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# Probabilistic evaluation of deterioration processes with maintenance activities

## Keywords

deterioration modelling, probabilistic methods, maintenance policy, risk assessment.

## Abstract

Reliable operation of contemporary complex systems depends on selecting efficient maintenance policy, which often must take into account not only the reliability, but also economic factors. In this work, we present an approach which allows evaluation of various possible maintenance scenarios with respect to these two areas. The method is based on the concept of a life curve and discounted cost used to study the effect of equipment aging under different maintenance strategies. The deterioration process is first described by a Markov model and then its various characteristics are used to develop the equipment life curve and to quantify other reliability parameters. Based on these data, effects of various "what-if" maintenance scenarios can be examined and their efficiency compared. Simple life curves are combined to model equipment deterioration undergoing diverse maintenance actions, while computing other parameters of the model allows evaluation of additional critical factors, such as probability of equipment failure. Additionally, the paper deals with the problem of the model adjustment so that the computed frequencies are close to the historical values, which is very important in practical applications of the method.

## 1. Introduction

Selection of an efficient maintenance strategy plays a very important role in the management of today's complex systems. When searching for an optimal strategy, numerous issues must be taken into account and, among them, reliability and economic factors are often equally important. On the one hand, for obvious reasons, in successful system operation failures should be avoided and this opts for extensive and frequent maintenance activities. On the other, superfluous maintenance may result in large and unnecessary costs. Finding a reasonable balance between these two factors is the key point in efficient maintenance management and to facilitate finding such a balance some measures should be available that allow for quantitative evaluation of the deterioration process of a system which is subjected to various maintenance actions (inspections, repairs, replacements, etc.).

The purpose of the development described in this work is to provide a computer tool for a person deciding about the maintenance activities, which would help to evaluate both the risks and the costs associated with the selection of various possible maintenance strategies. Rather than searching for a solution to a problem: "what maintenance strategy would lead to the best reliability and dependability parameters of the system operation", in this approach different maintenance scenarios can be examined in the "what-if" type of studies and then, using the tool, their reliability and economic effects can be automatically estimated so that the persons managing the maintenance is assisted in making informed decisions ([10], [17]).

Our method has been presented initially in [4] and its specific extensions were further described in [13], [14] and [15]. In this work, we summarize the current state of the development and include an original study of an application that illustrates potential of this method in practical applications.

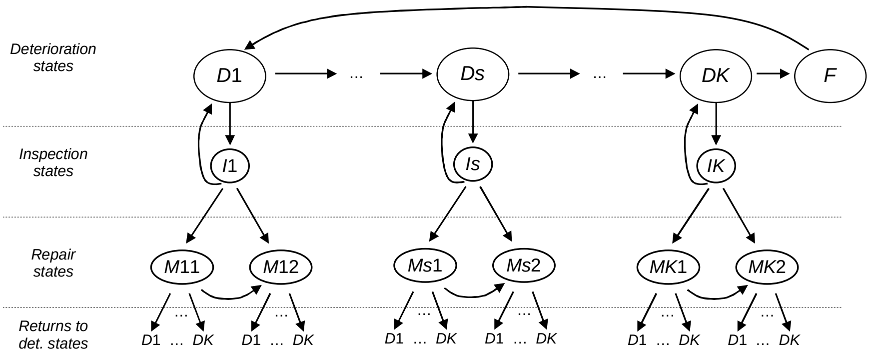


*Figure 1.* Structure of the state-transition model which represents the deterioration process together with inspection and repair events (an example with two types of repairs is shown).

## 2. Modeling deterioration and maintenance

There are three major factors that decide about equipment deterioration: its physical characteristics, operating practices, and the maintenance policy. Of these three aspects, especially the last one relates to the events and actions that should be properly modeled.

The proposed method uses the model that assumes that the equipment will deteriorate in time and, if not maintained, will eventually fail. If the deterioration process is discovered, preventive maintenance is performed which can restore the condition of the equipment. Such a maintenance activity will return the system to a specific state of deterioration, whereas repair after failure will restore to “as new” condition ([1], [9]). The maintenance policy components that must be recognized are: monitoring or inspection (how the equipment state is determined), the decision process (which determines the outcome of the decision), and finally, the maintenance actions (or possible decision outcomes).

