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# Integrated software tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety
# Part 2
# Integrated software tools application – Exemplary system operation and reliability unknown parameters identification

**Keywords**

system operation process, reliability parameters, identification, software tools

**Abstract**

There is presented the application of the integrated software tools to the operation and reliability models of an exemplary complex technical system unknown parameters identification. There are performed in the paper, the exemplary system operation and reliability analysis and modelling. The identification of the probabilities of transitions this system operation process between the operation states and the conditional mean values of this process sojourn times at the particular operation states because of the lack of statistical data is performed throw the arbitrary fixing their values assumption. Next using the computer program CP 8.3 the automatic evaluation of the system components unknown intensities of departures the reliability state subsets and the identification of the exponential forms of their multistate reliability functions on the arbitrarily fixed statistical data coming from the system components states changing processes are performed as well.

## 3. The exemplary system operation process unknown parameters identification

### 3.1. The exemplary system analysis

We analyze [8] the reliability of an exemplary system $S$ that consists of two subsystems $S_1$, $S_2$. The subsystem $S_1$ is composed of two series subsystems, each of them composed of 3 components, denoted respectively by

$$E_{ij}^{(1)}, \quad i = 1,2, \quad j = 1,2,3,$$

with the reliability structure presented in *Figure 1.*

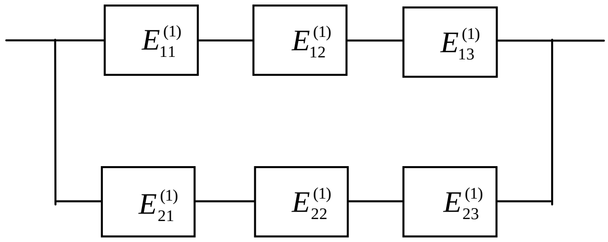



*Figure 1.* The scheme of the system $S_1$ reliability structure

The subsystem $S_2$ is composed of four series subsystems, each of them composed of 2 components, denoted respectively by

$$E_{ij}^{(2)}, \; i = 1,2,3,4 \quad j = 1,2,$$

with the reliability structure presented in *Figure 2*.

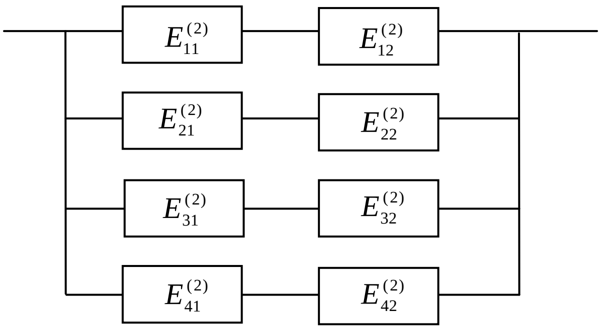



*Figure 2.* The scheme of the system $S_2$ reliability structure

The subsystems $S_1$, $S_2$, illustrated in *Figures 1–2* are forming a series reliability structure presented in *Figure 3*.

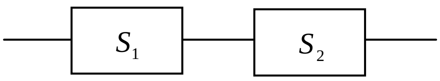



*Figure 3.* The general scheme of the system $S$ reliability structure

## 3.2. The exemplary system operation process modelling

Under the assumption that the exemplary system structure and the subsystem components reliability depend on its changing in time operation states, we arbitrarily fix the number of the system operation process states $\nu = 4$ and we distinguish the following as its four operation states [7], [8]:

- an operation state $z_1$ – the system is composed of the subsystem $S_1$, with the scheme showed in *Figure 1*, that is a series-parallel system,
- an operation state $z_2$ – the system is composed of the subsystem $S_2$, with the scheme showed in *Figure 2* that is a series-parallel system,
- an operation state $z_3$ – the system is composed of the subsystems $S_1$ and $S_2$, with the scheme showed in *Figure 3* that are series-parallel system with the schemes given in *Figures 1-2,*
- an operation state $z_4$ – the system is composed of the subsystem $S_1$ and $S_2$, with the scheme showed in *Figure 3,* while the subsystem $S_1$ is a series-parallel system with the scheme given in *Figure 1* and the subsystem $S_2$ is a series-"2 out of 4" system.

Moreover, we assume that there are possible the transitions between all system operation states. Thus, according to Section 2 of [IS&RDSS], the parameters of the system operation process semi-Markov model are [5]:

– the initial probabilities $p_b(0)$, $b = 1,2,3,4,$ of the system operation process $Z(t)$ staying in the particular states $z_b$ at the moment $t = 0$,
– the matrix $[p_{bl}]_{4x4}$ of probabilities of the exemplary system operation process $Z(t)$ transitions between the operation states,
– the matrix $[H_{bl}(t)]_{4x4}$ of conditional distribution functions of the exemplary system operation process $Z(t)$ conditional sojourn times $\theta_{bl}$ in the operation states,
– the mean values of the conditional sojourn times $\theta_{bl}$.

