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Recent soft computing methods in software reliability engineering

Keywords

fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy numbers, subjective data analysis

Abstract

This paper presents recent approaches in soft computing to manage imprecision and uncertainty which appear in software reliability engineering. Firstly, recent approaches like imprecise probabilities, generalized intervals, fuzzy sets and intuitionistic-fuzzy sets are shortly described, and the usage of intuitionistic fuzzy numbers for system reliability computation is shown. Intuitionistic-fuzzy approaches for software reliability growth models are proposed and experimental results are given.

1. Introduction

Recent definitions of imprecision include aspects related to vagueness ("vague; indistinct; not perfectly apprehended") and chance dependency ("dependent on chance or unpredictable factors; doubtful; of unforeseeable outcome or effect"). According to [20], [21], the approximate reasoning is used to manage those situations when experts use vague concepts for the evaluation, observation and decision on a system evolution based on different models likes: arithmetic intervals, fuzzy numbers, intuitionistic fuzzy numbers, fuzzy logic, and fuzzy devices. In order to deal with uncertainty, probabilities are attached with objects under manipulation (probabilistic trees, probabilistic networks, probabilistic generative mechanisms, probabilistic thinking). In large, probabilistic reasoning [13], [26] and fuzzy logic [32], [33], [34] were identified as possible approaches. If subjective probabilities, fuzzy sets/logic, neural networks, evolutionary computing and hybrid approach, such a framework is called *soft computing* based [20], [21].
The mentioned references proposed a soft computing framework for software reliability engineering. Software reliability modelling and prediction in soft computing environments are considered also by [15] and [16]. This paper continues the developments considering the new approaches based on intuitionistic fuzzy sets and numbers.
The next section is dedicated to various models useful for decision making under incomplete information: imprecise probabilities, generalized intervals, fuzzy sets, vague sets, and intuitionistic-fuzzy sets.
The usage of triangular intuitionistic-fuzzy numbers in system reliability computing is described in the third section. Similar computation will be necessary for trapezoidal intuitionistic-fuzzy numbers.
The fourth section proposes some methodologies for intuitionistic-fuzzy ranking of the software reliability growth models. The methodologies consider single and multi-expert models, consensus establishing and distance-based classification.
Experimental results on the applicability of the proposed approaches are described in the fifth section, and concluding remarks are presented in the end.

2. Recent approaches in imprecision and uncertainty modelling

The uncertainty appears when the knowledge is incomplete (missing pieces of knowledge, low plausibility, wrong or incomplete hypothesis). Using statistical approach this track was followed in [24], [28]. A priori and a posteriori probabilities are used to derive distribution functions based on minimum information principle.

Interval representation provides a way to consider the crisp or fuzzy membership. However, the interval interpretation can be different. One interpretation of intervals is based on imprecise probabilities [9], [10], [29], [34]. According to [10], "imprecise probability is used as a generic term to cover all mathematical models which measure chance or uncertainty without sharp numerical probabilities". From this point of view researchers already identified various models: comparative probability orderings, interval-valued probabilities (given by upper and lower probabilities), fuzzy measures, possibility measures, etc. From a mathematical point of view, all the models listed above are equivalent to special kinds of upper or lower previsions.

Another interpretation generates approaches based on possibility and necessity degrees. As Dubois [12] mentioned, "assigning an interval [a; b] to a quantity x", not necessarily random (the real value of x is can be precise, but unknown), "means that x is known to take one and only one value in [a; b], but it is not known which one." The size of the interval should be small in order to be more informative. The *possibilistic* interpretation asserts that x in [a; b] describes that any value outside [a; b] is impossible for x. Both possibility and necessity degrees can be used under this interpretation. According to [12], the "possibility degree of an event expresses the extent to which this event is plausible", and the "necessity degrees express the certainty of events".

Computing with intervals is used to deal with uncertainty by the interval analysis method, a method developed by mathematicians since the 1950s as an approach to give bounds on rounding errors and measurement errors in mathematical computation. After the computational framework was established various numerical methods that yield reliable results were designed and implemented in software. For reliability analysis tasks, the generalized intervals [30] were considered.

An alternative to the interval approach is the usage of fuzzy numbers, which are special cases of fuzzy sets. In this case, an interval is given to describe a real number by means of a membership function.