In practical circumstances, an important requirement for the determination of the remaining life of the equipment is establishing its current state of deterioration. Even though at the present state of development no perfect diagnostic test exists, monitoring and testing techniques may permit approximate quantitative evaluation of the state of the system. It is assumed that four deterioration states can be identified with reasonable accuracy: (a) normal state, (b) minor deterioration, (c) significant (or major) deterioration, and (d) equipment failure. Furthermore, the state identification is accomplished through the use of scheduled inspections. Decision events generally correspond to the inspection events, but can be triggered by observations acquired through continuous monitoring. The decision process will be affected by what state the equipment is in, and also by the external factors such as economics, current load level of the equipment, its anticipated load level and so on.

### 2.1. Construction of the model

All the necessary assumptions about the aging process and maintenance activities can be incorporated in an appropriate state-space (Markov) model ([6], [7], [8], [11], [12], [16]). It consists of the states the equipment can assume in the process, and the possible transitions between them. In a Markov model, the rates associated with the transitions are assumed to be constant in time.

The method described in this work uses a model of the Asset Maintenance Planner (AMP) ([2], [3]). The AMP model is designed for equipment exposed to deterioration but undergoing maintenance at prescribed times. It computes the probabilities, frequencies and mean durations of the states of such equipment. The basic ideas in the AMP model are the probabilistic representation of the deterioration process through discrete stages, and the provision of a link between deterioration and maintenance. For structure of a typical AMP model see Figure 1.

In the model, the deterioration progress is represented by a chain of *deterioration states D1 … DK* which then leads to the *failure state F*. In most situations, it is sufficient to represent deterioration by three stages: an initial (D1), a minor (D2), and a major (D3) stage ($K = 3$). This last is followed, in due time, by equipment failure (F) which requires extensive repair or replacement.

In order to slow deterioration and thereby extend equipment lifetime, the operator will carry out maintenance according to some pre-defined policy. In the model of *Figure 1*, regular inspections (*I*s) are performed which result in decisions to continue with minor (*Ms*1) or major (*Ms*2) maintenance or do nothing (more than two types of repairs can also be included). The expected result of all maintenance activities is a single-step improvement in the deterioration chain; however, allowances are made for cases where no improvement is achieved or even where some damage is done through human error in carrying out the maintenance, which results in returning to the stage of more advanced deterioration.

The choice probabilities (at transitions from inspection) and the probabilities associated with the various possible outcomes are based on user input and can be estimated, e.g., from historical records or operator expertise.

Mathematically, the model in *Figure 1* can be represented by a semi-Markov process, and solved by the well-known procedures. The solution will yield all the state probabilities, frequencies and mean durations. Another technique, employed for computing the so-called first passage times (FPT) between states, will provide the average times for first reaching any state from any other state. If the end-state is *F*, the FPTs are the mean remaining lifetimes from any of the initiating states.

## 2.2. Using the model to estimate the life curve and the probability of failure

A convenient way to represent the deterioration process is by the *life curve* of the equipment ([1]). Such a curve shows the relationship between asset condition, expressed in either engineering or financial terms, and time. For examples please refer to *Figure 2* in chapter 4 where life curves will be used in a case study presenting various types of analysis carried out for evaluation of the maintenance scenarios.

As pointed out above, computing the average first passage time (FPT) from the first deterioration state (*D1*) to the failure state (*F*) yields an average lifetime of the equipment, i.e., the length of its life curve. On the other hand, solving the model for the state probabilities makes possible computing the expected state durations, which are used to determine the shape of the curve (some additional decisions are required as to how the deterioration states are mapped to ranges of the asset condition values). Simple life curves obtained for different maintenance policies can be later combined if constructing composite life curves which describe various maintenance scenarios are required (*Figure 3* in Chapter 4).