To identify all these parameters of the exemplary system operation process the statistical data about this process is needed and we can do it automatically using the computer program CP 8.1 "Identification of the operation processes". As the considered system is an exemplary one and its operation process parameters are arbitrarily assumed then we do not have the statistical data collected that are needed for estimating these parameters.

## 3.3. The exemplary system operation process identification

In this case, we do not have statistical data on the exemplary system operation process and we fix the process parameters defined by (2.1) and (2.3) in IS&RDSS 2 [5] arbitrarily.

The arbitrarily fixed transient probabilities from the operation state $z_b$ into the operation state $z_l$, defined by $p_{bl}$ (2.1), are given in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0.22 & 0.32 & 0.46 \\ 0.20 & 0 & 0.30 & 0.50 \\ 0.12 & 0.16 & 0 & 0.72 \\ 0.48 & 0.22 & 0.30 & 0 \end{bmatrix}.$$

As we do not have the realizations of the conditional sojourn times $\theta_{bl}$, $b,l = 1,2,3,4,$ of the exemplary system operation process at the particular operation states, then it is not possible to identify their distributions defined by (2.2) in [IS&RDSS 2].
The arbitrarily fixed conditional mean values $M_{bl} = E[\theta_{bl}],$ $b,l = 1,2,3,4,$ defined by (2.3) in [IS&RDSS 2], of the system sojourn times in the particular operation states are as follows:

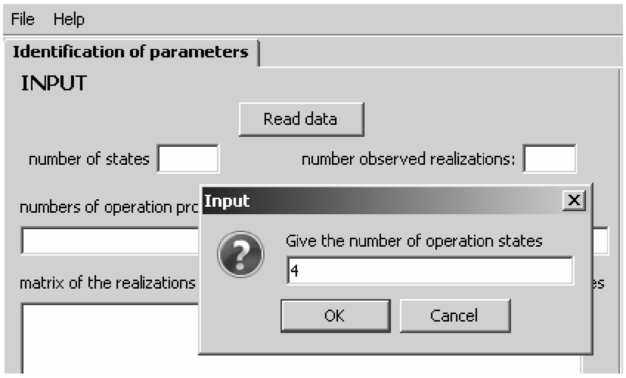
$$M_{12} = 192,\ M_{13} = 480,\ M_{14} = 200,$$

$$M_{21} = 96,\ M_{23} = 81,\ M_{24} = 55,$$

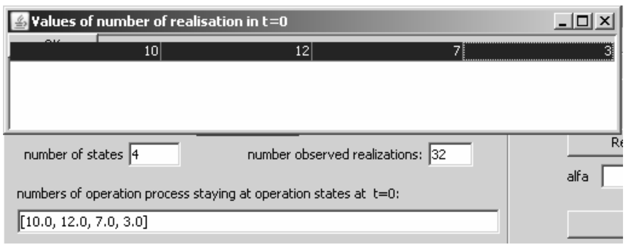
$$M_{31} = 870,\ M_{32} = 480,\ M_{34} = 300,$$

$$M_{41} = 325,\ M_{42} = 510,\ M_{43} = 438.$$

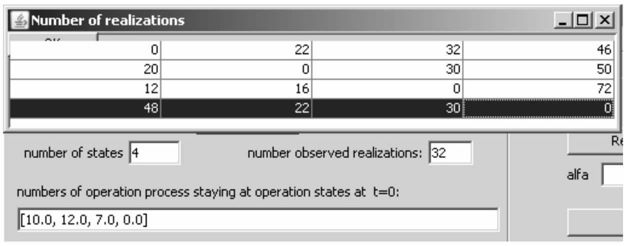
In case we have in disposal data about the system operation process we can use the computer program CP 8.1 to determine the parameters of the system operation process. Below there is given illustration of this computer program running [3].
First the computer program is reading in [6]:
– the number of operation states of the operation process,
– the number of the observed realisation of the operation,



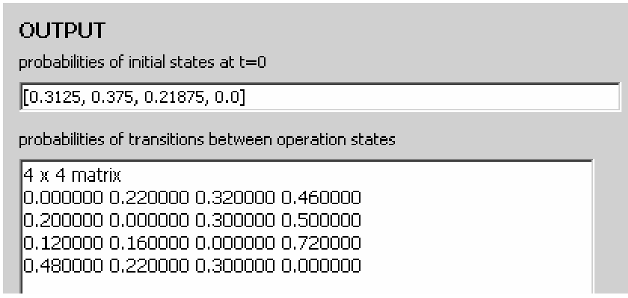

– the vector of the realizations of the numbers of staying of the operation process in the operation states at the initial moment,



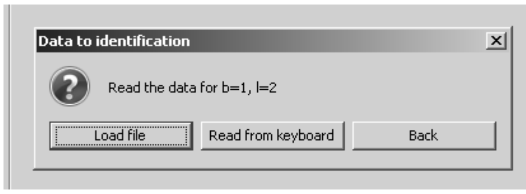

– the matrix of the realizations of the numbers of the system operation process transitions between the operation states.