A fuzzy set (firstly introduced by Zadeh [33]) is called a fuzzy number if its membership function increases monotonously to a unique maximum degree equal to 1 and then decreases monotonously. Fuzzy numbers can be used to introduce fuzzy probabilities. If X is a discrete random variable with *n* realization $X_1, X_2, \ldots, X_n$, then $A_1, A_2, \ldots, A_n$ which are fuzzy numbers having zero degree membership outside of the interval [0, 1], and unique maximum degree $\varphi_{Ai}(p_i) = 1$, for some p_i belonging to [0, 1], with the sum of all p_i ($i = 1, 2, \ldots, n$) being unity defines the fuzzy probabilities $A_1, A_2, \ldots, A_n$. In the case of intuitionistic-fuzzy numbers, both a membership and a non-membership function are given. Generalized fuzzy numbers are also used for imprecise modelling [1, 5, 22].

In fuzzy computing with intuitionistic fuzzy numbers [5, 22], the most used membership (and non-membership) functions have triangular or trapezoidal shape. The output is an interval, a membership (and a non-membership) function, the value being obtained by a defuzzyfication procedure. If *A* is a TIFN (Triangular Intuitionistic Fuzzy Number) then *A* is described by five real numbers $(a_1, a_2, a_3; a', a'')$, $a' \le a_1 \le a_2 \le a_3 \le a''$, and two triangular functions. The first one – the membership function – is given by:

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, \text{for } a_2 \le x \le a_3 \\ 0, \text{otherwise} \end{cases}.$$

The second one – the non-membership function – has the following definition:

$$\nu_A(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a'}, \text{for } a' \le x \le a_2 \\ \dfrac{x - a_2}{a'' - a_2}, \text{for } a_2 \le x \le a'' \\ 1, \text{otherwise} \end{cases}.$$

Let be valid the relation: $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4'$. A Trapezoidal Intuitionistic Fuzzy Number A in $\boldsymbol{R}$ (TrIFN), written as $(a_1, a_2, a_3, a_4; a_1', a_2, a_3, a_4')$, has the membership function

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, \text{for } a_1 \le x \le a_2 \\ 1, \text{for } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, \text{for } a_3 \le x \le a_4 \\ 0, \text{otherwise} \end{cases},$$

and the non-membership function

$$\nu_A(x) = \begin{cases} \frac{a_2 - x}{a_2 - a_1^{'}}, \text{for } a_1^{'} \le x \le a_2 \\ 0, \text{for } a_2 \le x \le a_3 \\ \frac{x - a_3}{a_4^{'} - a_3}, \text{for } a_3 \le x \le a_4^{'} \\ 1, \text{otherwise.} \end{cases}$$

The concept of probability based on frequency can be used to model some kind of uncertainty only for the case of large number of repeatable circumstances [26]. One extension of classical probability considers subjective probabilities containing no formal calculations and reflecting only the subject's opinions (based on past experience), and embedding a high degree of personal bias. Other extensions are based on imprecise probabilities [9], [10], [11], [29], [30], [34].

Let us consider the following inference problem. Given a serial system S having two components C_1 and C_2, α the probability of C_1 to fail before m hours of working time, and the mean-time-to failure of C_2 belongs to the interval [n, p] (hours), is asked to infer γ the probability of S to fail after q hours of working time, where q is greater than maximum of m, n, and p. The solution can be obtained on the base of interval approaches.

The uncertainty is modelled by a family F of probability distributions, and lower and upper probability bounds are defined by: $P\_(A)$ = *inf*{(P(A), P in F}, $P_+(A)$ = *sup*{P(A), P in F}, with $P\_(A) = 1-P_+(C_A)$, where C_A is the complement of A. This kind of model can be extended to the P-box model. A p-box is defined by a pair of cumulative distributions (F_L, F_U) on the real line such that $F_L \le F_U$, bounding the cumulative distribution of an imprecisely known probability function with density p. According to [12], “A p-box is a covering approximation of a parameterized probability model whose parameters (like mean and variance) are only known to belong to an interval.” This approach is different from confidence bands obtained by various methods, including bootstrap [4].

Imprecise reliability is still at stage of development; even some research was done according to [9], and [29].

A successful approach uses generalized intervals [30]. A generalized interval x := [x_1; x_2] (x_1, x_2 in R) is determined by a pair of real numbers x_1 and x_2; x is *proper* when $x_1 \le x_2$ and is *improper* when $x_2 \le x_1$. When $x_1 = x_2$, x is a *pointwise* interval.

