediction

Using the results of the exemplary system reliability prediction given by (69)-(70) and the results of the classical renew theory presented in [1], we may predict the renewal and availability characteristics of this system in the case when it is repairable and its time of renovation is either ignored or non-ignored.

First, assuming that the system is repaired after the exceeding its reliability critical state r = 2 and that the time of the system renovation is ignored and applying *Proposition 8.1* from [1], we obtain the following results:

a) the time $S_N(2)$ until the *Nth* exceeding by the system the reliability critical state r = 2, for sufficiently large N, has approximately normal distribution $N(357.68N, 275.18\sqrt{N})$, i.e.,

$$F^{(N)}(t,2) = P(S_N(2) < t) \cong F_{N(0,1)}(\frac{t-357.68N}{275.18\sqrt{N}}),$$
$$t \in (-\infty, \infty);$$

b) the expected value and the variance of the time $S_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$ are respectively given by

$$E[S_N(2)] \cong 357.68N, \ \ D[S_N(2)] \cong 75724.03N;$$

c) the number $N(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large t, approximately has the distribution of the form

$$P(N(t,2) = N) \cong F_{N(0,1)}(\frac{357.68(N+1)-t}{14.55\sqrt{t}})$$

$$- F_{N(0,1)}(\frac{357.68N - t}{14.55\sqrt{t}}), \ N = 0,1,...;$$

d) the expected value and the variance of the number $N(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large t, approximately are respectively given by

$$H(t,2) \cong 0.0028t, \ \ D(t,2) \cong 0.0016t. \qquad (76)$$

Further, assuming that the system is repaired after the exceeding its reliability critical state $r = 2$ and that the time of the system renovation is non-ignored and it has the mean value $\mu_0(2) = 10$ and the standard deviation $\sigma_0(2) = 5$ and applying *Proposition 8.2* from [1], we obtain the following results:

a) the time $\bar{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$, for sufficiently large N, has approximately normal distribution $N(357.68N + 10(N-1), \sqrt{75724.03N + 25(N-1)}$, i.e.,

$$\bar{F}^{(N)}(t,2) = P(\bar{S}_N(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 367.68N + 10}{\sqrt{75749.03N - 25}}), \ \ t \in (-\infty, \infty);$$

b) the expected value and the variance of the time $\bar{S}_N(2)$ until the *Nth* exceeding by the system the reliability critical state $r = 2$, for sufficiently large N, are respectively given by

$$E[\bar{S}_N(2)] \cong 357.68N + 10(N-1),$$

$$D[\bar{S}_N(2)] \cong 75724.03N + 25(N-1);$$

c) the number $\bar{N}(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large t, has approximately distribution of the form

$$P(\bar{N}(t,2) = N)$$
$$\cong F_{N(0,1)}(\frac{367.68(N+1) - t - 10}{14.35\sqrt{t+10}})$$

$$- F_{N(0,1)}(\frac{367.68N - t - 10}{14.35\sqrt{t+10}}),$$
$$N = 1,2,...;$$

d) the expected value and the variance of the number $\bar{N}(t,2)$ of exceeding by the system the reliability critical state $r = 2$ up to the moment $t, t \geq 0,$ for sufficiently large t, are respectively given by

$$\bar{H}(t,2) \cong \frac{t+10}{367.68}, \ \ \bar{D}(t,2) \cong 0.0015(t+10);$$

e) the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large N, has approximately normal distribution $N(367.68N, 275.23\sqrt{N})$, i.e.,

$$\bar{\bar{F}}^{(N)}(t,2) = P(\bar{\bar{S}}_N(2) < t)$$

$$\cong F_{N(0,1)}(\frac{t - 367.68N)}{275.23\sqrt{N}}), \ \ t \in (-\infty, \infty);$$

f) the expected value and the variance of the time $\bar{\bar{S}}_N(2)$ until the *Nth* system's renovation, for sufficiently large N, are respectively given by

$$E[\bar{\bar{S}}_N(2)] \cong 367.68N, \ \ D[\bar{\bar{S}}_N(2)] \cong 75749.03N;$$

g) the number $\overline{\overline{N}}(t,2)$ of the system's renovations up to the moment $t, t \geq 0$, for sufficiently large *t*, has approximately distribution of the form

$$P(\overline{\overline{N}}(t,2) = N) \cong F_{N(0,1)}(\frac{367.68(N+1)-t}{14.35\sqrt{t}})$$

$$- F_{N(0,1)}(\frac{367.68N-t}{14.35\sqrt{t}}),\ N = 0,1,...;$$

h) the expected value and the variance of the number $\overline{\overline{N}}(t,2)$ of system's renovations up to the moment $t, t \geq 0$, for sufficiently large *t*, are respectively given by

$$\overline{\overline{H}}(t,2) \cong 0.0027t,\ \overline{\overline{D}}(t,2) \cong 0.0015t; \qquad (77)$$

i) the steady availability coefficient of the system at the moment $t, t \geq 0$, for sufficiently large *t*, is given by

$$A(t,2) \cong 0.97,\quad t \geq 0;$$

j) the steady availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0$, for sufficiently large *t*, is given by

$$A(t,\tau,2) \cong 0.0027\int_{\tau}^{\infty} \boldsymbol{R}(t,2)dt,\quad t \geq 0,\quad \tau > 0,$$

where $\boldsymbol{R}(t,2)$ is given by (65).

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