Having the model and the life curve, one can compute the probability of failure (*PoF*) within given time period *T* for the equipment which is in some specific asset condition. The procedure is as follows:

(1) For the current asset condition, find from the life curve the corresponding deterioration state *Ds* and then compute a state progress *SP* (%), i.e. estimate how long the equipment has already been in the *Ds* state.

(2) Running FPT analysis on the model, find the distributions $D_s(t)$ and $D_{s+1}(t)$ of first passage time from the current state *Ds* and the subsequent deterioration state $D(s+1)$, to the failure state *F*.

(3) Interpreting the state progress as a weight which balances the current equipment condition between *Ds* and $D(s+1)$, estimate the final value of the probability as:

$$PoF = D_s(T)\cdot(1 - SP) + D_{s+1}(T)\cdot SP \qquad (1)$$

## 3. Automatic adjustment of the model

Preparing the Markov model for some specific equipment is not an easy task and requires expert intervention. The goal is to create the model representing closely the real-life deterioration process known from the records that usually describe equipment operation under a regular maintenance policy with some specific frequencies of inspections and repairs. The model itself permits calculation of the repair frequencies and compliance of the computed and recorded frequencies is a very desirable feature that verifies its trustworthiness of the model.

In this point, we will describe briefly a method of model adjustment proposed in [13] and [14] that aims at reaching such a compliance. It can be used also for a different task: fully automatic generation of a model for a new maintenance policy with modified frequencies of repairs which is required very often during the evaluation of various maintenance scenarios.

## 3.1. The method

Let *K* represents the number of deterioration states and *R* the number of repairs in the model under consideration. Also, let $\mathrm{P}^{sr}$ = probability of selecting maintenance *r* in state *s* (assigned to decision after state I*s*) and $\mathrm{P}^{s0}$ = probability of returning to state D*s* from inspection I*s* (situation when no maintenance is scheduled as a result of the inspection). Then, for all states $s = 1 \ldots K$:

$$\mathrm{P}^{s0} + \sum_r \mathrm{P}^{sr} = 1 \qquad (2)$$

Let $\mathrm{F}^r$ represents the frequency of repair *r* acquired through solving the model. The problem of model tuning can be formulated as follows:

> Given an initial Markov model $M_0$, constructed as above and producing the frequencies of repairs $\mathbf{F}_0 = \left[\mathrm{F}_0^0, \mathrm{F}_0^1, \ldots \mathrm{F}_0^R\right]$, adjust the probabilities $\mathrm{P}^{sr}$ so that some goal

frequencies $\mathbf{F}_G$ are achieved.

The vector $\mathbf{F}_G$ usually represents the observed historical values of the frequencies of various repairs. In the proposed solution, a sequence of tuned models $M_0$, $M_1$, $M_2$,… $M_N$ is evaluated with each consecutive model approximating desired goal with a better accuracy. The procedure consists of the following steps:

1° For model $M_i$ compute the vector of repair frequencies $\mathbf{F}_i$.

2° Evaluate an error of $M_i$ as a distance between vectors $\mathbf{F}_G$ and $\mathbf{F}_i$.

3° If the error is within the user-defined limit, consider $M_i$ as the final model and stop the procedure ($N = i$); otherwise proceed to the next step.

4° Create a new model $M_{i+1}$ through tuning values of $P_i^{sr}$, then correct $P_i^{s0}$ according to (2).

5° Proceed to step 1° with the next iteration.

The error computed in step 2° can be expressed in may ways. As the frequencies of repairs may vary in a broad range within one vector $\mathbf{F}_i$, yet the values of all are significant in model interpretation, the relative measures work best in practice:

$$\left\|\mathbf{F}_G - \mathbf{F}_i\right\| = \frac{1}{R}\sum_{r=1}^{R}\left|F_i^r / F_G^r - 1\right| \tag{3}$$

or

$$\left\|\mathbf{F}_G - \mathbf{F}_i\right\| = \max_r \left|F_i^r / F_G^r - 1\right| \tag{4}$$

The latter formula is more restrictive and it was used in the numerical implementation of the method.