The computer program estimates the following parameters of the operation process [3]:
i) the vector of the probabilities $p_b(0)$, $b = 1,2,...,\nu,$ of the initial states of the system operation process,
ii) the matrix of the probabilities $p_{bl}$, $b,l = 1,2,...,\nu,$ of the system operation process transitions from the operation state $z_b$ to the operation state $z_l$.



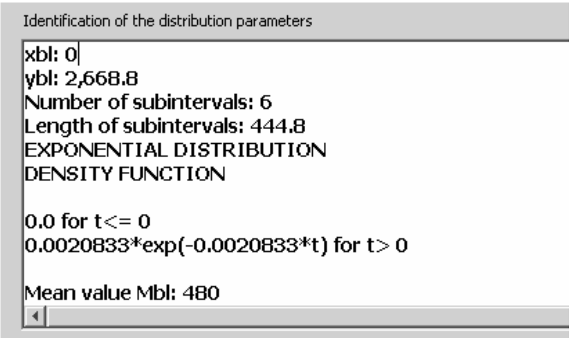

Then the computer program is reading in:
- the realizations $\theta_{bl}^k$, $k$ = 1,2, …, $n_{bl}$, $b$, $l$ = 1,2,...,$\nu$, $b \neq l$, of the conditional sojourn times $\theta_{bl}$ of the system operations process at the operation state $z_b$ when the next transition is to the operation state $z_l$.




After reading the realizations the computer program CP 8.1 [6]:
– testifies successively the hypotheses that the form of the distribution functions of the conditional sojourn times $\theta_{bl}$, $b,l = 1,2,...,\nu,$ $b \neq l,$ at the operation state $z_b$ when the next transition is to the operation state $z_l$, is one of the following: uniform distribution, triangular distribution, double-trapezium distribution, quasi-trapezium distribution, exponential distribution, Weibull distribution, normal distribution, chimney distribution,
– determines the best fitting distribution and gives its name,

– determines the mean value from the fitted distribution $M_{bl}$, $b,l = 1,2,...,\nu,\ b \neq l,$ ( in case of the hypothesis acceptance),
– determines the empirical values of the mean values $\bar{\theta}_{bl}$, $b,l = 1,2,...,\nu,\ b \neq l,$ (in case of the hypothesis rejecting).

The exemplary results of the computer program CP 8.1 are given in the window below.




## 4. The exemplary system reliability model unknown parameters identification

### 4. 1. The exemplary system components reliability modelling

We assume that the exemplary system and its components have four reliability states 0, 1, 2 3, i.e. $z = 3$. And consequently, at all operation states $z_b$, $b = 1,2,3,4,$ we arbitrarily distinguish the following reliability states of the system and its components:

- a reliability state 3 – the system operation is fully effective,
- a reliability state 2 – the system operation is less effective because of ageing,
- a reliability state 1 – the system operation is less effective because of ageing and more dangerous,
- a reliability state 0 – the system is destroyed.

We assume that there are possible the transitions between the components reliability states only from better to worse ones and we fix that the system and components critical reliability state is $r = 2$. Moreover, we assume that the changes of the operation states of the system *S* operation process *Z*(*t*) have an influence on the system reliability structure and the system multi-state components reliability as well.

The system operation process influence on the system reliability structure is expressed as follows.

At the system operation state $z_1$, the system is composed of the series-parallel subsystem $S_1$ containing two series subsystems ($k = 2$), each composed of three components ($l_1 = 3,\ l_2 = 3$) with the reliability structure showed in *Figure 1*.

At the system operation state $z_2$, the system is composed of the series-parallel subsystem $S_2$ containing four series subsystems ($k = 4$), each composed of two components ($l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$) with the reliability structure showed in *Figure 2*.

At the system operational state $z_3$, the system is a series system with the reliability structure showed in *Figure 3*, composed of two series-parallel subsystems $S_1$, $S_2$ illustrated in *Figures 1-2*.

The subsystem $S_1$ consists of two series subsystems ($k = 2$), each composed of three components ($l_1 = 3,\ l_2 = 3$) with the reliability structure showed in *Figure 1*. The subsystem $S_2$ consists of four series subsystems ($k = 4$), each composed of two components ($l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$) with the reliability structure showed in *Figure 2*.

At the system operation state $z_4$, the system is a series system with the scheme showed in *Figure 3*, composed of the subsystem $S_1$ and $S_2$ illustrated in *Figures 1-2*, whereas the subsystem $S_1$ is a series-parallel system and the subsystem $S_2$ is a series-"2 out of 4" system.