Generalized interval calculus is based on the Kaucher’s arithmetic [30]. Given a generalized interval x = [x_1, x_2], two operators *pro* and *imp* return *proper* and *improper* values, defined by *pro*(x) := [min(x_1, x_2); max(x_1, x_2)] and *imp*(x) := [max(x_1, x_2); min(x_1, x_2)] respectively. The relationship between proper and improper intervals is established by duality with the operator *dual* as *dual*(x) := [x_2, x_1].

Given a sample space Ω and algebra of random events over Ω, it is possible to define the generalized interval probability which obeys the axioms of Kolmogorov, reconsidered if interval probabilities are used. An interval probability p = [p_1, p_2] is a generalized interval without the restriction of $p_1 \le p_2$. The probability of the union of two events E_1 and E_2 is defined as $P(E_1 \cup E_1) = p(E_1) + p(E_2) - \text{dual}(p(E_1 \cap E_2))$, where x+y is defined by x+y = [x_1, x_2]+[y_1, y_2] = [x_1+y_1, x_2+y_2], x-y = [x_1-y_2, x_2-y_1], x.y and x/y being defined by special rules depending on the sign of the parts. The inclusion relationship between generalized intervals (and the binary relation ≤) corresponds to the geometrical segment inclusion. The imprecise model based on interval probabilities is based on [3, 9, 10, 29, 30]:

- $P(C_A)$ = 1 – dual (p(A)); equivalent with $p_1(C_A) = 1 - p_2(A)$, and $p_2(C_A) = 1 - p_1(A)$;
- For a mutually disjoint event partition the sum of interval probabilities is equal to unity.
- The Bayes' rule with generalized intervals is defined as

  $$P(E_i / A) = \frac{p(A / E_i) p(E_i)}{\sum_{j=1}^{n} dualp(A / E_j) dualp(E_j)},$$

  where E_i are mutually disjoint partition of the set Ω.

In this way, the reliability formulas developed in the classical framework will be applied taking into account the new operators. However, a final interpretation is required similar to defuzzification approach used during the application of the fuzzy, vague, or fuzzy-intuitionistic models.

Vague modelling [7, 8, 17] is considered in the following and the fuzzy-intuitionistic computing methodology is shown. When the universe of discourse X is a non empty and finite set, a vague set A of the universe of discourse U can be represented by a true-membership function t_A and a false-membership function f_A. A vague number is a vague subset in the universe of discourse X that is both normal (the maximum value of the true membership function is 1) and convex (similar to fuzzy convex sets).

Vague calculus uses triangular vague sets, trapezoidal vague sets, and general vague sets. As an example, if the triangular vague sets are used (A = (a_1, a_2, a_3; t_A, f_A), B = (b_1, b_2, b_3; t_B, f_B)) then:

$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3; t_{A+B}, f_{A+B})$;
$A - B = (a_1-b_3, a_2-b_2, a_3-b_1; t_{A-B}, f_{A-B})$;
$A * B = (a_1b_1, a_2b_2, a_3b_3; t_{AB}, f_{AB})$,

and
A / B = (a_1/b_3, a_2/b_2, a_3/b_1; $t_{A/B}$, $f_{A/B}$), with functions *t* and *f* defined correspondingly.

Based on vague calculus the system reliability can be obtained for all kind of systems: series, parallel, and hybrid.
The operations on IFS (Intuitionistic Fuzzy Sets) can be introduced according to the generalized fuzzy set theory. Some examples follow [2, 22]:

$A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))), x \in X\};$
$A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))), x \in X\};$
$A + B = \{(x, \mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)), x \in X\};$
$AB = \{(x, \mu_A(x)\mu_B(x), \nu_A(x)+\nu_B(x)-\nu_A(x)\nu_B(x)), x \in X\};$