## 3.1. Approximation of the model probabilities

Of all the steps outlined in the previous section, it is clear that adjusting probabilities $P_i^{sr}$ in step 4° is the heart of the whole procedure.

In general, the probabilities represent $K{\cdot}R$ free parameters and their uncontrolled modification could lead to serious deformation of the model. To avoid this, a restrictive assumption is made: if the probability of some particular maintenance must be modified, it is modified proportionally in all deterioration states, so that at all times

$$P_0^{1r} : P_0^{2r} : \ldots : P_0^{Kr} \;\sim\; P_i^{1r} : P_i^{2r} : \ldots : P_i^{Kr} \tag{5}$$

for all repairs ($r = 1\ldots R$).

This assumption also significantly reduces dimensionality of the problem, as now only $R$ scaling factors $\mathbf{X}_{i+1}=[X_{i+1}^1, X_{i+1}^2, \ldots X_{i+1}^R]$ must be found to get all new probabilities for the model $M_{i+1}$:

$$P_{i+1}^{sr} = X_{i+1}^r \cdot P_0^{sr}, \quad r = 1\ldots R, \; s = 1\ldots K \tag{6}$$

Moreover, although the frequency of a repair $r$ depends on the probabilities of all repairs (modifying probability of one repair changes, among others, state durations in the whole model; thus, it changes the frequency of all states) it can be assumed that, in a case of a single-step small adjustment, its dependence on repairs other than $r$ can be considered negligible and

$$F_i^r = F_i^r\left(X_i^1, X_i^2 \ldots X_i^R\right) \approx F_i^r\left(X_i^r\right) \tag{7}$$

With these assumptions, generation of a new model is reduced to the problem of solving $R$ non-linear equations in the form of $F_i^r\left(X_i^r\right) = F_G^r$. This can be accomplished with one of the standard root-finding algorithms.

Development described in this work has been implemented and verified on practical examples with the following three approximation algorithms: Newton method working on a linear approximation of $F_i^r()$, the secant method and the false position (*falsi*) method.

(A) Newton method on linear approximation (NOLA)

In this solution, it is assumed that $F_i^r()$ is a linear function defined by points $F_i^r(X_i)$ (obtained after solving the model in step 1°) and $F_i^r(0)$ (which is always zero). Then simply

$$X_{i+1}^r = F_G^r / F_i^r \tag{8}$$

It should be noted that important advantage of this approach lies in the fact that no other point than the current frequency $F_i^r(X_i)$ is required to compute the next approximation, so errors of the previous steps do not accumulate and the convergence is good from the first iteration.

(B) The secant method

In this standard technique, the function is approximated by the secant defined by the last two approximations in points $X_{i-1}^r$, $X_i^r$ and a new one is computed as:

$$X_{i+1}^r = X_i^r - \frac{X_i^r - X_{i-1}^r}{F_i^r - F_{i-1}^r}\left(F_i^r - F_G^r\right) \tag{9}$$

After that, $X_{i-1}^r$ is discarded and $X_{i+1}^r$ and $X_i^r$ are considered in the next iteration.
To begin the procedure two initial points are needed. In this method, the first point is chosen as the initial frequency of the model $M_0$ ($X_0^r$=1), while the second point is computed as in NOLA method above: $X_1^r = F_G^r / F_0^r$.

(C) The false position (*falsi*) method

In this approach $X_{i+1}^r$ is computed as in (9) but the difference lies in choosing the points for the next iteration. While in (B) $X_{i-1}^r$ is always dropped, now $X_{i+1}^r$ is paired with that one of $X_i^r$, $X_{i-1}^r$ which lies on the opposite side of the root. In this way, when (9) is applied, the solution is bracketed between $X_i^r$ and $X_{i-1}^r$ (which is the essence of the *falsi* method).
As in (B), the two initial points are needed but now they must lie on both sides of the root, i.e.