The subsystem $S_1$ consists of two series subsystems ($k = 2$), each composed of three components ($l_1 = 3,\ l_2 = 3$) with the reliability structure showed in *Figure 1*. The subsystem $S_2$ consists of four series subsystems ($k = 4$), each composed of two components ($l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$) and is a series-"2 out of 4" system ($m = 2$).

The system operation process influence on the system components reliability is expressed by the assumption that the subsystems $S_\upsilon$, $\upsilon = 1,2,$ are composed of four-state, i.e. $z = 3$, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,$ having the conditional four-state reliability functions

$$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)}$$
$$= [1, [R_{ij}^{(\upsilon)}(t,1)]^{(b)}, [R_{ij}^{(\upsilon)}(t,2)]^{(b)}\ [R_{ij}^{(\upsilon)}(t,3)]^{(b)}],$$
$$t \geq 0,\ \ b = 1,2,3,4,\ \ \upsilon = 1,2,$$

with the exponential co-ordinates

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)} t],$$

$[R_{ij}^{(\upsilon)}(t,3)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(3)]^{(b)} t],$

$t \geq 0,\ b = 1,2,3,4,\ \upsilon = 1,2,$

different in various operation states $z_b$, where $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$, $b = 1,2,3,4$, $\upsilon = 1,2,$ are the subsystems components unknown intensities of departures respectively from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$.

### 4.2. The exemplary system components reliability identification

#### 4.2.1. Data collections coming from system components reliability state changing processes

To estimate existing in the formulae (5.1)-(5.2) the subsystems components unknown intensities $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$, $b = 1,2,3,4$, $\upsilon = 1,2,$ of departure respectively from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, we suppose that we have in disposal data collected from the system components reliability states changing processes due to the experiment *Case 2* described in Section 6.1.1 of IS&RDSS 6 [6]. Namely, we have in disposal the following data for particular components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,$ of the system [5]:

- the numbers of identical experiment posts $n^{(b)} = n_{ij}^{(b)}$,
- the observation times $\tau^{(b)} = \tau_{ij}^{(b)}$,
- the numbers $m^{(b)}(u) = m_{ij}^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,3\}$, $u = 1,2,3,$
- the sets $A_{ij}^{(b)}(u) = \{t_i^{(b)}(u):\ i = 1,2,...,m^{(b)}(u)\}$ of realizations $t_i^{(b)}(u) = t_{ij}^{(b)}(u)$ of the component lifetimes $T_{ij}^{(b)}(u)$ in the reliability states subset $\{u, u+1,...,3\}$, $u = 1,2,3,$ at the operation state $z_b$, $b = 1,2,3,4$.

The data for all components are presented in [7].

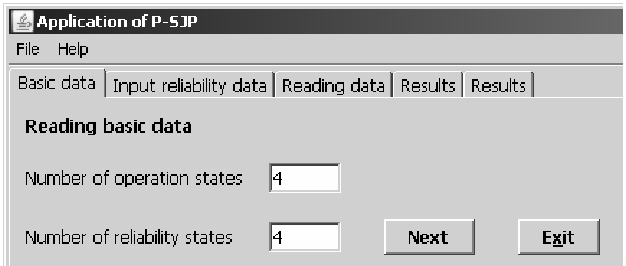
#### 4.2.2. Estimating system components intensities of departures from reliability state subsets

As there are data collected from the exemplary system components reliability states changing processes, then their reliability functions unknown parameters identification using the methods described in Section 6.2.1.1 of IS&RDSS 6 [6] is possible. To find the approximate values $[\hat{\lambda}_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\hat{\lambda}_{ij}^{(\upsilon)}(2)]^{(b)}$ and $[\hat{\lambda}_{ij}^{(\upsilon)}(3)]^{(b)}$ of the subsystems $S_\upsilon$, $\upsilon = 1,2,$ components unknown intensities $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$ and $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$ of departure respectively from the reliability states subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, while the system is operating in the operation state $z_b$, $b = 1,2,3,4,$ existing in (5.1)-(5.2), we can use statistical data presented in Section 6.1 and the formula (6.9) from IS&RDSS 6 [6]. We can also use the formula (6.10) from IS&RDSS 6 [6] to get their pessimistic evaluations.

Using the computer program CP 8.3 "Reliability models identification of the components" we can find automatically these evaluations and the results are presented below [4].