The arithmetic operation, denoted generically by *, of two IFNs (Intuitionistic Fuzzy Numbers) is a mapping of an input subset of ***R**x**R*** (with elements *x* = (x_1, x_2)) onto an output subset of ***R*** (with elements denoted by *y*). Let A_1 and A_2 be two IFN, and $(A_1 * A_2)$ the resultant of the operation *. Then:

$$(A_1 * A_2)(y) = \left\{ \begin{pmatrix} y, \\ \vee_{y=x_1*x_2} [A_1(x_1) \wedge A_2(x_2)], \\ \wedge_{y=x_1*x_2} [A_1(x_1) \vee A_2(x_2)] \end{pmatrix}^T \forall x_1, x_2, y \in R \right\},$$

with

$$\mu_{(A_1*A_2)}(y) = \vee_{y=x_1*x_2} [A_1(x_1) \wedge A_2(x_2)],$$

and

$$\nu_{(A_1*A_2)}(y) = \wedge_{y=x_1*x_2} [A_1(x_1) \vee A_2(x_2)].$$

The arithmetic operations on IFNs can be defined using the (α, β) – cuts method. Let $\alpha, \beta \in [0, 1]$ be fixed numbers such that $\alpha + \beta \leq 1$. A set of (α, β) - cut generated by an IFS A is defined by: $A_{\alpha,\beta} = \{(x, \mu_A(x), \nu_A(x)), x \in X, \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$.
The (α, β) – cut of a TIFN is given by $A_{\alpha,\beta} = \left\{ [A_1(\alpha), A_2(\alpha)], [A_1'(\beta), A_2'(\beta)] \right\}$, where:

- $A_1(\alpha)$, and $A_2'(\beta)$ are continuous, monotonic increasing functions of α, respective β;
- $A_2(\alpha)$, and $A_1'(\beta)$ are continuous, monotonic decreasing functions of α, respective β;
- $A_1(1) = A_2(1)$, and $A_1'(0) = A_2'(0)$.

When using the 5-tuple notation, we obtain:

$$A_1(\alpha) = a_1 + \alpha(a_2 - a_1),$$

$$A_2(\alpha) = a_3 - \alpha(a_3 - a_2),$$
$$A_1'(\beta) = a_2 - \beta(a_2 - a'),$$

and

$$A_2'(\beta) = a_2 + \beta(a'' - a_2).$$

To fulfill the aim of this paper, the following properties are necessary [22]:

1. If TIFN *A* = $(a_1, a_2, a_3; a', a'')$, and $k > 0$, then the TIFN *kA* is given by $(ka_1, ka_2, ka_3; ka', ka'')$.
2. If TIFN *A* = $(a_1, a_2, a_3; a', a'')$, and $k > 0$, then the TIFN *kA* is given by $(ka_3, ka_2, ka_1; ka'', ka')$.
3. If $A = (a_1, a_2, a_3; a', a'')$ and $B = (b_1, b_2, b_3; b', b'')$ are TIFNs, then the TIFN $A \oplus B$ is defined by $(a_1+b_1, a_2+b_2, a_3+b_3; a'+b', a''+b'')$;
4. If $A = (a_1, a_2, a_3; a', a'')$ and $B = (b_1, b_2, b_3; b', b'')$ are TIFNs, then the TIFN $A \otimes B$ is defined by $(a_1b_1, a_2b_2, a_3b_3; a'b', a''b'')$.

The above results can be proved using the (α, β) – cuts method.

3. Intuitionistic-fuzzy reliability modelling

As a general rule, systems (hardware, software, distributed, embedded etc.) are composed by various components connected according to some architecture depending on the requirements, environment and the new design/development paradigms. In the following, different aspects on reliability engineering will be considered when soft computing approaches are used. We are using system reliability as a consequence of the cybernetic nature of systems based on both hardware and software.

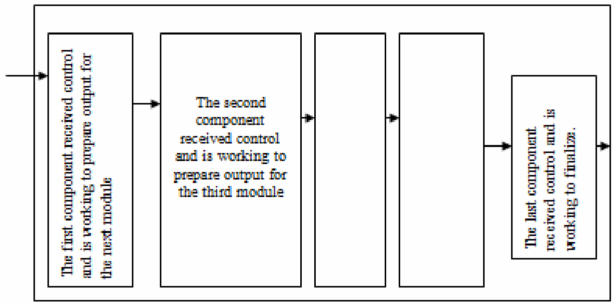

Figure 1. A serial system

If S is a system integrating a number of components (off-the-shelf components) according to a serial architecture (the above diagram) and if R_j is the intuitionistic fuzzy reliability of the j^{th} component, and R_S is the intuitionisc fuzzy reliability of the entire system (with *n* items), and $R_j = (r_{j1}, r_{j2}, r_{j3}; r_j', r_j'')$, then

$$R_S = R_1 \otimes R_2 \otimes \cdots \otimes R_n,$$

being defined by $(r_1, r_2, r_3; r', r'')$, with:

$$r_i = \prod_{j=1}^{n} r_{ji},\ i = 1, 2, 3,\ r' = \prod_{j=1}^{n} r_j',\ \text{and}\ r'' = \prod_{j=1}^{n} r_j''.$$