$$\left(F_0^r - F_G^r\right)\cdot\left(F_1^r - F_G^r\right) < 0 \tag{10}$$

Choosing such points may pose some difficulty. To avoid multiple sampling, it is proposed to select $X_0^r$ = 1 (as previously) and then to compute $X_1^r$ like in NOLA method but with some "overshoot" that would guarantee (10):

$$X_1^r = \left(F_G^r / F_0^r\right)^{\alpha} \tag{11}$$

with parameter $\alpha$ > 1 controlling the overshoot effect. The overshot must be sufficient to ensure (10) but, on the other hand, should not produce too much of an error as this would deteriorate approximation process during the initial steps and would produce extra iterations of the method. If (10) is not satisfied by the initial value of $X_1^r$, (11) can be re-applied with an increased value of $\alpha$, although it should be noted that each such correction requires solving a new $M_1$ model and in effect this is the extra computational cost almost equal to that of the whole iteration.

## 3.3. Comparison of the approximation methods

For a detailed evaluation of the three proposed approximation methods please refer to [13] and [14]. It is shown that although simplifications of the NOLA solution may seem critical, in practice it works quite well. As it was noted above, this method has one advantage over its more sophisticated rivals: since it does not depend on previous approximations, selection of the starting point is not so important and the accuracy during the first iterations is often better than in the secant or *falsi* methods. Superiority of the latter methods, especially of the *falsi* algorithm, manifests itself in the later stages of the approximation when the potential problems with an initial selection of the starting points have been diminished.
Another important issue is how the adjustment modifies behavior of the Markov model in addition to reaching the desired repair frequencies and how the model should be constructed in order to accommodate the modifications without undesired side effects. For discussion of this topic please refer to [15].

## 4. Evaluating reliability and cost for different maintenance strategies

The methodology presented above will be illustrated by an example of some specific piece of equipment with a model which has been created and fine-tuned so that it represents the actual reliability and maintenance parameters found in the authentic historical records. According to them, the average equipment life has been found to reach 18.7 years of operation before failure. The model includes three deterioration states and represents the default maintenance policy with three possible repair types corresponding to, respectively, minor, medium and major repairs.

## 4.1. Life curves

*Figure 2* presents life curves computed for this equipment with various repair policies. The rightmost one represents the standard (historical) policy with all three repairs implemented with their typical frequencies, while the leftmost one – corresponding to the average equipment life of approx. 10 years – has been created from the model with all repairs removed (so called “do nothing” policy). As it is shown, in this specific case, turning off all the maintenance actions results in shortening of the equipment life by 46% and this fact can be compared to expected economic savings. The other three curves represent the following mid-range scenarios which were selected in this work as typical examples of the solutions that may be considered in the real-world applications:
- turning off the major repair without changing the frequencies of the remaining two ones (minor and medium), which has been evaluated to reduce the average equipment life to 14.7 years (i.e. by 21%),
- keeping only the medium repair with minor and

major ones removed (equipment life reduced by 28% to 13.4 years),
- reducing by half the frequencies of all three repairs (equipment life reduced by 40% to 11.3 years).

It should be stressed that in the three mid-range cases the curves have been computed using models that were tuned to required repair frequencies with the numerical procedure described in the previous section.

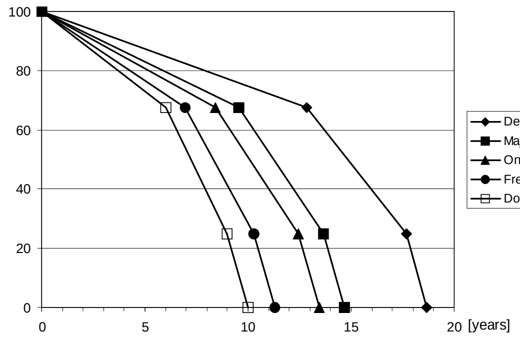


*Figure 2.* Life curves for equipment with different repair policies.

Having such models not only the shape and length of the curves can be evaluated, but also other significant reliability characteristics, with the probability of failure within the specific time horizon being one of the most important ones in further analysis.