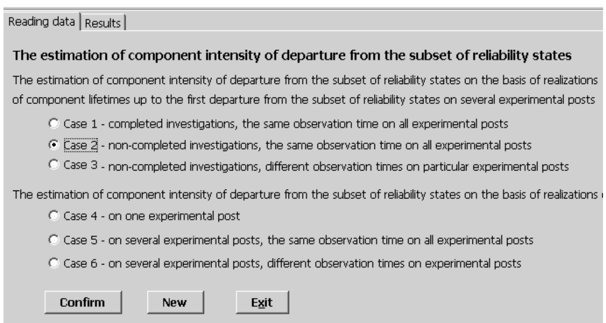
First the computer program is reading in:

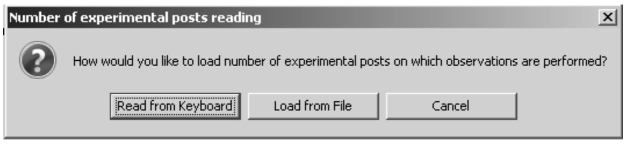
- the number of operation states $v$,
- the number of reliability states $z+1$.




As we have in disposal data coming from components reliability states changing process due to the experiment *Case 2* the computer program is reading in [1]:

- the number of experimental posts $n^{(b)}$ on which the observation is performed at the operation state $z_b$, $b = 1,2,...,\nu$, the observation time $\tau^{(b)}$, $\tau^{(b)} > 0$, of components at the operation state $z_b$, $b = 1,2,...,\nu$,
- the numbers of components $m^{(b)}(u)$, $m^{(b)}(u) \leq n^{(b)}$, that have left the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ at the operation state $z_b$, $b = 1,2,...,\nu$,
- the moments $t_i^{(b)}(u)$, $i = 1,2,...,m^{(b)}(u)$, of departure of the component on the $i-$th observational post from the reliability states subset $\{u, u+1,..., z\}$ at the operation state $z_b$, $b = 1,2,...,\nu$.






Then after reading all necessary data mentioned above, in case we have in disposal data coming from components reliability states changing process, the computer program CP 8.3 determines [1]:

– the maximum likelihood evaluation of the unknown component intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the operation state $z_b$, $b = 1,2,...,\nu$, (in all cases),
– the pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the operation state $z_b$, $b = 1,2,...,\nu$, (in all cases except case 1),

The results are presented below:

– for the components $E_{11}^{(1)}$ and $E_{21}^{(1)}$ of the subsystem $S_1$;

**The maximum likelihood evaluation of component intensity of departure from the subset of reliability states**

The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z1: 0.0008378498677768177.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z1: 0.0008730526832728563.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z1: 0.0009131638272978911.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0009352349777881693.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0009742104835872595.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0010185892538833714.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0013469080537178624.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.001434417584272033.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0015389489883673561.

**The pessimistic evaluation of component intensity of departure from the subset of reliability states**

The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z1: 0.0010473123347210223.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z1: 0.0010913158540910703.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z1: 0.001141454784122364.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0010391499753201882.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0010824560928747327.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0011317658376481907.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0013865229964742701.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.0014766063367506223.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0015842121939075725.

– for the components $E_{12}^{(1)}$ and $E_{22}^{(1)}$ of the subsystem $S_1$;

**The maximum likelihood evaluation of component intensity of departure from the subset of reliability states**

The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z1: 0.0010953759486738127.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z1: 0.0011412113142950304.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z1: 0.0011934191458528684.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0011903837693760836.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.001239791930571652.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.001296030203138647.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0015403451953078715.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.0016401715871814282.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0017592926741248646.

**The pessimistic evaluation of component intensity of departure from the subset of reliability states**

The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z1: 0.0013040189865164437.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z1: 0.0013585848979702742.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z1: 0.001420737078396272.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.001293895401495743.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0013475999245344042.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0014087284816724425.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0015798412259567913.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.001682227268904029.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.001804402742692169.

– for the components $E_{13}^{(1)}$ and $E_{23}^{(1)}$ of the subsystem $S_1$;

**The maximum likelihood evaluation of component intensity of departure from the subset of reliability states**

The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z1: 0.001058121079283501.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z1: 0.0011008308652006396.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z1: 0.0011493309252113948.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0011481915982327834.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0011962655709567523.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0012485411068588333.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0015004039549109376.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.001594896331738437.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0017073063958324213.

**The pessimistic evaluation of component intensity of departure from the subset of reliability states**

The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z1: 0.0012596679515279772.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z1: 0.0013105129347626661.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z1: 0.0013682511014421366.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0012480343459051993.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0013002886640834266.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0013571098987596015.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0015388758511907053.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.00163579110947532.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0017510834829050474.

– for the components $E_{11}^{(2)}$ and $E_{21}^{(2)}$ of the subsystem $S_2$;

**The maximum likelihood evaluation of component intensity of departure from the subset of reliability states**

The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z2: 0.0012714558169103624.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z2: 0.0013543253766717454.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z2: 0.001453356344808793.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0009352349777881693.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0009742104835872595.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0010185892538833714.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0013469080537178624.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.001434417584272033.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0015389489883673561.

**The pessimistic evaluation of component intensity of departure from the subset of reliability states**

The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z2: 0.0013906547997457087.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z2: 0.0014812933807347216.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z2: 0.001589608502134617.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0010391499753201882.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0010824560928747327.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0011317658376481907.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0013865229964742701.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.0014766063367506223.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0015842121939075725.