If S is a system composed by n items running in parallel, using the above notations, the intuitionistic fuzzy reliability of S is given by [22]:

$$R_S = 1 \Theta \prod_{j=1}^{n} (1 \Theta R_j),$$

and is defined by $(r_1, r_2, r_3; r', r'')$, with: $r_i = 1 - \prod_{j=1}^{n} (1 - r_{ji})$, $i = 1, 2, 3$, $r' = 1 - \prod_{j=1}^{n} (1 - r'_j)$, and $r'' = 1 - \prod_{j=1}^{n} (1 - r''_j)$.

Other soft computing approaches were proposed in [15], [16]. Optimal software reliability allocation using intuitionistic-fuzzy sets is described in [2].

Using the above methodology intuitionistic-fuzzy reliability formulas can be derived for hybrid architectures like neural architectures for evolutionary computing systems.

4. An intuitionistic-fuzzy method for ranking software reliability growth models

According to Musa [23], "software reliability engineering is based on a solid body of theory that includes operational profiles, random process software reliability models, statistical estimation, and sequential sampling theory" consisting of five activities: "define <<necessary reliability>>, develop operational profiles, prepare for test, execute test, and apply failure data to guide decisions."

Also, software reliability engineering includes effects of product and development process metrics and factors (on operational software behaviour. Finally, the software reliability engineering provides guidelines for software development, acquision, use, and maintenance [18], [19].

Being defined as a probability, the reliability analyse uses uncertainty models. However, sometimes, the data cannot be measured and recorded precisely. In this case imprecise models are necessary to be used. The existence of fuzziness is modelled by membership functions, respective membership and non-membership functions when intuitionistic-fuzzy approach is used. Cai [6] identifies fuzzy success states and fuzzy failure states of a system under operation. Wu [31] considers the systems' fuzzy reliability estimation using Bayesian approach making use of fuzzy random variables. Nonlinear models for the evaluation of the reliability using fuzzy sets are described in [14]. A fuzzy-probabilistic approach was described in [25].

In the following, we describe an intuitionistic-fuzzy approach for ranking software reliability growth models (SRGM) from a nonempty set of alternatives $\{A_1, A_2, \ldots, A_m\}$.

Two types of analysis are considered. For the first one, one expert has to select the most appropriate SRGM to be used during failure-data analysis. The second case addresses the existence of *p* experts $\{E_1, E_2, \ldots, E_p\}$ evaluating every SRGM used during data analysis.

The selection is based on a nonempty set of criteria/attributes $\{C_1, C_2, \ldots, C_n\}$ taking into consideration weights indicating the importance (priority) of every criterion. Linguistic variables like: *very low, low, medium, high, very high* (or *very poor, poor, average, good, very good*), are used to describe the performance of every SRGM related to every criterion.

For every linguistic variable there are defined a membership and a non-membership function. As defined above, if Ω is the universe of discourse, an intuitionistic fuzzy set X in Ω is given by X = $\{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A(.)$: X $\to$ [0, 1] gives the degree of membership, and $\nu_A(.)$: X $\to$ [0, 1] gives the degree of non-membership of x to A. The expression $1 - \mu_A(x) - \nu_A(x)$, denoted by $\tau_A(x)$, is called the *hesitancy degree*.

Let us consider, in the following, as universe of discussion the set of SRGMs denoted by $\{A_1, A_2, \ldots, A_m\}$. The matrix of linguistic performance is obtained and should be used to choose the "awarded" SRGM. In this manner a three dimensional array of linguistic values is obtained: ACE = $\{(\mu_{ijk}, \nu_{ijk}); i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, p\}$ describing both the degree of acceptance and the degree on non-acceptance (rejection) by the expert E_k, of the SRGM indexed by *i*, related to the criterion C_j. Every expert E_k associates a weight (a positive real number) for every criterion C_j according to his/her belief for the importance of the criterion: a matrix W = $(w_{ik}; i = 1, 2, \ldots, p; k = 1, 2, \ldots, n)$ is built. When p = 1 it is obtained the single expert model (useful when one expert has to select one SRGM), and if p > 1 the multi-expert model is obtained. The weighted average of the linguistic performance value is calculated for each SRGM indexed by *i* as follows:

$$\lambda_{ik} = \sum_{j=1}^{n} \mu_{ijk}\theta_{kj},\ \eta_{ik} = \sum_{j=1}^{n} \nu_{ijk}\theta_{kj},\ \varepsilon_{ik} = 1 - \lambda_{ik} - \eta_{ik},$$

standing for a weighted average membership, non-membership, and hesitation to be considered by the expert E_k, where

$$\theta_{kj} = \frac{w_{kj}}{\sum_{j=1}^{n} w_{kj}}.$$

The single expert model will select the SRGM having the largest weighted average membership

degree (*optimistic* scenarios), the smallest weighted average non-membership degree (*pessimistic* scenarios), or the smallest weighted average hesitation degree (*prudent* scenarios).
When there are available many experts then if there is a model A_{i_0} having the weighted average performance acceptable (with the largest/smallest/ smallest membership/ non-membership/ hesitation degree) by all experts then A_{i_0} will be selected as the winner. Otherwise, a consensual ranking is required. As a natural fact, experts are resistant to option changing, and the model should consider both membership and non-membership degrees:

$$f_k(x, y; \lambda_{ik}, \eta_{ik}) = 1 - \frac{1}{e^{(x-\lambda_{ik})^2 + (y-\mu_{ik})^2}},$$

where (x, y) describes the weighted average linguistic performance when consider all experts and SRGMs. The target is to identify those cases with small resistance to option changing.
Distance–based similarity study is another approach. Different distances have been defined in literature as described in [1]. These are based on geometrical representation of intuitionistic fuzzy sets (2D, 3D, spherical). For the case study discussed in this paper, the normalized Euclidian distance was used, namely, if A and B are intuitionistic fuzzy sets in X, the normalized Euclidean distance d(A, B) is given by:

$$d(A,B) = \sqrt{\frac{1}{2m}\sum_{i=1}^{m}\left[\begin{array}{l}\alpha^2(\mu_A(o_i) - \mu_B(o_i))^2 + \\ \beta^2(\nu_A(o_i) - \nu_B(o_i))^2 + \\ \gamma^2(\tau_A(o_i) - \tau_B(o_i))^2\end{array}\right]},$$

where α, β, and γ give the importance of the membership, non-membership, and hesitation function during analysis process. The case $\alpha = \beta = \gamma = 1$ was considered, but variations can be used for simulation reason.

Table 1. Intuitionistic-fuzzy values

	E_1	E_2	…	E_k	…	E_p
A_1	(λ_{11}, μ_{11})	(λ_{12}, μ_{12})	…	(λ_{1k}, μ_{1k})	…	(λ_{1p}, μ_{1p})
A_2	(λ_{21}, μ_{21})	(λ_{22}, μ_{22})	…	(λ_{2k}, μ_{2k})	…	(λ_{2p}, μ_{2p})
⋮	⋮	⋮	⋱	⋮	⋱	⋮
A_i	(λ_{i1}, μ_{i1})	(λ_{i2}, μ_{i2})	…	(λ_{ik}, μ_{ik})	…	(λ_{ip}, μ_{ip})
⋮	⋮	⋮	⋱	⋮	⋱	⋮
A_m	(λ_{m1}, μ_{m1})	(λ_{m2}, μ_{m2})	…	(λ_{mk}, μ_{mk})	…	(λ_{mp}, μ_{mp})

Applying the aggregation method already presented, the *Table 1* is obtained.
The following matrix of distances between models considering preferences of all experts is obtained and used for similarity analysis [1]: $D_A = (d_{ij})_{1 \le i,j \le m}$, where

$$d_{ij} = \sqrt{\frac{1}{2p}\sum_{k=1}^{p}\left[(\lambda_{ik} - \lambda_{jk})^2 + (\mu_{ik} - \mu_{jk})^2 + (\varepsilon_{ik} - \varepsilon_{jk})^2\right]}.$$

Similarly, a matrix D_E containing the distance computed between experts when considering models as their attributes can be generated and analyzed.