## 4.2. Maintenance scenarios

The models and the life curves for different repair policies can be used in evaluation of various maintenance scenarios. As examples, we will consider a situation when, with initial equipment deterioration estimated as 80% of "as new" condition, some specific actions – a repair or just a change in maintenance policy – will take place after a 3 year delay while the effects will be evaluated for a 10 year time period. The actions in the scenarios will be as follows:
- adopting "do nothing" policy, which means just stopping all inspections and repairs; in case of failure the equipment will be repaired and its condition restored to 85%,
- replacing the equipment with "as new" one and then switching to the "do nothing" policy,
- performing a major refurbishment of the equipment which restores its condition to 85% and then continuing with a medium repair only.

*Figure 3* shows the composite life curves created over a period of 10 years for the above scenarios and compares them to the "continue as before" policy. The composite curves were constructed with the appropriate segments of the basic curves form Figure 2. Starting form the initial asset condition of 80% of the initial asset value, which corresponds to the equipment ca. 8 year old, the curves run down to 72% during the first three years and then split at the moment of the action. For the "do nothing" action

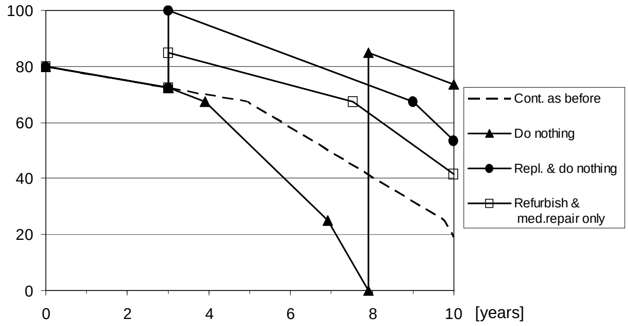


*Figure 3.* Life curves for different maintenance scenarios over a time horizon of 10 years.

deterioration rate speeds up, while for the two other actions the asset condition is first increased as a result of the replacement or refurbishment and, then, a new reduced repair policy is applied, which again causes a higher rate of deterioration. The shapes of the curves make possible a quantitative comparison of these processes and allow evaluation of their effects.

It can be noted that, in the case of "do nothing" action, it is predicted that the equipment will fail within the time horizon under consideration. While in such a case, different actions (repairs or replacements) may take place, in this specific scenario it is assumed that the equipment will be repaired with its condition restored to 85%,, but other courses of action can also be modeled.

## 4.3. Probability of failure

Probability of failure within the time horizon computed for the strategies under consideration is

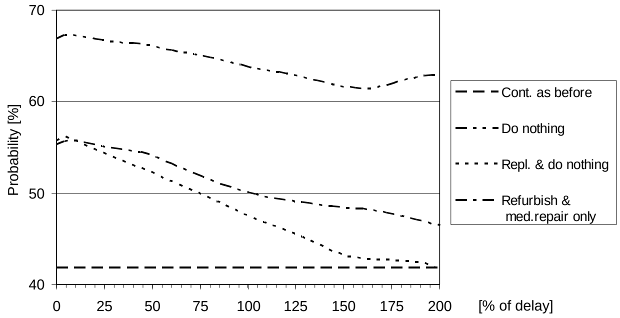


*Figure 4.* Probability of equipment failure within a period of 10 years as a function of action delay.

shown in *Figure 4.* Values on the graphs are presented as functions of the action delay time (100% = 3 years) and they are compared against the probability of failure for the unmodified standard maintenance ("continue as before"). The value of this probability has been computed to be 42%.

It can be seen in case of all three scenarios that, since the new maintenance policy after the action is more or less reduced, the more the action is delayed, the less probable equipment failure becomes. For evident reasons adopting "do nothing" policy leads to the highest values of the failure probability, while replacing the equipment and "doing nothing" afterwards turned out to be a less dangerous strategy (in terms of failure probability) than refurbishing and then keeping only the medium repair. Whether the differences in the economic expenses of these two possible strategies justify this discrepancy in the reliability parameter or not – remains an open question in further cost analysis and generally depends on the costs associated with the equipment failures.