– for the components $E_{12}^{(2)}$ and $E_{22}^{(2)}$ of the subsystem $S_2$.

**The maximum likelihood evaluation of component intensity of departure from the subset of reliability states**

The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z2: 0.0014628553354683114.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z2: 0.0015576979749926325.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z2: 0.0016709569615679899.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.001188845527614814.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0012381234355233763.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0012942070168528261.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0015388864775283115.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.0016384489350081922.
The maximum likelihood evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.0017573108637858785.

**The pessimistic evaluation of component intensity of departure from the subset of reliability states**

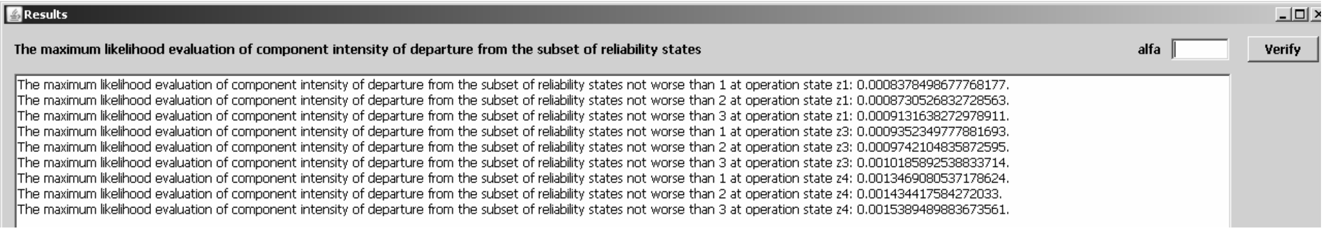
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z2: 0.0015814652275333097.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z2: 0.0016839978108028459.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z2: 0.001806439958451881.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z3: 0.0012922233995813195.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z3: 0.0013457863429601916.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z3: 0.0014067467574487241.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 1 at operation state z4: 0.0015783451051572426.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 2 at operation state z4: 0.0016804604461622484.
The pessimistic evaluation of component intensity of departure from the subset of reliability states not worse than 3 at operation state z4: 0.001802370116703465.

### 4.2.3. Identifying system components exponential reliability functions

As there are data collected from the system components reliability states changing processes, then it is possible to verify the hypotheses on the exponential forms of the system components conditional reliability functions. To this end, we use the procedure given in Section 6.2.2.1 of IS&RDSS 6 [6] and data collected in Section 6.1.

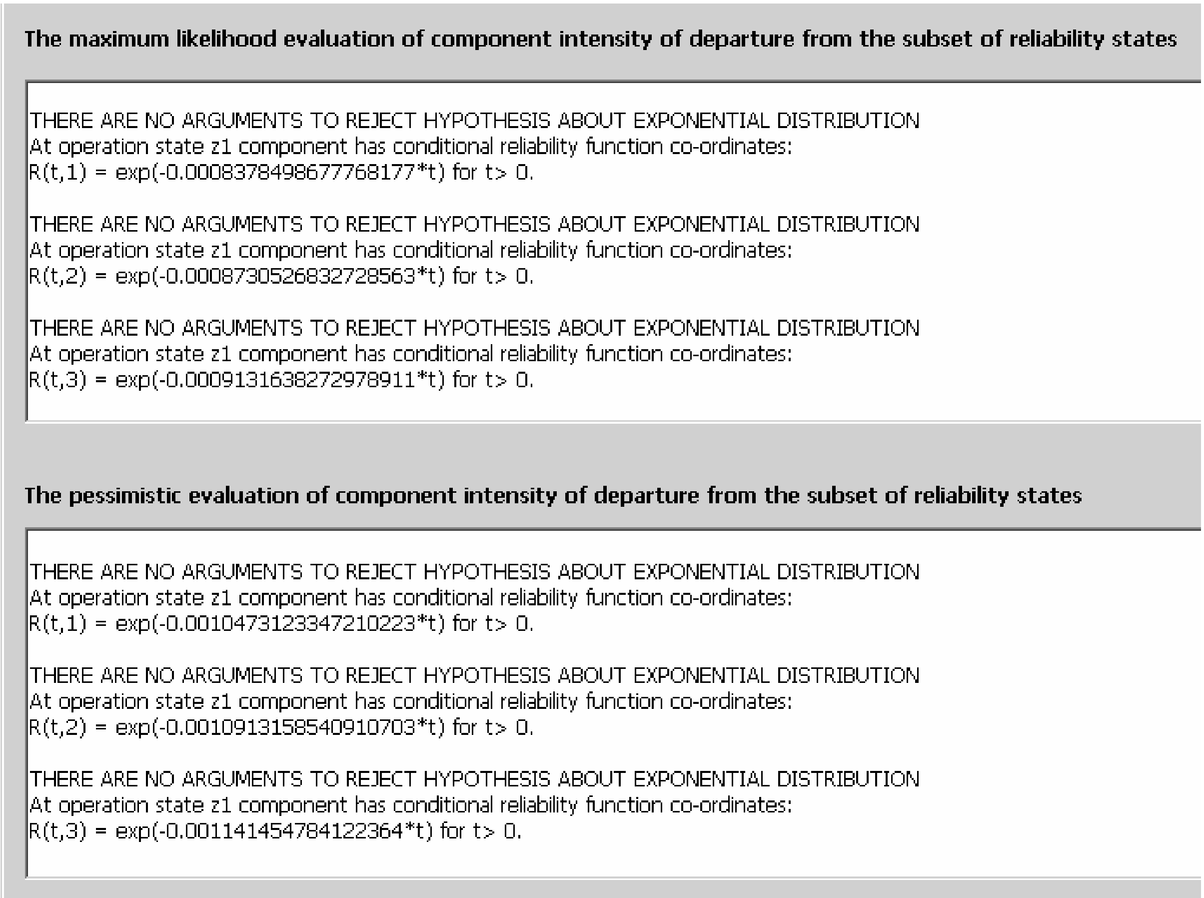
We may verify the hypotheses on the conditional exponential four-state exemplary components reliability functions $[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)}$, $\upsilon = 1,2$, $b = 1,2,3$, at the particular operation states $z_b$, $b = 1,2,3$.