5. Experimental studies

Various SRGMs exist to estimate the expected number of total defects of the expected number of remaining defects in software. A systematic strategy in failure-data analysis using SRGMs has to determine the best models as testing phase progresses. Also it is necessary to establish the best moment for ending the test phase [18], [19], [27].
In order to apply the intuitionistic-fuzzy approach described above, the set of models should be organized in subsets based on similarities. For instance, the Goel-Okumoto (G-O) model, the delayed S-shaped (S) model, the Gompertz (G) model, and the Yamada exponential (Y) model could belong to the same subset because all of them assume that testing takes into account an operational profile. Static defect estimation models like capture-recapture models, curve-fitting methods and experience-based methods will belong to a different subset. A full classification scheme [23] identifies five attributes to be considered: time domain (calendar, execution), category (finite failures, infinite failures), type (Poisson, Binomial, other types), class (exponential, Weibull, Pareto, Geometric, Inverse linear, Inverse polynomial, Power), and family (distribution dependent type). It is important to apply the ranking approach to models having similar assumptions or conditions.
Taking into consideration the existence of a large variety of software differing by size, structure, function, operational environment, development life-cycles, Musa [23] considers like evaluation criteria: projective validity, quality of assumptions, applicability and simplicity. The practice proved that the best model can be dependent on system/release and is not a good idea to use it for new releases and other systems without a new investigation.
During our investigation the first subset was considered for analysing failure data collected during software testing. Inspired by [27], the criteria used for model evaluation can be: the *goodness-of-fit* level (not convergence, very low, low, medium, good,

very good), the *prediction stability* (very unstable, unstable, stable, very stable), and *predictive ability* (low ability, average ability, high ability, very high ability). The predictive ability can have similar meaning as projective validity of Musa [23].

There are many statistical tests useful to evaluate the goodness of fit (how the model fits the collected data under analyse): χ^2, Kolmogorov-Smirnov, Cramer-von Mises, Anderson-Darling, and R^2. During experiments, the R^2 measure was selected for usage.

A prediction is α-stable if the prediction *k* is within α% of the prediction *k-1*. The ranking procedure can be applied daily, weekly, or monthly. For simplicity, we apply the methodology every week during the testing period. The prediction is *very stable* for α = 10, *stable* for α = 20, *unstable* for α = 40, and *very unstable* for α = 60. However, this threshold is subjective and can be defined different from project to project.

The predictive ability is measured in terms of error (the difference between estimate and actual data) and relative error (the ratio error/actual). The linguistic variable approach is used in the context of relative error usage. The model proves β-ability if the absolute value of relative error is less then β%. *Very high* ability is obtained for β = 1, *high* ability is considered when β = 5, *average* ability is compatible with β = 10 and *low* ability is considered when β>10. Also the choice of the threshold β is a subjective task.

The applicability of the distance-based approach is easy to be understood and is not described.

For the ranking methodology, the experiment was conducted using failure data collected during the development of some Java-based software for time series analysis. The development period was one year. Data from testing phase where collected for 6 month (24 weeks). A fault was immediately removed and appropriate updates in software were made accordingly, if required. The SRGMs where applied, without rejection, starting with the fourth week after software stabilization. We present the evaluation results obtained in the week 20 (the number of failures found was 19):

Table 2. Subjective evaluation

Model	Failure estimate	Goodness of fit	Prediction stability	Predictive ability
G-O	12	good	very stable	average
S	18	very good	stable	very high
G	17	good	stable	high
Y	25	medium	stable	high

We found that Yamada exponentional model overestimates the number of failures, and Goel-Okumoto underestimates this number. The delayed S-shaped model and the Gompertz model give similar predictive results, the difference consisting in different levels of subjective evaluation criteria.

When define the linguistic variables in the intuitionistic-fuzzy environment, the last three columns are presented in the format of *Table 1*. The group of experts has the possibility of establish priorities over goodness-of-fit, prediction stability, and the predictive ability. After the aggregation procedure only one column/expert is obtained, and the consensus strategy will be applied if experts gave different rankings. Three evaluators with experience in software reliability growth modelling participate during our experiment. Being a small software project, the multi-expert ranking model was simple to be applied for our experiment.

6. Conclusion

Software reliability engineering helps the software management team to obtain reliable software. This paper reviews various models useful for decision making under incomplete information (imprecise probabilities, generalized intervals, fuzzy sets, vague sets, and intuitionistic-fuzzy sets) and describes the usage of triangular intuitionistic-fuzzy numbers in system reliability computing. Some methodologies for intuitionistic-fuzzy ranking of the software reliability growth models are proposed. The methodologies consider single and multi-expert models, consensus establishing and distance-based classification.

Experimental results on some Java-based software for time series analysis show the applicability of the proposed approaches.

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