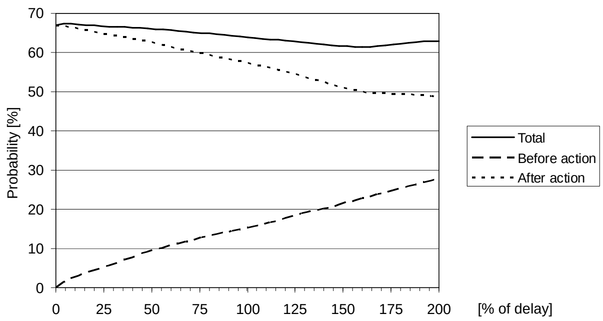


*Figure 5.* Probability of equipment failure before and after the action for "do nothing" scenario.

One interesting observation can be made about the curve for "do nothing" strategy: its decrease is not strictly monotonic and there is a local minimum at the level of 61% for the delay equal to 164% (4.9 years) after which the probability begins to rise slowly. To explain this rise, the two components: the probability of failure before and after the action should be investigated and they are shown in *Figure 5*. In general, these two components behave as expected: the later the action takes place, the higher the probability of failure before and the lower probability of failure after the action but the rates of these two flows – increasing and decreasing – are not constant and do not sum up into a monotonic decrease. In this case, the probability of failure after the action falls down to some extent slower after the point of 164% and this causes the local minimum in the total probability of failure.

## 4.4. Cost analysis

In financial evaluations the costs are expressed as present value (PV) quantities and this approach should also be used in this kind of studies because maintenance decisions on aging equipment include timing, and the time value of money is an important consideration in any decision analysis. The cost difference is often referred to as the Net Present Value (NPV). In the case of maintenance, the NPV can be obtained for several re-investment options which are compared with the "Continue as before" policy.

Cost evaluation for any maintenance scenario involves calculation of the following three fundamental classes of components:

1. cost of the maintenance activities,
2. cost of the selected action (i.e. refurbishment or replacement),
3. cost associated with failures (cost of repairs, system cost, penalties).

To compute the PV, inflation and discount rates are required for the specified time horizon. The cost of maintenance over the time horizon is the sum of the maintenance costs incurred by the original maintenance policy for the duration of the delay period (up to the action), and the costs incurred by the new policy for the remainder of the time horizon (after the action). The costs associated with the equipment failure over the time horizon can be computed similarly except that the failure costs before and after the action should be multiplied by the respective probabilities of failures, and the two products added.

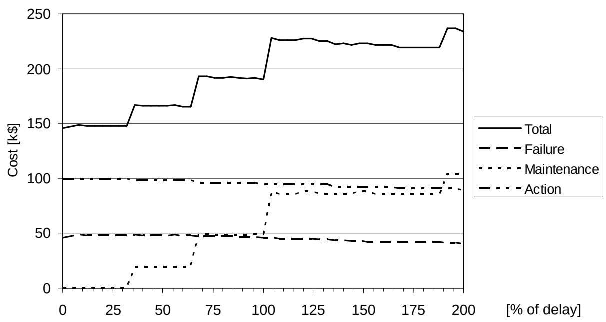


*Figure 6.* Estimated cost of "replace & do nothing" scenario (total value and the three components).

*Figure 6* presents the plots showing the cost analysis for the scenario "replace and then do nothing". Again (as it was in the case of probability of failure) the values are visualized as functions of the action delay varying in the range 0 ÷ 200% of user-specified reference value. The cost of replacement ("Action"), although does not depend on the delay, is not constant on the plot due to the PV calculations. It is also evident that delaying the action causes more repairs to be performed as elements of the present repair policy before "do nothing" becomes effective, hence several noticeable leaps appear in the maintenance cost flow.

## 5. Conclusion

The purpose of the method presented in this paper is to help the maintenance supervisor in choosing an effective yet cost-efficient maintenance policy. Based on the Markov models representing deterioration process, the equipment life curve and other reliability parameters can be evaluated. Once a database of equipment models is prepared, the end-user can perform various studies with different maintenance strategies and compare expected outcomes. As the results are visualized through the relatively simple concept of a life curve, no detailed expert knowledge about internal reliability parameters or configuration is required.

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