In order to perform this verification we can use the second part of the program CP 8.3 "Reliability models identification of the components". Then the significance level $\alpha$ ($\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test in the field "alfa" should be given and next after pressing the button "Verify" the program verify the hypothesis.



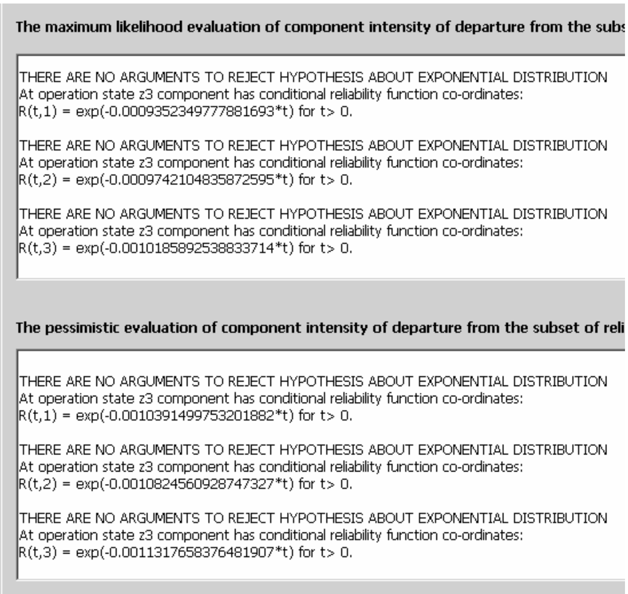

As a result the computer program [4]:

- testifies the hypothesis that the coordinates of the conditional multistate reliability function of the system component are the exponential reliability functions,
- gives the answer whether the hypothesis is rejected or there are no arguments to reject it,
- if there are no arguments to reject the hypothesis about the exponential distribution gives the coordinates of the conditional multistate reliability function of the system component.



In *Case 2*, the computer program also testifies the hypothesis about the conditional multistate reliability function of the system components for the pessimistic evaluation of the intensities of departure.

Below there are presented the results of the verification the hypothesis that coordinates of the conditional four-state reliability function of the system component $E_{11}^{(1)}$ are exponential reliability functions.


The maximum likelihood evaluation of component intensity of departure from the subs

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z3 component has conditional reliability function co-ordinates:
R(t,1) = exp(-0.000935234977881693*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z3 component has conditional reliability function co-ordinates:
R(t,2) = exp(-0.000974210483587259 5*t) for t> 0.

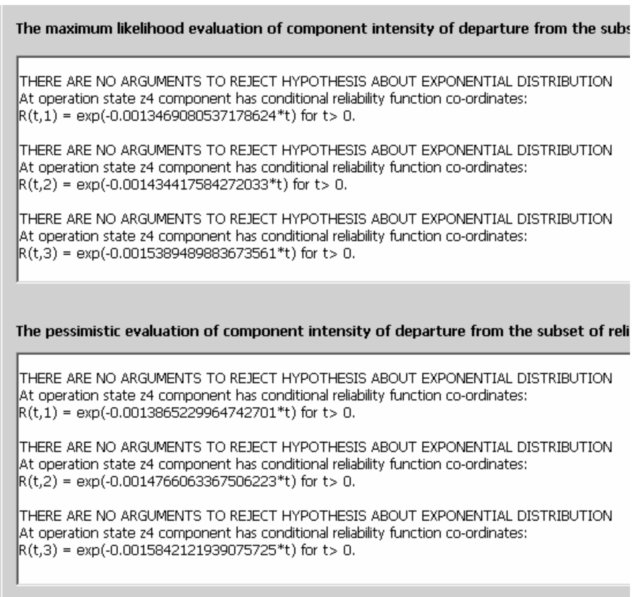
THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z3 component has conditional reliability function co-ordinates:
R(t,3) = exp(-0.00101858925388 33714*t) for t> 0.

The pessimistic evaluation of component intensity of departure from the subset of reli

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z3 component has conditional reliability function co-ordinates:
R(t,1) = exp(-0.00103914997532018 82*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z3 component has conditional reliability function co-ordinates:
R(t,2) = exp(-0.00108245609287473 27*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z3 component has conditional reliability function co-ordinates:
R(t,3) = exp(-0.00113176583764819 07*t) for t> 0.


The maximum likelihood evaluation of component intensity of departure from the subs

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z4 component has conditional reliability function co-ordinates:
R(t,1) = exp(-0.0013469080537178624*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z4 component has conditional reliability function co-ordinates:
R(t,2) = exp(-0.001434417584272033*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z4 component has conditional reliability function co-ordinates:
R(t,3) = exp(-0.0015389489883673561*t) for t> 0.

The pessimistic evaluation of component intensity of departure from the subset of reli

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z4 component has conditional reliability function co-ordinates:
R(t,1) = exp(-0.0013865229964742701*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z4 component has conditional reliability function co-ordinates:
R(t,2) = exp(-0.0014766063367506223*t) for t> 0.

THERE ARE NO ARGUMENTS TO REJECT HYPOTHESIS ABOUT EXPONENTIAL DISTRIBUTION
At operation state z4 component has conditional reliability function co-ordinates:
R(t,3) = exp(-0.0015842121939075725*t) for t> 0.

The hypotheses verification for the remaining components of the subsystems at various operation sates is analogous as for the component $E_{11}^{(1)}$ [7].

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## References

[1] Blokus-Roszkowska, A., Guze, S., Kołowrocki, K., Jurdziński, M., Kwiatuszewska-Sarnecka, B., Milczek, B., Soszyńska, J., Salahuddin Habibullah M. & Fu, X. (2009). Methods of unknown parameters of complex technical systems operation, reliability, availability, safety models evaluation. Task 7.3 in WP7: Integrated package of solutions for complex industrial systems and processes safety and reliability optimization. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[2] Blokus-Roszkowska, A., Guze, S., Kołowrocki, K., Kwiatuszewska-Sarnecka, B., Milczek, B & Soszyńska, J. (2009). Methods of complex technical systems reliability, availability and safety evaluation and prediction. Task 7.2 in WP7: Integrated package of solutions for complex industrial systems and processes safety and reliability optimization. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[3] Blokus-Roszkowka, A., Guze, S., Kołowrocki, K. & Soszyńska, J. (2010). The computer program for identification of the operation processes of complex technical systems. Task 8.1 in WP8: Packages of Tools for Complex Industrial Systems and Processes Safety and Reliability Optimization. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[4] Blokus-Roszkowka, A., Guze, S., Kołowrocki, K. & Soszyńska, J. (2010). The computer program for reliability models identification of the components of complex technical systems. Task 8.3 in WP8: Packages of Tools for Complex Industrial Systems and Processes Safety and Reliability Optimization. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[5] Kołowrocki, K. & Soszyńska, J. (2010). Integrated Safety and Reliability Decision Support System – IS&RDSS. Tasks 10.0-10.15 in WP10: Safety and Reliability Decision Support Systems for Various Maritime and Coastal Transport Sectors. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[6] Kołowrocki, K. & Soszyńska, J. (2009). Methods of complex technical systems operation processes modeling. Task 7.1 in WP7: Integrated package of solutions for complex industrial systems and processes safety and reliability optimization. Poland-Singapore Joint Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[7] Kołowrocki K. & Soszyńska, J. (2010). The exemplary system operation, reliability, risk, availability and cost identification, prediction and optimization - Testing IS&RSS. Task 9.6 in WP9: Applications and Testing of Packages of Tools in Complex Maritime Transportation Systems and Processes Safety and Reliability Optimization. Poland-Singapore Joint Research Project. MSHE Decision No. 63/N-Singapore/2007/0. Gdynia Maritime University.

[8] Kołowrocki, K., Soszyńska-Budny, J. & Ng Kien Ming. (2011). Integrated package of tools supporting decision making on operation, identification, prediction and optimization of complex technical systems reliability and safety. Part 2. IS&RDSS Application – Exemplary system operation and reliability unknown parameters identification. *Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association,* Issue 5, Vol. 2, 373-386.